Stability analysis of delay-parameter space

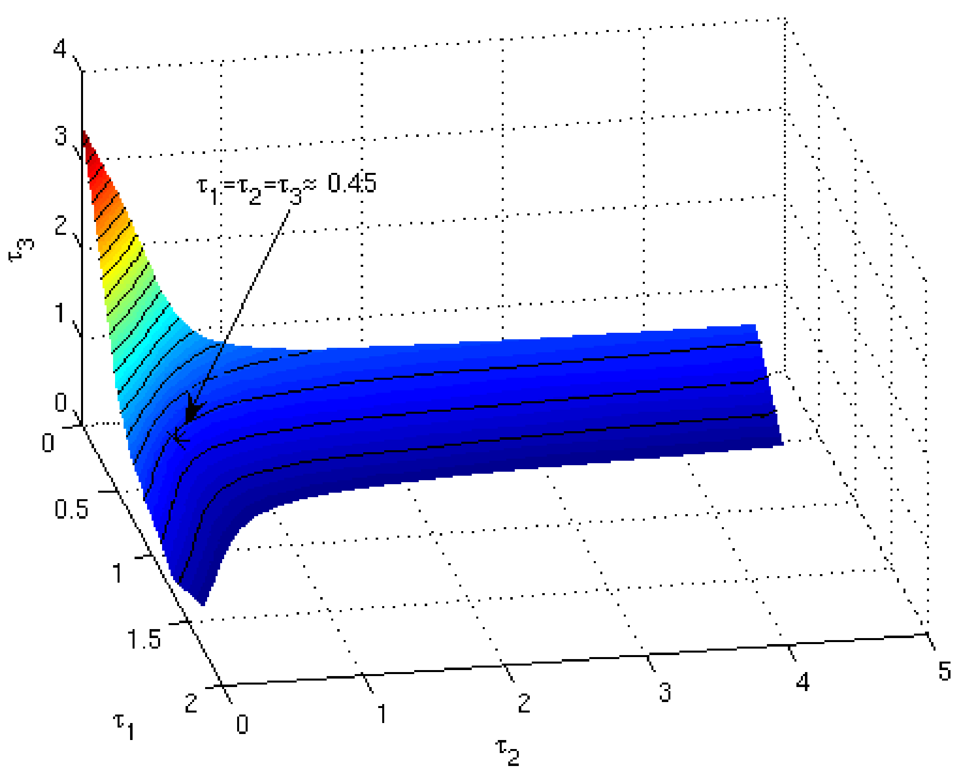
We consider a stability analysis of a multi-dimensional delay-differential equation

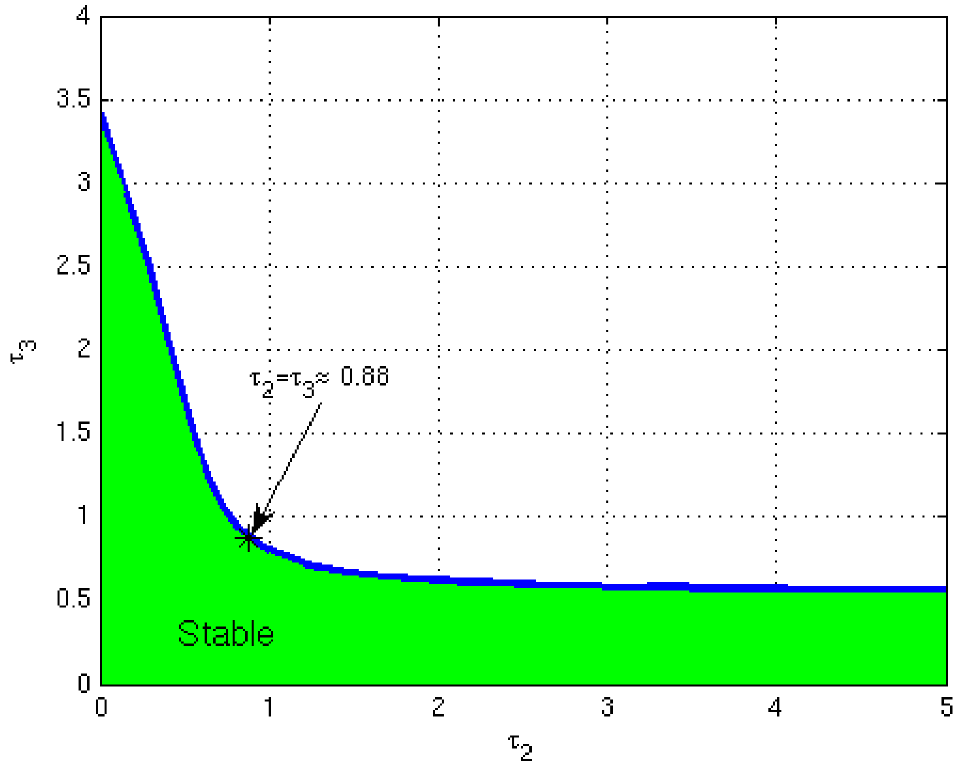
$$\dot{x}(t) = A_0 x(t) + A_1 x(t-\tau_1) + A_2 x(t-\tau_2) + A_3 x(t-\tau_3)$$

where

$$A_0 = \begin{pmatrix} 3 & 0 \\ -1 & 1.5 \end{pmatrix}, \quad A_1 = \begin{pmatrix} -1 & -0.7 \\ 1 & 0.2 \end{pmatrix},$$
$$A_2 = \begin{pmatrix} -1.6 & -0.7 \\ 0.8 & 0 \end{pmatrix}, \quad A_3 = \begin{pmatrix} -0.7 & 0.2 \\ -0.5 & -2 \end{pmatrix}.$$

Even though the stability analysis of time-delay systems with multiple delays like this has received a lot of attention in the time-delay community, we believe that it is not widely known that there are methods to plot the region in delay-parameter space in which the system is stable. We list the following publications as examples where a stability analysis is done using these type of plots, sometimes referred to as stability charts, [Beretta, Geometric stability switch criteria in delay differential systems with delay dependent parameters, SIAM J. Math. Anal. 33:1144-1165 2002][Mahaffy et al, A geometric analysis of stability regions for a linear differential equation with two delays, Int. J. Bif. Chaos Appl. Sci. Eng. 5:779-796 1995][Ahlborn et al, Controlling Dynamical Systems using Multiple Feedback Control, Phys. Rev. E 72, 016206 2005][Niculescu et al, On the Stability Crossing Boundaries of Some Delay Systems Modeling Immune Dynamics in Leukemia, Proc 17th Int. Symp. on Math. Theory of Networks and Systems, 2006]. The delay-space stability chart for this equation is visualized in the following figures.





To our knowledge the following types of modern approaches can be used for the computation of delay-space stability charts:

- *Computation of the stability switching curves and surfaces and points where there is an imaginary eigenvalues*

  [Gu, Niculescu, On stability crossing curves for general systems with two delays, J. Math. Anal. Appl. 311:231-253 (2005)]

  [Louisell, A matrix method for determining the imaginary axis eigenvalues of a delay system, IEEE Trans. Autom. Control 46:2008-2012 (2001)]

  [Sipahi, Olgac, Complete stability robustness of third-order LTI multiple time-delay systems, IEEE Trans. Autom. Conrol 50:1826-1831 (2005)]

  [Jarlebring, Computing the Stability Region in Delay-space of a TDS using Polynomial Eigenproblems, 6th IFAC Workshop on Time-Delay Systems, 2006]

  [Ergenc, et al. Kronecker Summation Method and Multiple Delay Systems, 6th IFAC Workshop on Time-Delay Systems 2006],

- *Computation of the rightmost eigenvalues on a grid of points in the stability chart*

  [Breda et al, Pseudospectral differencing methods for characteristic roots of delay differential equations, SIAM J. Sci. Comp. 27:482-495 2005]

  [Engelborghs, DDE-BIFTOOL A matlab package for bifurcation analysis of delay differential equations, Leuven 2000],

- *Exploitation of the fact that any multiple-delay system can be approximated to arbitrary accuracy with a commensurate system, where exact stability-conditions are known*

  [Chen et al, A new method for computing delay margins of linear delay systems, Syst. Contr. Letters, 26:107-117 1995]

  [Fu, et al, Stability of linear neutral time-delay systems: Exact conditions via matrix pencil solutions, IEEE Trans Autom. Control 51:1063-1069 2006].