A time-delay approach for the modeling and control of plasma instabilities in thermonuclear fusion

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Abstract

This letter presents a summary of [1], where we investigated the stability problems and control issues that occur in a *reversed-field pinch* (RFP) device, EXTRAP-T2R, used for research in fusion plasma physics and general plasma (ionized gas) dynamics. The plant exhibits, among other things, magnetohydrodynamic *instabilities* known as *resistive-wall modes* (RWMs), growing on a time-scale set by a surrounding non-perfectly conducting shell. We propose a new modeling approach that takes into account experimental constraints, such as the actuators dynamics and control latencies leading to a *multivariable time-delay model* of the system. The open-loop field-error characteristics are estimated and a stability analysis of the resulting closed-loop *delay differential equation* (DDE) emphasizes the importance of the delay effects. We then design an optimal PID controller (its structure being constrained by the *intelligent-shell* (IS) control architecture) that achieves a *direct eigenvalue optimization* of the corresponding DDE. The presented results are substantially based on and compared with experimental data.

I. Fusion, magnetohydrodynamics and EXTRAP-T2R facility

The control of magnetohydrodynamic (MHD) *instabilities* in toroidal devices for magnetic confinement is a crucial issue for thermonuclear fusion plasmas. Indeed, advanced plasma confinement scenarios, such as the ones considered for ITER experiment (a major step towards industrial fusion reactors) [2], motivate a better understanding of MHD phenomena and their regulation. The Reversed-Field Pinch (RFP) device EXTRAP-T2R (T2R), which we are considering in this work, is particularly well suited for MHD studies in general (one of the main focuses of this experiment) and more specifically for the active control of the MHD modes. Continuous research efforts have been done in this direction based on various physical approaches. We are now addressing the problem from a control-oriented point of view, highlighting the impact of the main dynamics and the key issues for the closed-loop stabilization.

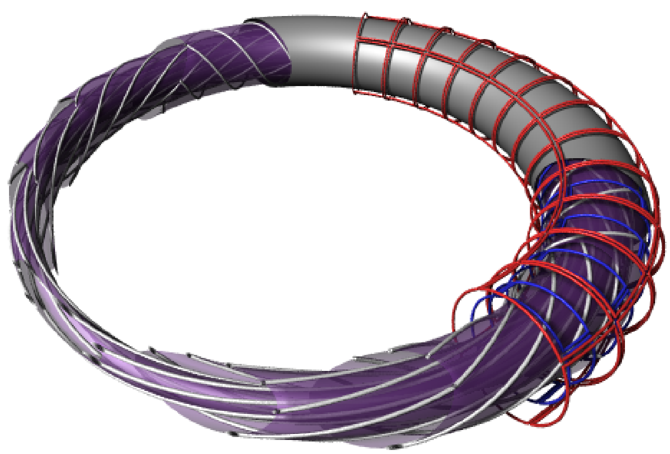
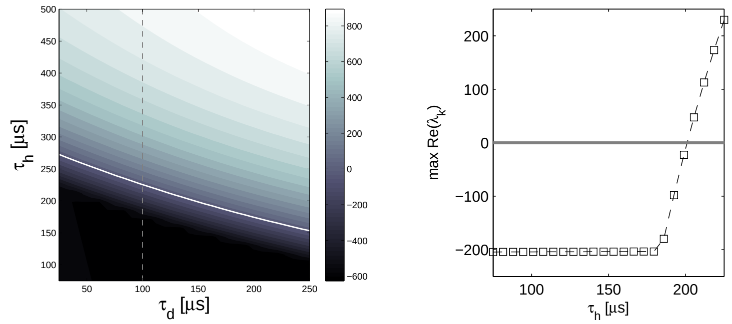


Fig. 1. Cartoon of RFP magnetic equilibrium structure, vacuum vessel, sensor (blue) and actuator (red) saddle coils.

The RFP device T2R, presented in Fig. 1, is a torus equipped with an equidistributed array of equally shaped 4×32 actuator saddle coils fully covering the surface outside the resistive wall (plasma shell), and a corresponding set of 4×32 sensor saddle coils inside the wall (with 50% coverage). The coils inputs and outputs are subtracted pairwise in a top-down and inboard-outboard fashion, effectively implying 64 control and 64 measurement signals. To illustrate the significance of Intelligent-Shell (IS) feedback MHD-stabilization for T2R, note that the plasma is confined during $\sim 10-12$ *ms* only *without* IS whereas a sustained plasma current is routinely achieved for over 70 *ms with* IS (limited by the experiment's power supply).

Controlling the MHD instabilities (described using a Fourier mode decomposition) requires to consider the aliasing of spatial harmonics induced by to the *periodic* arrays of actuators and sensors. It also requires to take into account the actuation dynamics and control latencies, which have a strong impact due to the fastness of the MHD dynamics. The aim of this paper is then to introduce and analyze a new model for describing the dynamics of T2R, by explicitly taking into account the sensors/actuators configuration (aliasing and additional dynamics) and the control implementation (mainly, time-delays). We present a new description of the plant from a control-oriented perspective that includes the peripheral dynamics, addresses the



(a) SISO-set stability in (τ_h, τ_d)-space. (b) MIMO-plant stability w.r.t. τ_h.

Fig. 2. Aspects of RWM stability.

issue of control latency, and employs a fixed-structure gain synthesis approach (presently instantiated for a classic PID) for T2R IS. The controller gains are directly optimized for the closed-loop Delay Differential Equation (DDE) model we develop. Experimental results illustrate the obtained performance improvements.

II. Physical model, time-delay approaches and closed-loop stability

Many physical models for MHD analysis are based on the single-fluid MHD model [3] that combines Ohm's law with Navier-Stokes and Maxwell's equations. In the so-called *ideal* case, the conductivity is supposed to be perfect. While this model is relevant for many physical phenomena, it does not take into account some critical phenomena such as transport processes [4]. However, for control applications, simplified linearized ideal-MHD models are the natural starting point; the conventional tool for investigating macroscopic stability of MHD-equilibria. Under the classical assumption of large aspect-ratio tokamak with circular cross-section, the modes write as $e^{i(m\theta - n\phi)}$, where m and n are the poloidal and toroidal mode numbers. The stability property of the plasma is then related to the modes dynamics, coupled with the steady-state physical variables (such as the safety factor or magnetic field), and the modes describe the Fourier harmonics of MHD perturbations.

In RFP experiments such as T2R, the time variation of the radial field perturbations is typically used to investigate the MHD behavior and the error field amplification is obtained according to Pustovitov model [5] (i.e. for $m = 1$)

$$\frac{\partial b_n}{\partial t} \quad = \quad \gamma_n b_n + \gamma_{n,w}\,,\, b_{n,w} \tag{1}$$

where $b_n(t)$ is the perturbed field, $b_{n,w}(t)$ is the field due to external sources, γ_n is the natural growth rate of the plasma mode and $\gamma_{n,w}$ is the diffusion rate of the harmonic at the thin wall in the absence of plasma. Another model of main interest is used in plasma control experiments such as [6], where the dynamics is described by the Ordinary Differential Equation (ODE) of the form:

$$\frac{db_{m,n}}{dt} = \gamma_{m,n} b_{m,n} + \frac{M_{m,n}}{\tau_{m,n}} I_{m,n} \tag{2}$$

where $\tau_{m,n}$ is the diffusion time, $M_{m,n}$ is a proportionality factor and $I_{m,n}(t)$ is the Fourier harmonics of the active coil currents (control input).

In [1], we extended the previous model by taking into account the experimental implementation in the system description. It was noted that the actuators dynamics and controller latency τ_h play an important role in the closed-loop response. Introducing the PI + time-delay control (time-delay implementation of the derivative term, with a controller cycle τ_d that can be considered as a tuning parameter) available architecture, the closed-loop dynamics was obtained with the following state-space description:

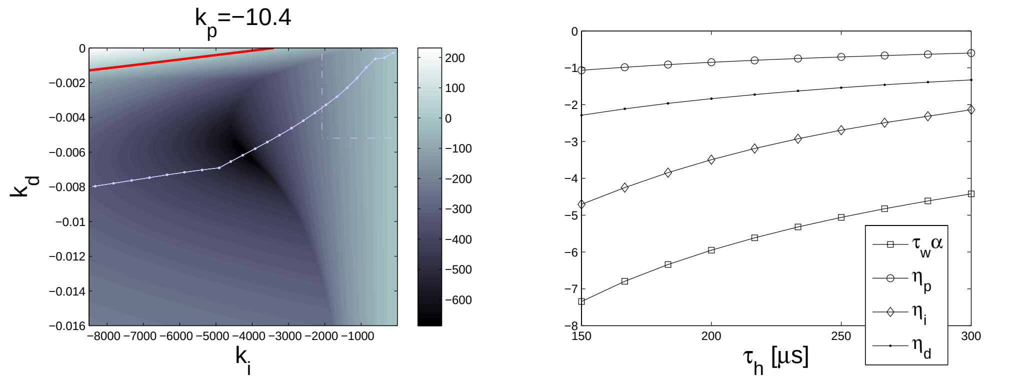
$$\dot{x}(t) \quad = \quad \mathcal{A}_0 x(t) + \mathcal{A}_1(\theta) x(t - \tau_h) + \mathcal{A}_2(\theta) x(t - \tau_h - \tau_d) + \mathcal{E} v_1(t)\,, \tag{3}$$

where $\mathcal{A}_0$, $\mathcal{A}_1$, $\mathcal{A}_2$, $\mathcal{E}$ are the state-space matrices and $v_1(t)$ is an external perturbation. Note that the control parameters $\theta = (K_p, K_i, K_d)$ enter affinely in $\mathcal{A}_1$, $\mathcal{A}_2$.

The (asymptotic) stability of the DDE-class (3) and the model sensitivity with respect to time-delays are established by using the corresponding characteristic equation (for $n = 2$)

$$\det \Delta(s) = \det\left(sI - \mathcal{A}_0 - \sum_{i=1}^{n} \mathcal{A}_i e^{-s\tau_i} \right) = 0. \tag{4}$$

It is well-known that (4) has an *infinite* (but countable) number of roots $s = \lambda_j$ and that (3) has a point spectrum. Furthermore, since the set $\left\{\lambda_j : \det \Delta(\lambda_j) = 0,\ \mathrm{Re}(\lambda_j) > a\right\}$ with a real is *finite* (see, e.g., [7] and the references therein), it follows that the



(a) Maximum value of the closed-loop spectrum for gain design *a)* in the (k_i, k_d)-space.

(b) Normalized PID gains for design *b)* (parameterized by τ_h).

Fig. 3. Closed-loop spectrum optimization.

stability problem is reduced to analyze the location of the *rightmost* characteristic roots with respect to the imaginary axis. This is illustrated in Fig. 2 where, for a given set of control parameters θ, the impact of the time-delays is shown both on a single channel and for the MIMO plant. The SISO-set stability property shows that the maximum admissible controller latency is inversely proportional to the derivative delay (stability limit represented by the white line on Fig. 2(a)). The MIMO analysis also provides for the maximum admissible value of τ_h. For further comments on the spectrum properties for DDEs, see, for instance, [7].

III. MODEL-BASED CONTROL AND DELAY COMPENSATION

Real-time control experiments suggest that the closed-loop performances and stability properties are affected by delays in the control inputs (i.e. due to the digital controller). Another source of mismatch between the model and experimental measurement may be due to the ODE representation (2) of the system. For example, it was shown that resistive equations similar to (1) can be related to variation laws with a delayed kernel [8].

Our aim is to select the controller gains for the DDE (3) that ensure MHD stability and minimize the closed-loop spectral abscissa under PID structure constraints. The PID structure in the actual experiment control system (IS) is regarded as fixed, imposing a structural constraint in the optimization problem. We then consider a fixed-order/fixed-structure controller synthesis approach, employed to select PID gains for IS operation of T2R. The method, as instantiated in this work, concerns model (3), i.e. it handles constant time-delays explicitly, which has at least two practical benefits:

1) developing control algorithms with varying computational complexity (varying τ_{CPU}) implies varying τ_h, which can easily be taken into account in the proposed approach;
2) a free τ_h can act as a fitting parameter to mimic experimentally observed instability onset.

As detailed in [7], the asymptotic damping maximization of (3) is formulated as minimizing the spectral abscissa of the characteristic equation with

$$\theta^* = \arg\min_{\theta} \max_{\lambda} \{\mathrm{Re}(\lambda) : \det \Delta(\lambda, \theta) = 0\}$$

This problem is generally both nonconvex and nonsmooth, which motivates a hybrid SISO/MIMO method, where we minimize the maximum spectral abscissas of the SISO sets over $\mathcal{K}$ with

$$\tilde{\theta}^* = \arg\min_{\theta} \max_{k \in \mathcal{K}} \max_{\lambda} \{\mathrm{Re}(\lambda) : \det \Delta_k(\lambda, \theta) = 0\} \tag{5}$$

where Δ_k denotes the characteristic matrix ($\in \mathbb{R}^{4\times4}$) for (3) for a single mode $k \in \{m, n\}$. We then propose a *suboptimal solution to the minimization problem*.

The problem mentioned above opens a lot of possibilities in deriving PID gains guaranteeing closed-loop stability and appropriate performances. As a first step, we investigate *two different parameterizations* of the controller synthesis, limiting closed-loop performance and control-input norm, respectively by:

a) varying k_p and searching for the optimal (k_i^*, k_d^*) for a nominal τ_h, and
b) varying τ_h and determining the full optimal PID $\tilde{\theta}^* = (k_p^*, k_i^*, k_d^*)$.

Other issues, as for example, *fragility* of the controller and the link with the closed-loop performances will be considered in forthcoming papers.

The performance of *a)* is illustrated in Fig. 3(a), which depicts the rightmost values of the closed-loop spectrum for a fixed k_p in the (k_i, k_d) space (optimum in the darkest region). The bold dotted line corresponds to the evolution of (k_i^*, k_d^*) when

k_p varies (going left when the magnitude of k_p is increased) and the red is the stability boundary. The second gain design strategy *b)* is illustrated in Fig. 3(b) where the evolution of the normalized closed-loop spectrum is given by $\tau_w\bar{\lambda}$, $\bar{\lambda}$ being the rightmost eigenvalue. This evolution shows that optimal performance is decreased when the time-delay increases. Note that the numerical values of the optimal PID gains obtained with both strategies are very close to each other, as expected.

IV. Experimental Results and Performance Improvement

The new control approach presented in the previous sections motivated new series of experiments on T2R: shot numbers #20743 – #20755 and #20824 – #20838. Experimental plasma equilibrium conditions were set with a toroidal plasma current $I_p \approx 85\ kA$, a shot length $\tau_p =\sim 50-70\ ms$ and *reversal* and *pinch* values (typically used to characterize RFP equilibrium $(F,\Theta) \approx (-0.27, 1.72)$.

TABLE I
T2R experimental results.

Shot#	K_p	K_i	K_d	J_y	J_u	Comment
20743	150	16000	0.05	0.464	1.66	old setting 1
20744	160	16000	0.04	0.509	1.80	old setting 2
20746	106	37500	0.061	0.259	2.12	series *a)*
20747	126	47500	0.073	0.304	1.94	*a)*
20827	150	16000	0.05	0.501	1.60	old setting 1
20833	119.6	46800	0.065	0.304	1.77	*b)*
20835	106.8	39860	0.058	0.288	1.64	*b)*

The overall controller performance is summarized with the general quadratic measure:

$$J_\nu(\theta) \equiv \frac{1}{t_1 - t_0}\int_{t_0}^{t_1} \nu^T(\tau,\theta)\nu(\tau,\theta)d\tau, \tag{6}$$

where $\nu \doteq (\mathbf{y}\ or\ \mathbf{u})$, and θ denotes the dependence on the controller setting. The nature of T2R shots suggests a split of the timespan $[t_0, t_1]$ into two parts, corresponding to the *transient* (first 10 *ms*) and *steady-state* behaviours (between 10 and 50 *ms*). The performance improvements are summarized in Table I for the steady-state interval 10 – 45 ms, where the cost function (6) allows to compare the controllers used in the experiments. Performance over the interval $0-10\ ms$ is almost completely determined by the proportional controller gain K_p since the active coils are too weak to compensate for exogenous signals during the transient. Both optimized controllers clearly reveal a significant 44% reduction of average field energy at the sensors during steady-state period. This is at the expense of a higher input power, increased by 28%. The proposed model reproduces key aspects of T2R feedback stabilization, which implies that the "old" PID coefficients are significantly suboptimal in both model (decoupled modes hypothesis) and experiment (with the physical coupling) compared to the "new" PID coefficients.

V. Conclusions

A new model for MHD instabilities in T2R, explicitly including important geometrical and engineering aspects was presented. Direct closed-loop PID gain optimization for the corresponding DDE model was shown to provide useful results for experimental intelligent-shell feedback in a RFP fusion research device. Simulations and experiments for the EXTRAP T2R device have shown qualitative agreement, further indicating the applicability of the model to real experimental conditions. These results strongly encourage future work, theoretically and experimentally, in both physical modeling and multivariable control. In short, the time-delay approach considered here provides some new results in the analysis and control of the plasma MHD instabilities, which are critical for the development of a new sustainable energy with controlled thermonuclear fusion.

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