Santo Domingo – Julio 27, 28 2012

Identificacion de Sistemas Estructurales y su Uso en la Ingeneria Estructural Moderna

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Detección de Daño

Inspeccionando los parámetros de modelos

Inspeccionando residuos

Comprobación (estadística) de Hipótesis

Hipótesis nula. El sistema esta SANO.

Si no la aceptamos y el sistema esta sano – **ERROR Tipo I**

Si la aceptamos y el sistema tiene daño – **ERROR Tipo II**

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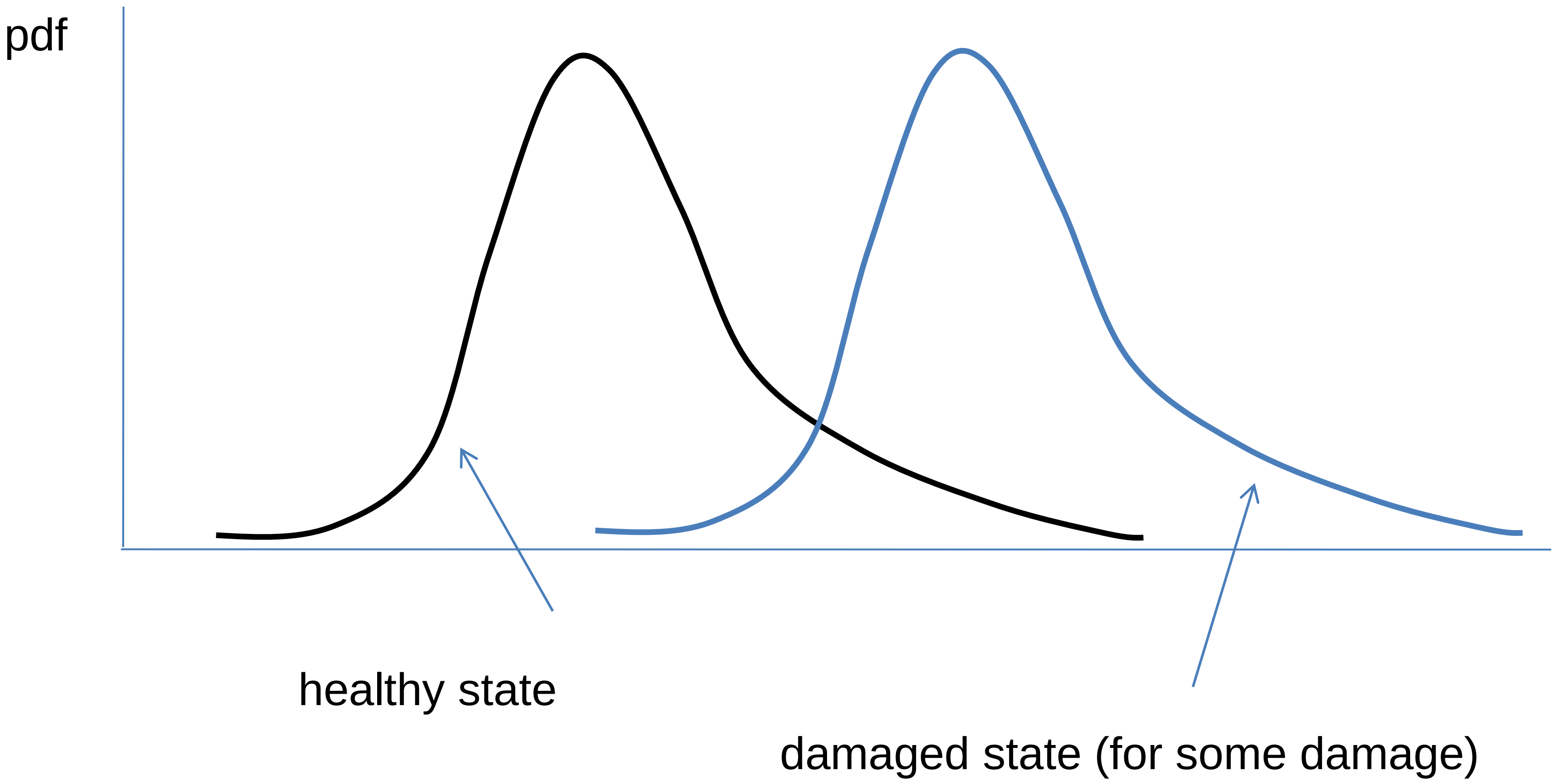


Es necesario que el daño produzca un cambio en la distribución probabilística de la métrica

Nos enfocamos en un método de residuos – lo llamaremos el método de Subespacios.

FORMULANDO EL MODELO

$$R_j \overset{\text{def}}{=} E(y_{k+j} y_k^T) \qquad R_j = \frac{1}{N} \sum_{k=1}^{N} y_{k+j} y_k^T$$

$$H_{p+1,q} = \begin{pmatrix} R_1 & R_2 & \cdots & R_q \\ R_2 & R_3 & \cdots & R_{1+q} \\ \vdots & \vdots & \ddots & \vdots \\ R_{p+1} & \cdots & \cdots & R_{p+q} \end{pmatrix}$$

Matriz de Hankel

$$H_{p+1,q} = \begin{bmatrix} U & S \end{bmatrix} \begin{bmatrix} s & \\ & \cong 0 \end{bmatrix} \begin{bmatrix} V & N \end{bmatrix}^T$$

Por las propiedades de la descomposición de valores singulares

$$S^T U = 0$$

p >= orden entre el # de salidas

q = p+1

$$\kappa_N = \mathrm{vec}(S^T U)$$

Condición Referencia

Nueva Data

El subíndice N enfatiza que las propiedades estadísticas del residuo dependen de la longitud de la señal que se utiliza para calcular las funciones de correlación.

La covarianza del residuo converge.

$$\varsigma_N = \sqrt{N}\kappa_N$$

$$\varsigma_N = \sqrt{N}\kappa_N$$

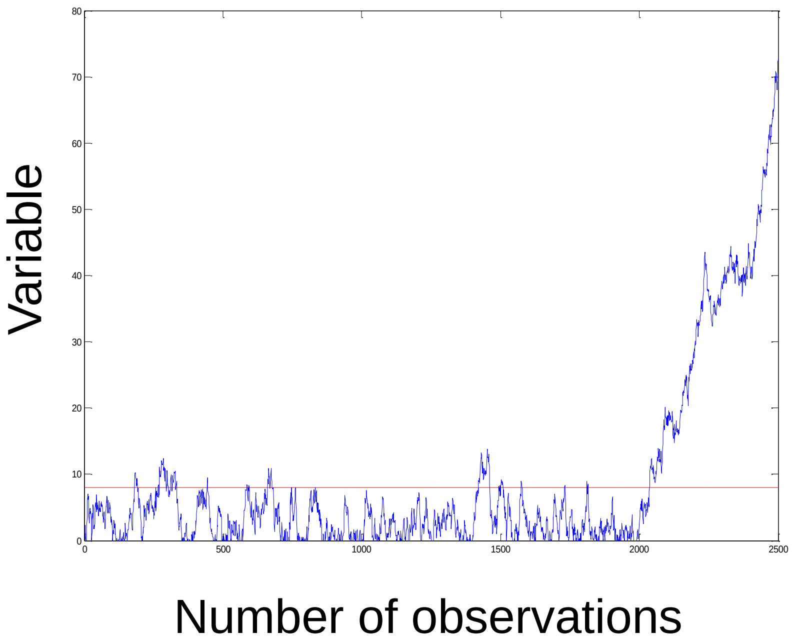
$$\Sigma_\varsigma = \frac{1}{nt}\sum_{j=1}^{nt} \varsigma_N^{(j)} \varsigma_N^{(j)T}$$

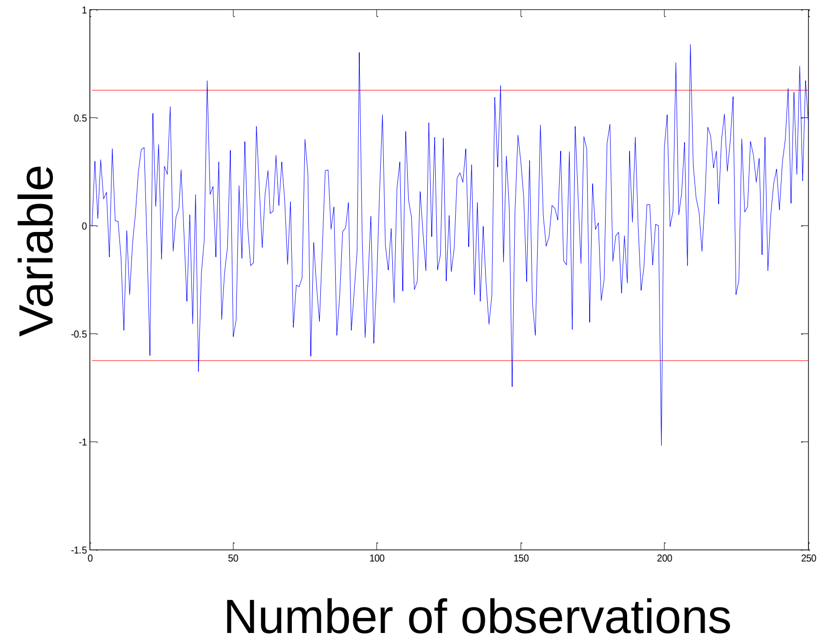
Metrica – chi square distributed

$$\chi_N^2 = \zeta_N^T \Sigma_\zeta^{-1} \zeta_N$$

Cartas de Control

- Es importante sacar ventaja de que el daño es permanente por lo que la información acumulada puede darnos seguridad de un evento que viendo un punto es difícil.





Tipos
(univariate observations)

- $\overline{X}$ and R Shewhart charts

- Exponentially Weighted Moving Average (EWMA)

- Cumulative Sum (CUSUM)

Carta CUSUM

- Contrasta la hipótesis nula (H_0) con la alternativa (H_1), y adiciona información a la llegada de cada X_i

- CUSUM presume una subida a bajada de la media de la distribución, normalmente especificada como porcentaje de la desviación estándar.

- Aumento

$$C_0^+ = 0$$

$$C_i^+ = \max\{0, C_{i-1}^+ + X_i - k\}$$

- Disminución

$$C_0^- = 0$$

$$C_i^+ = \min\{0, C_{i-1}^- + X_i - k\}$$

Diseño de una Carta CUSUM

Parámetros a seleccionar:

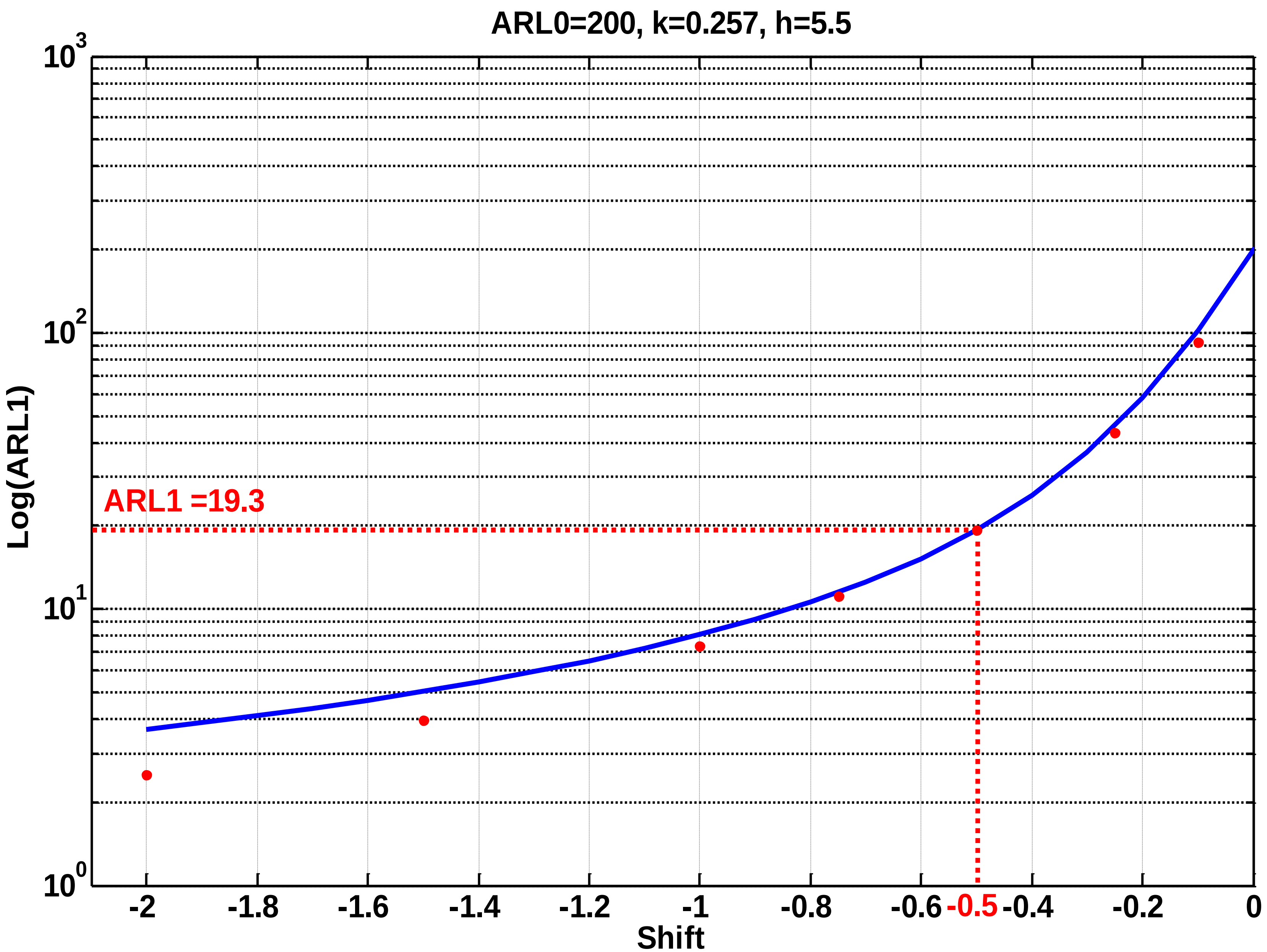
- En control Average Run Length "ARL_0"
- Movimiento de la media donde la prueba es optima

Con estos valores se obtienen

- k
- h

Optimal points for ARL_0=200, 500 and 1000

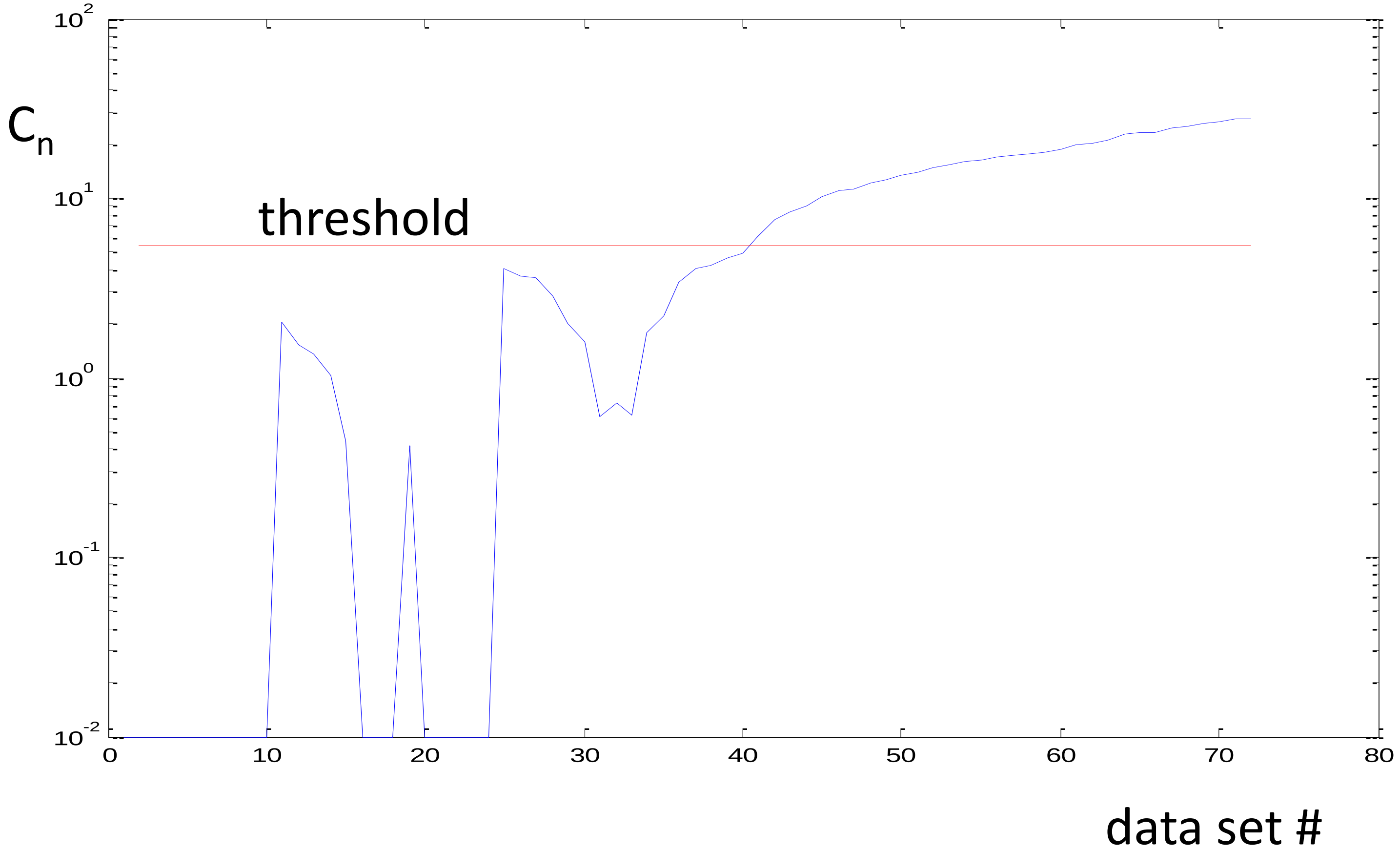
ARL_0= 200							
Shift	0.1	0.25	0.5	0.75	1	1.5	2
h	11	8	5.5	4.5	3.5	2.5	2
k	0.034	0.118	0.257	0.353	0.498	0.738	0.924
ARL_1	92.6	43.3	19.3	11.1	7.3	3.9	2.5
ARL_0= 500							
Shift	0.1	0.25	0.5	0.75	1	1.5	2
h	15.5	10.5	7.5	5.5	4.5	3	2.5
k	0.046	0.13	0.237	0.373	0.483	0.766	0.92
ARL_1	165.5	64.6	25.8	14.2	9.1	4.75	3
ARL_0= 1000							
Shift	0.1	0.25	0.5	0.75	1	1.5	2
h	19.5	12.5	8.5	6.5	5	4	3
k	0.051	0.136	0.253	0.366	0.507	0.654	0.883
ARL_1	241	83.1	31	16.6	10.5	5.4	3.3



ARL0=200, k=0.257, h=5.5
Log(ARL1)
Shift
ARL1 =19.3
-0.5
10^3
10^2
10^1
10^0
-2
-1.8
-1.6
-1.4
-1.2
-1
-0.8
-0.6
-0.4
-0.2
0

Ejemplo

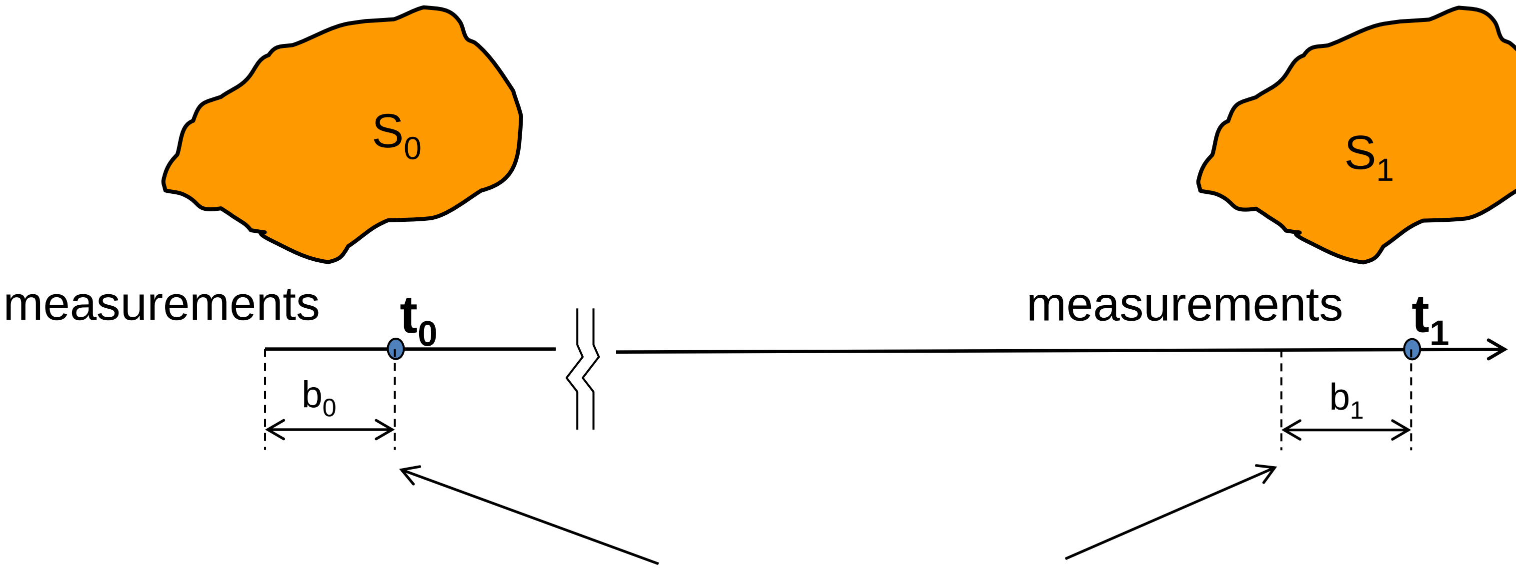
Consider a uniform 12 DOF chain system with unit masses and stiffness such that the first mode frequency is 1Hz. Damping is taken as 2% in every mode. The system is exited with white noise (at all the coordinates) and damage is simulated as 5% reduction in the stiffness of the second spring. The output signals are contaminated with white noise with a SNR of 5%. The model (the matrix S) is formulated using 30 minutes of acceleration data recorded of at 50Hz. The covariance of the residual is computed from a second record of 30 minutes that is divided into 200 sets of 9 secs duration (450 points). The order of the system is 24 but we select it as 10 to illustrate the fact that the approach can operate robustly with a highly truncated model. Once the covariance is computed we compute the chi square metric for data sets that are 30 minutes long. Damage is introduced in data set #31 (and subsequent ones). We select an $ARL_0 = 200$ and optimal behaviour at a shift in the mean of 0.5 standard deviations and thus, from Table 1 find that $k = 0.257$ and $h = 5.5$ with an expected $ARL_1 = 19.3$.



C_n
threshold
10^2
10^1
10^0
10^-1
10^-2
0
10
20
30
40
50
60
70
80
data set #

Localización de Daño

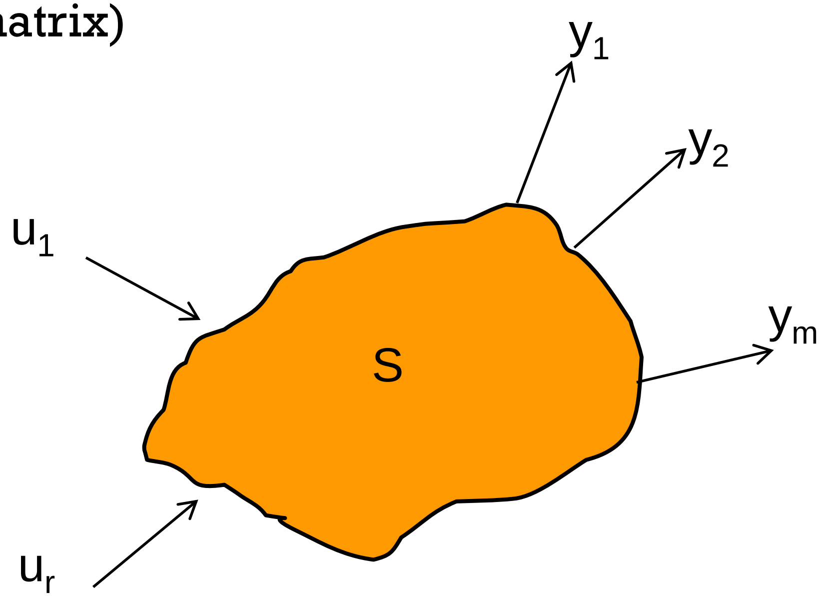
Operating Conditions



some ROM (S)

$$S_0 \neq S_1$$

Selected Characterization (S) (transfer matrix)



$$y(s) = G(s)u(s)$$

$\Delta G(s)$ from data.

Interrogation regarding damage location

Transfer Matrix Synthesis

$$\dot{x} = A_c x + B_c u$$

$$y = C_c x$$

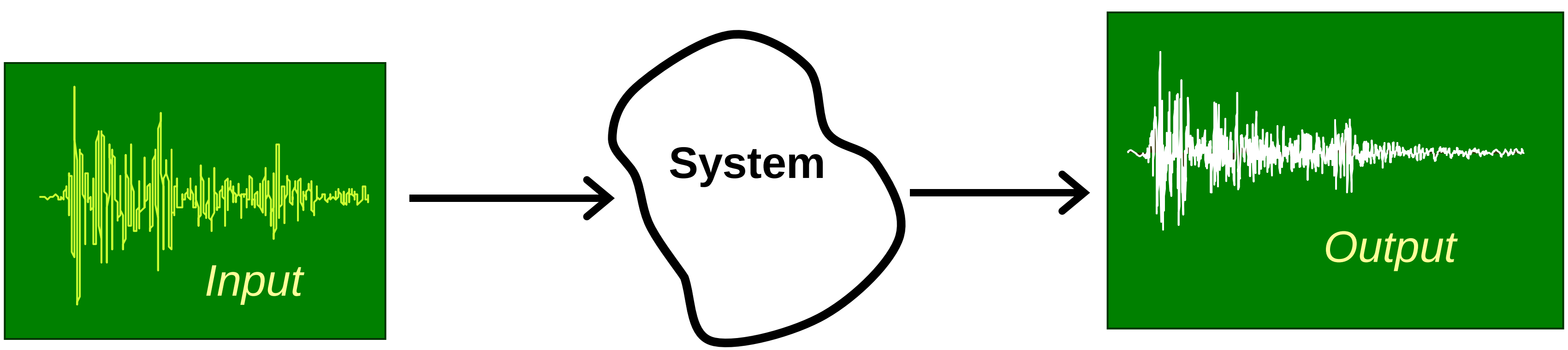
Identification → $\{A_c \; B_c \; C_c\}$

$$x_{(s)} = \left[I \cdot s - A_c\right]^{-1} B_c u_{(s)}$$

$$y_{(s)} = C_c \left[I \cdot s - A_c\right]^{-1} B_c u_{(s)}$$

→

$$G_{(s)} = C_c \left[I \cdot s - A_c\right]^{-1} B_c$$



Interrogation at the Origin

$$G_{(s)} = C_c \left[I \cdot s - A_c \right]^{-1} B_c$$

s=0

$$G_{(s=0)} = F = -C_c A_c^{-1} B_c$$

Static Flexibility.

F_u = static flexibility in reference state

F_d = static flexibility in damaged state

} **from data**

$$DF = F_d - F_u$$

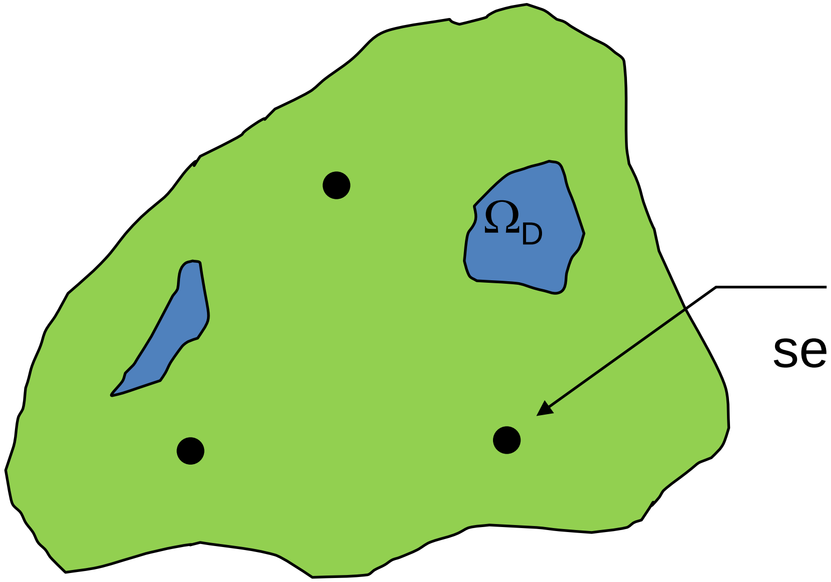
Let DF be rank deficient and L be the Kernel

$$DF \cdot L = 0 \quad \Longrightarrow \quad F_d \cdot L = F_u L$$

So?

it can be proven that …

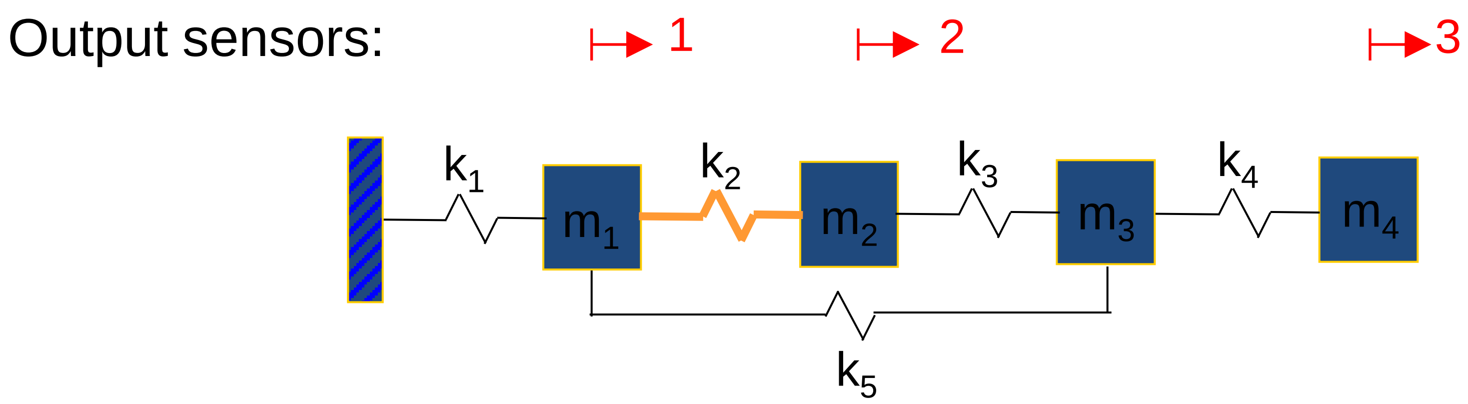
The stress fields for loads in the span of L are zero over a closed region Ω_D (not necessarily simply connected) that contains the damage



L = null space of DF

sensor location = fictive load point

DLV Approach (a simple illustration)



	k_1	k_2	k_3	k_4	k_5
Before Damage	1	1	1	1	1
After Damage	1	**0.5**	1	1	1

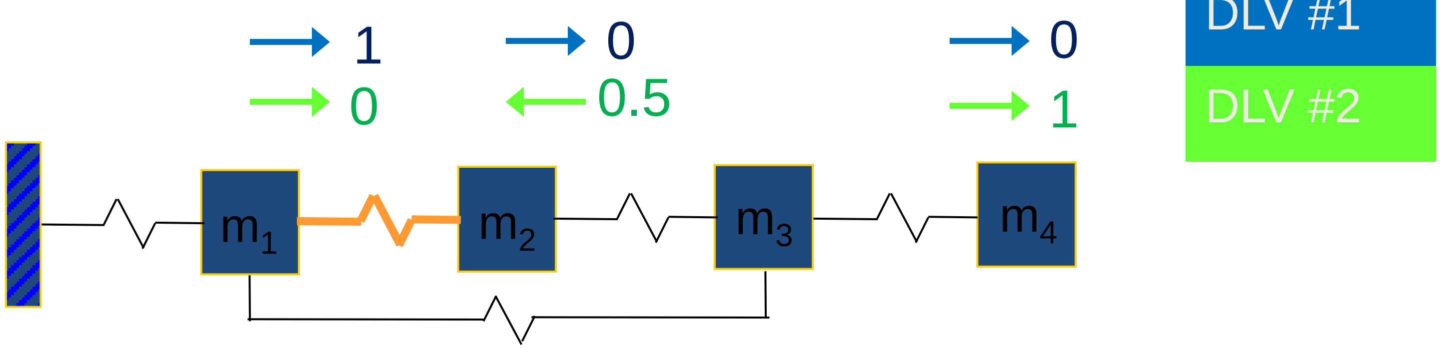
- F_U and F_D (in a real case synthesized from data)

$$F_U = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 5/3 & 4/3 \\ 1 & 4/3 & 8/3 \end{bmatrix} \qquad F_D = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1.5 \\ 1 & 1.5 & 2.75 \end{bmatrix}$$

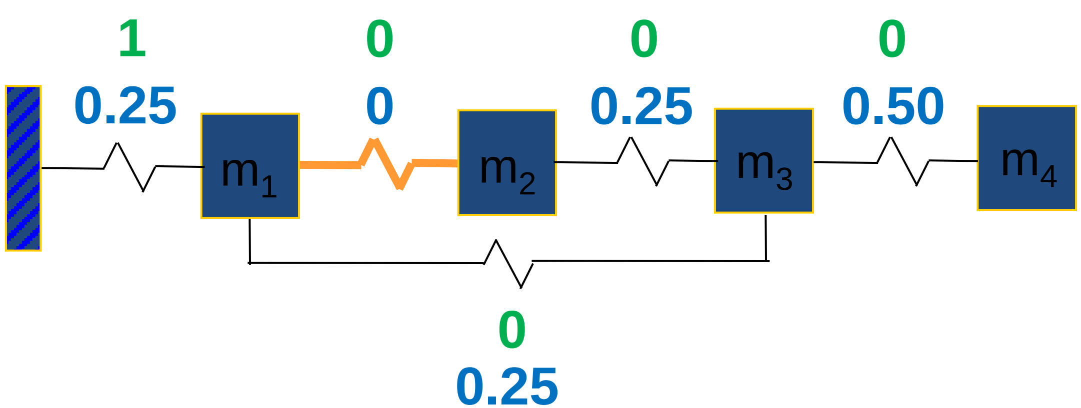
(cont.)

- $$DF = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1/3 & 1/6 \\ 0 & 1/6 & 1/12 \end{bmatrix} \Rightarrow SV(DF) = \begin{Bmatrix} 0.42 \\ 0 \\ 0 \end{Bmatrix} \quad N(DF) = \begin{bmatrix} 1 & 0 \\ 0 & -0.5 \\ 0 & 1 \end{bmatrix}$$

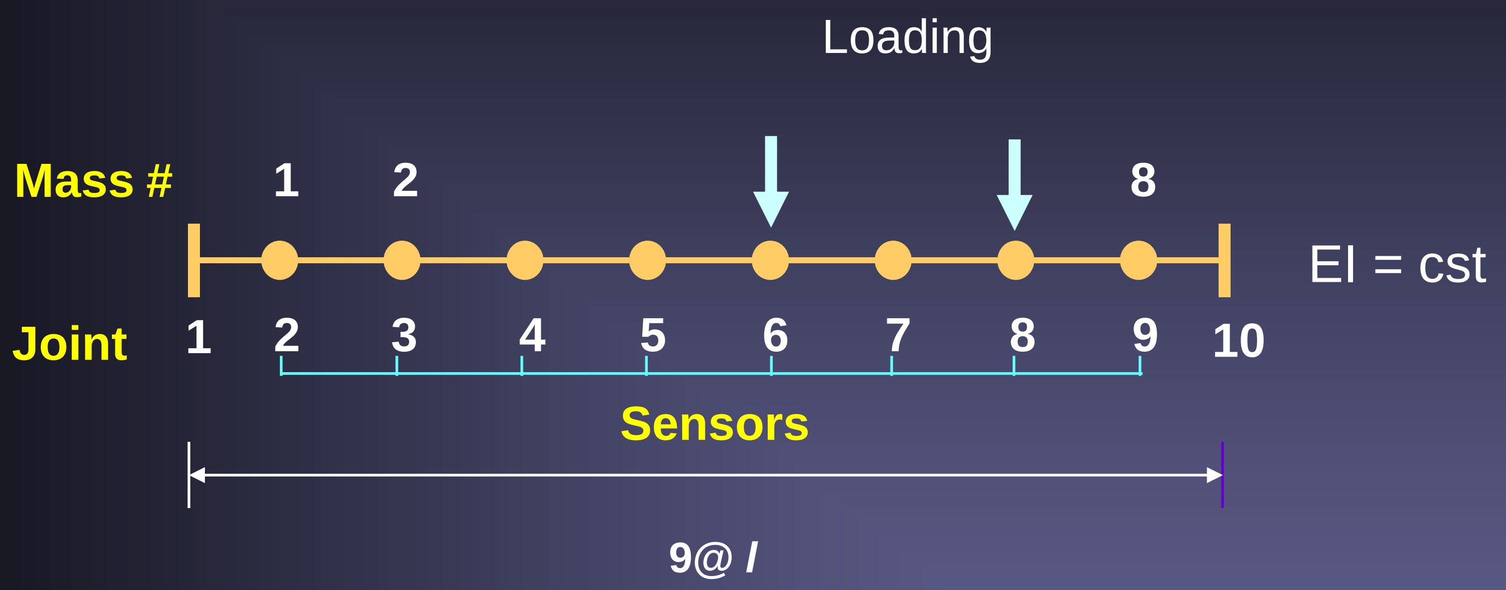
- Null Space as Loads:



- Stresses :



Example



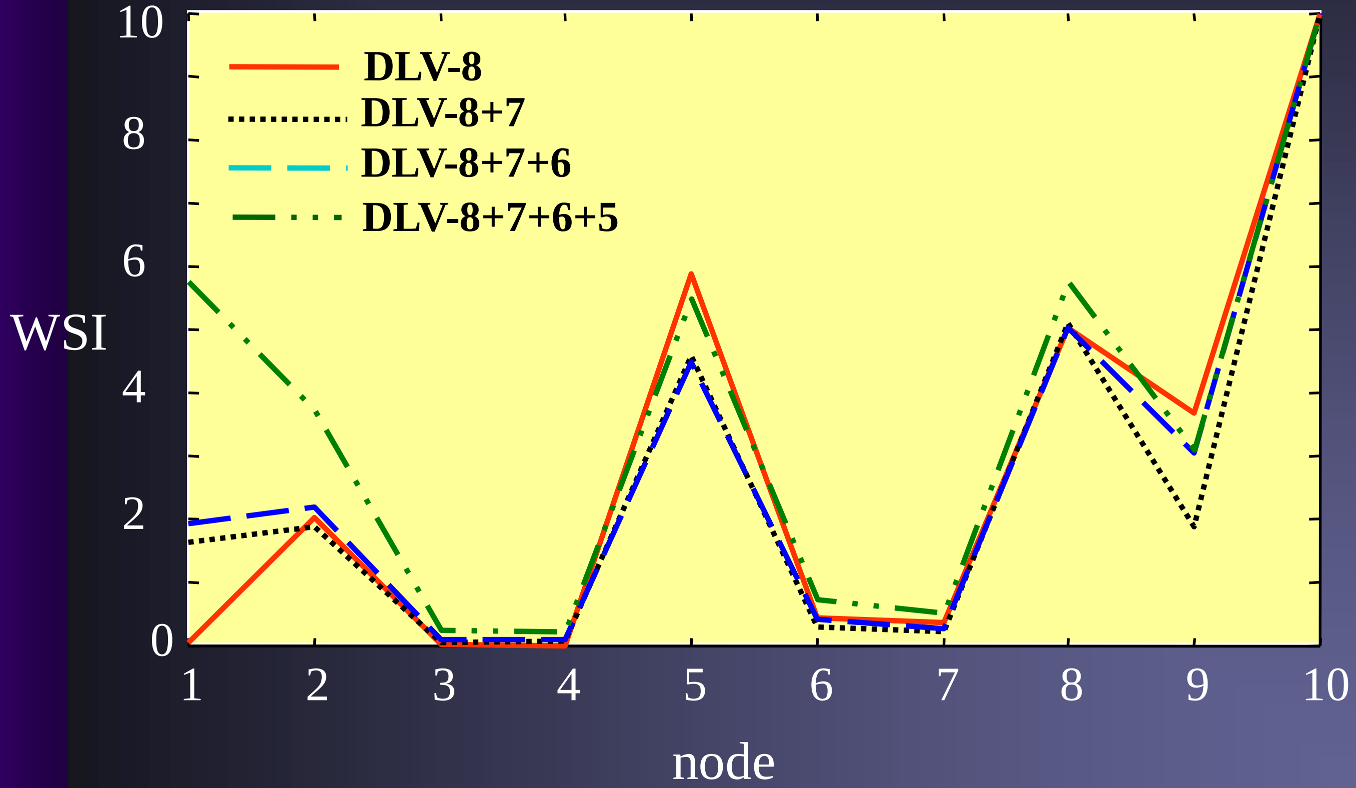
Input Noise RMS 5%

Output Noise RMS 10% (sensor #1)

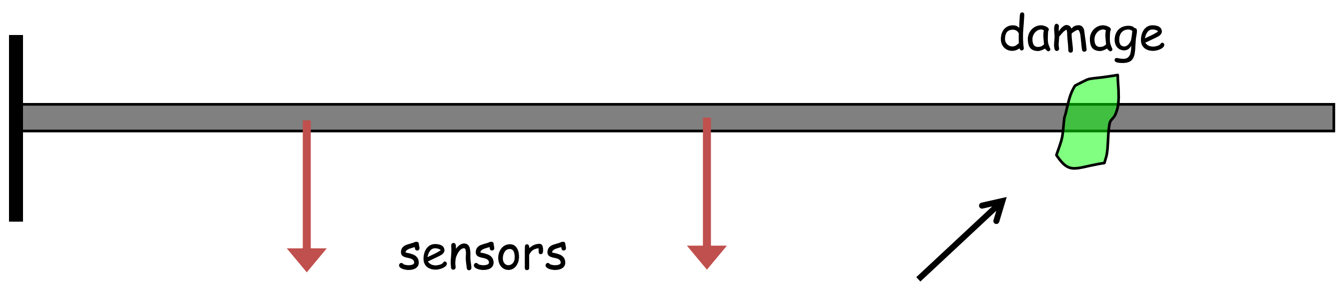
Damage: Segment 3-4 25% reduction in EI

Segment 6-7 50% reduction in EI

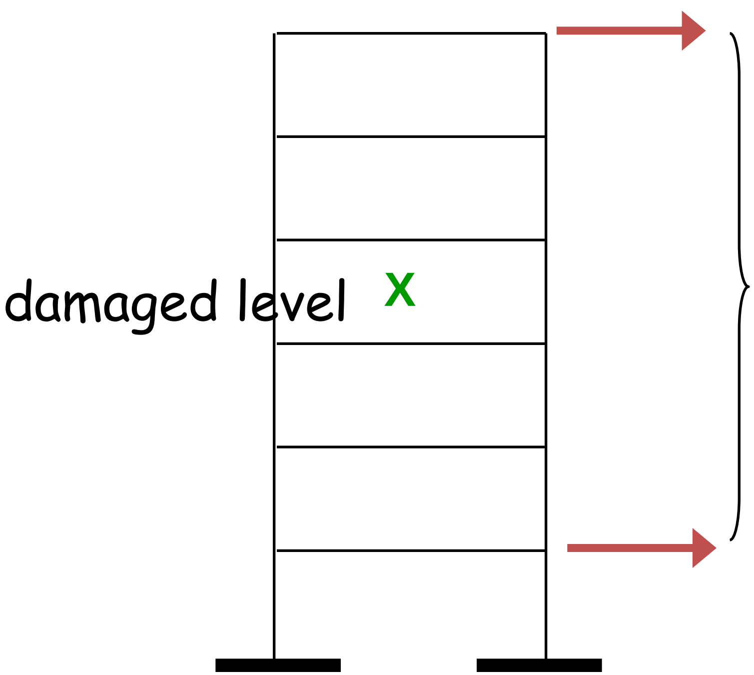
Weighted Stress Index



Resolution Limits at s = 0

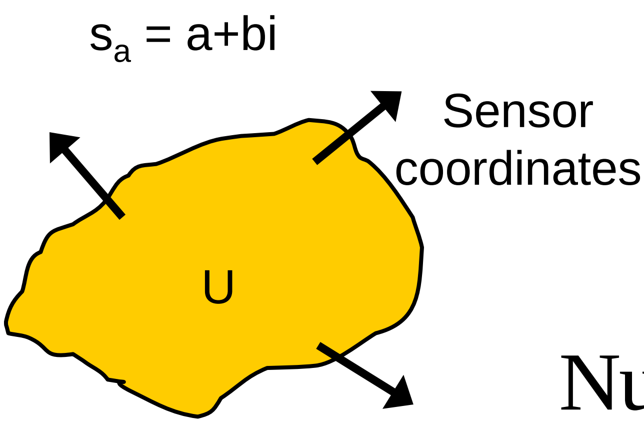


Unobservable from changes in static flexibility



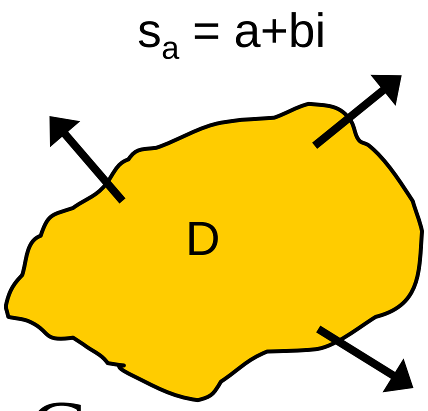
changes in static flexibility can't resolve beyond stating something happened in this region

Arbitrary Point in $\mathbb{C}$



$$G_{u,(s=s_a)}$$

$$\mathrm{Null}(\Delta G_{(s=s_a)}) = L$$

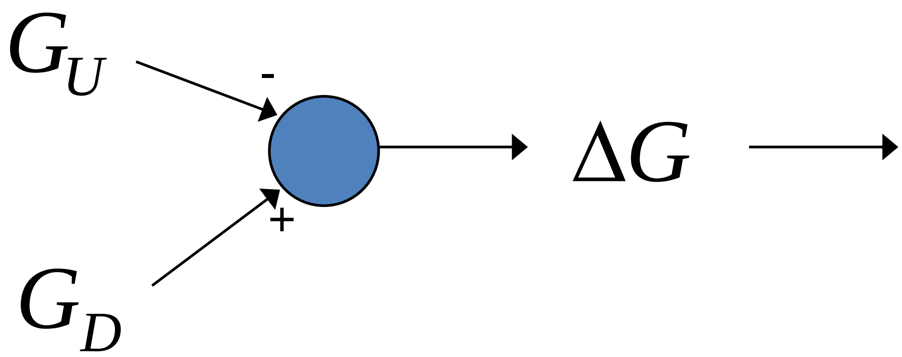


$$G_{d,(s=s_a)}$$

$$L = \left\lfloor \ell_1 \quad \cdot \quad \ell_q \right\rfloor \qquad \ell_j = \int_0^\infty f_{(t)} e^{-(a+bi)t} dt$$

dynamic response for f(t) is rigid body motion over the damaged region

dDLV Implementation



$$Null(\Delta G) = \ell_{(s)}$$

$$L_{(s)} = \ell_{(s)}$$ (expanded with zeros to the model coordinates)

$$L_{st(s)} = \underbrace{KG_{(s)}^{model}}_{} \cdot L_{(s)}$$

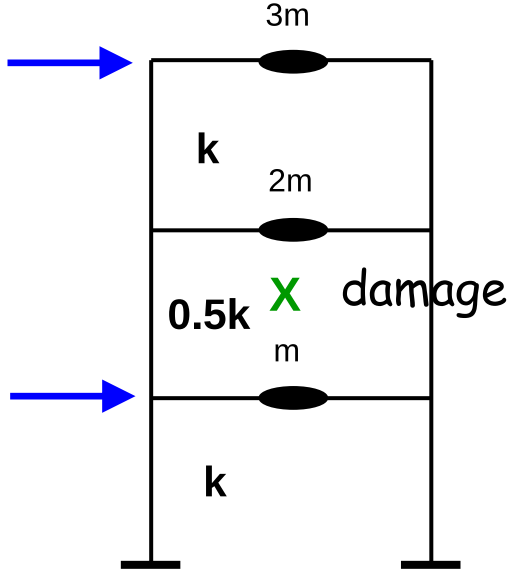
at s=0 this product = I

Complex stress field due to L_{st} is identically zero over damaged region

Illustration (at s = 0)

$$F_U = \frac{1}{k}\begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix} \qquad F_D = \frac{1}{k}\begin{bmatrix} 1 & 1 \\ 1 & 4 \end{bmatrix}$$

sensors



$$\Delta F = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \qquad dDLV = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$$

Static flexibility can't discriminate between #2and #3

Interrogation at 3+4i (arbitrary)

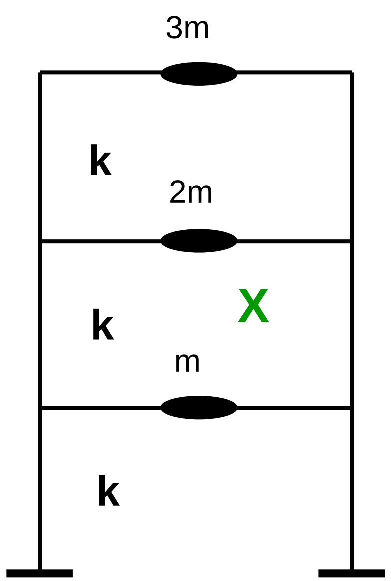
(*rhs* of the s-plane has no bearing here)

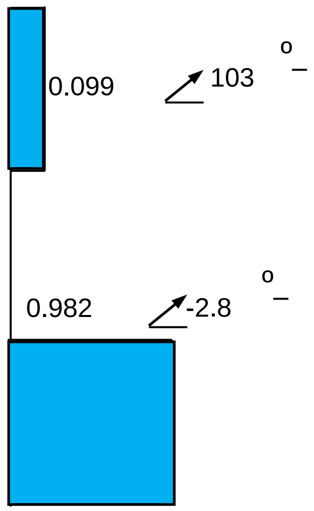
$$\Delta G = \begin{bmatrix} -0.0649 + 0.0892i & -0.4161 - 0.1469i \\ -0.4161 - 0.1469i & 0.0882 - 1.7623i \end{bmatrix}$$

$$dDLV = \begin{Bmatrix} 0.9701 \\ -0.0692 + 0.2325i \end{Bmatrix}$$

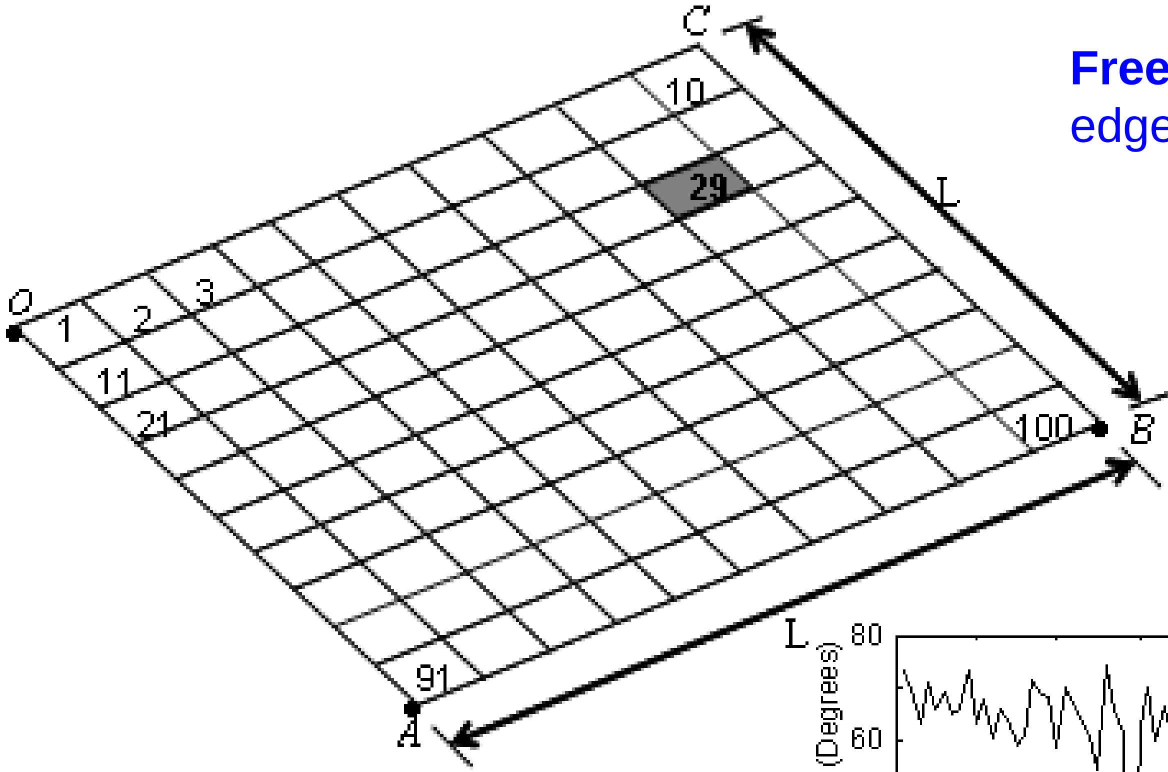
$$L_{st(s)} = KG_{(s)}^{model} \cdot L_{(s)}$$

$$L_{st} = \begin{Bmatrix} 0.9810 - 0.0484i \\ 0.0224 - 0.0959i \\ -0.0224 + 0.0959i \end{Bmatrix}$$



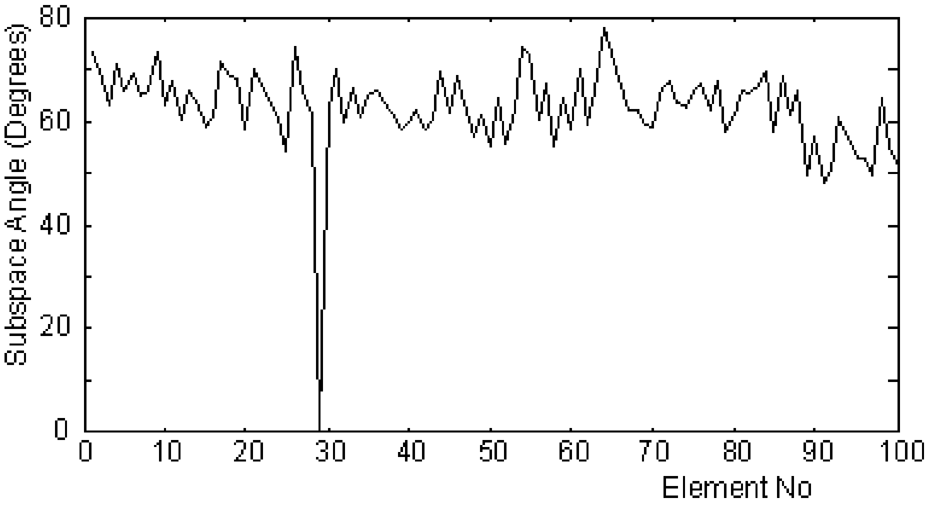


A More Complex Model



Free-free plate – sensors on the edge from 91 to 100

for details see [1]



Stochastic Extension

Idea: what is needed in the null space approach is not ΔG but a basis for its null space – therefore,

$$Q = T \cdot \Delta G$$

can serve as a surrogate provided T is full rank.

Constraints between the matrices of the state-space relation allow the estimation of Q - *without having ΔG or T explicitly*

Constraints

$$y = C_c^{dis} x$$

$$\dot{y} = C_c^{dis} A_c x + C_c^{dis} B_c u$$

$$\ddot{y} = C_c^{dis} A_c^2 x + C_c^{dis} A_c B_c u + C_c^{dis} B_c \dot{u}$$

$$\dot{x} = A_c x + B_c u$$

$$C_c^{dis} B_c = 0 \qquad D_c = C_c^{dis} A_c B_c$$

$$C_c^{vel} = C_c^{dis} A_c \qquad C_c^{acc} = C_c^{vel} A_c$$

$$C_c^{dis} B_c = C_c^{vel} A_c^{-1} B_c = C_c^{acc} A_c^{-2} B_c = 0$$

$$C_c^{dis} A_c B_c = C_c^{vel} B_c = C_c^{acc} A_c^{-1} B_c = D_c$$

$$C_c A_c^{-p} B_c = 0$$

$$C_c A_c^{1-p} B_c = D_c$$

$$H_p B_c = L D_c$$

where

$$L = \begin{bmatrix} I \\ 0 \end{bmatrix} \qquad H_p = \begin{bmatrix} C_c A_c^{1-p} \\ C_c A_c^{-p} \end{bmatrix}$$

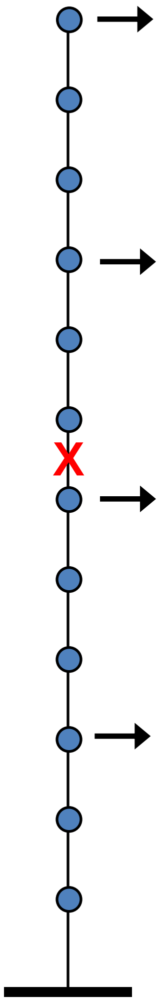
$$\Delta G = D_c \Delta R^T$$

$$R = C_c A_c^{-b} \left[I \cdot s - A_c \right]^{-1} H_p^{\dagger} L$$

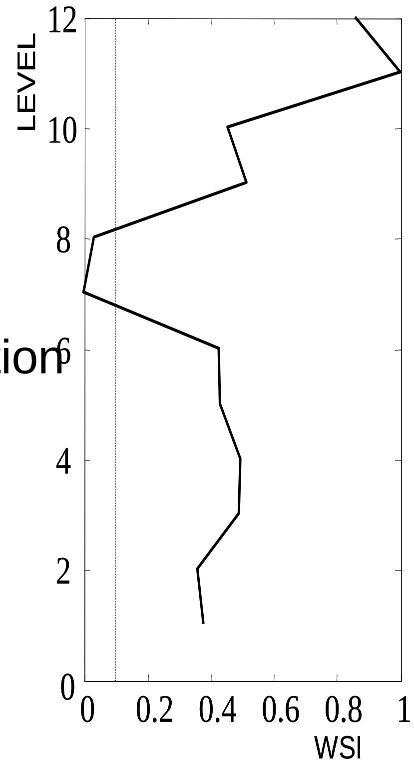
$$H_p = \begin{bmatrix} C_c A_c^{1-p} \\ C_c A_c^{-p} \end{bmatrix}$$

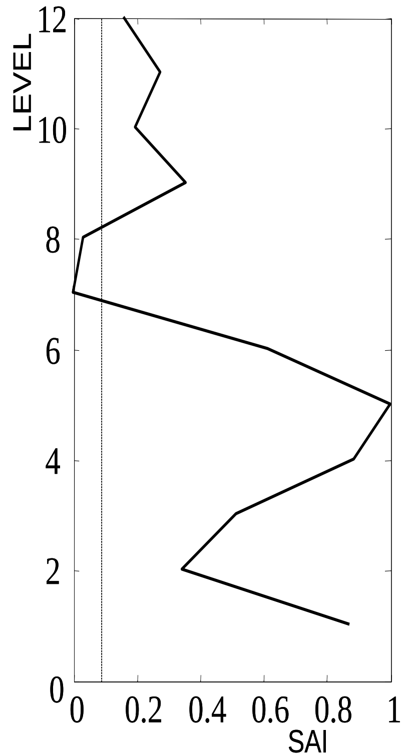
where the self-referencing direct transmission is guaranteed to be full rank

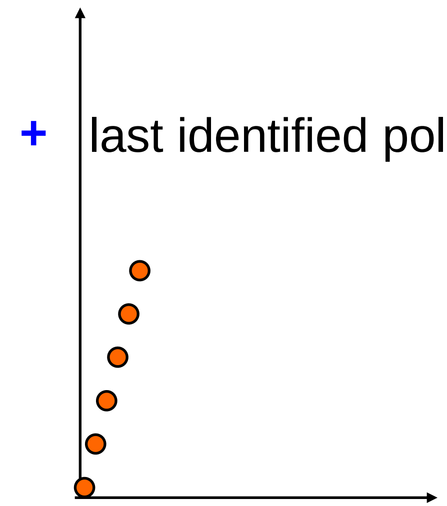
Illustration



25% reduction in stiffness

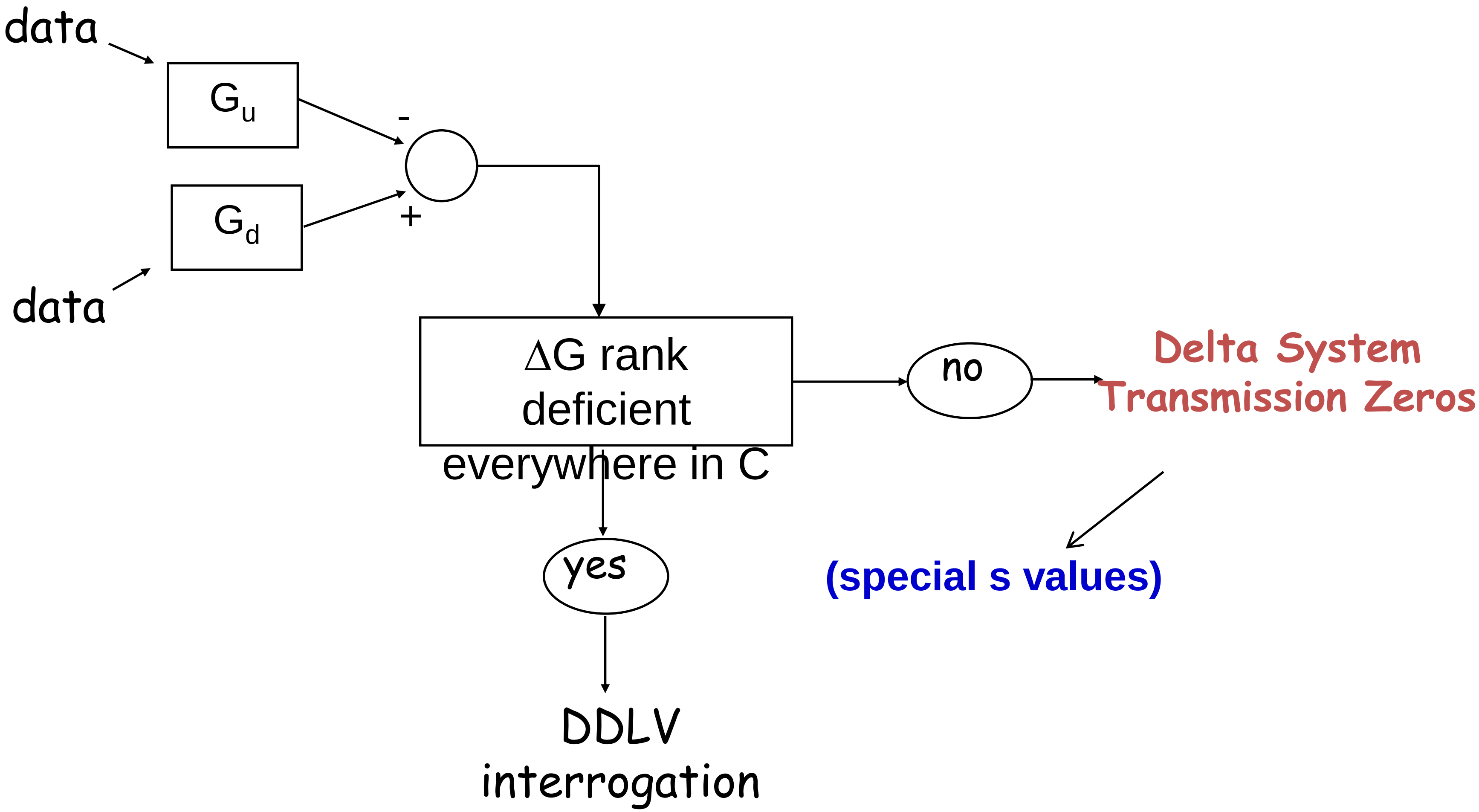




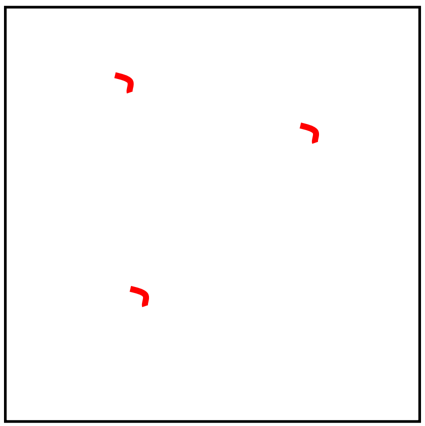


Four modes used in formulating H.

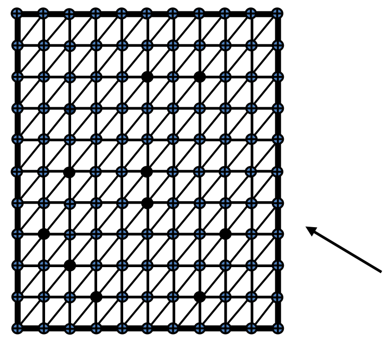
Saturation


data
G_u
-
+
G_d
data
ΔG rank
deficient
everywhere in C
no
Delta System
Transmission Zeros
yes
(special s values)
DDLV
interrogation

Illustration

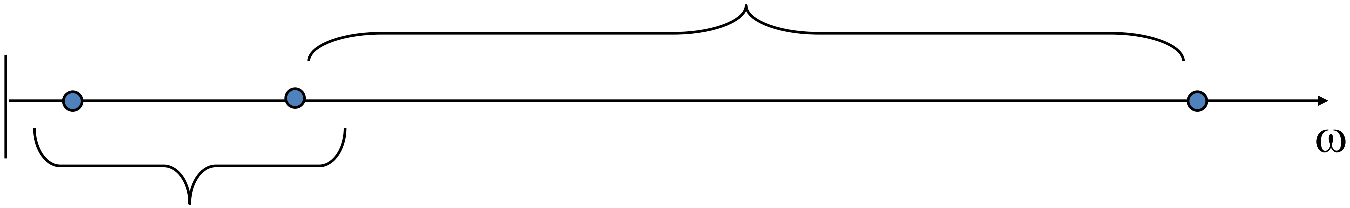


Free-Free Square Plate – 3 cracks



Damage simulated as 50% loss of E in the FE that contains the crack

306 truncated space

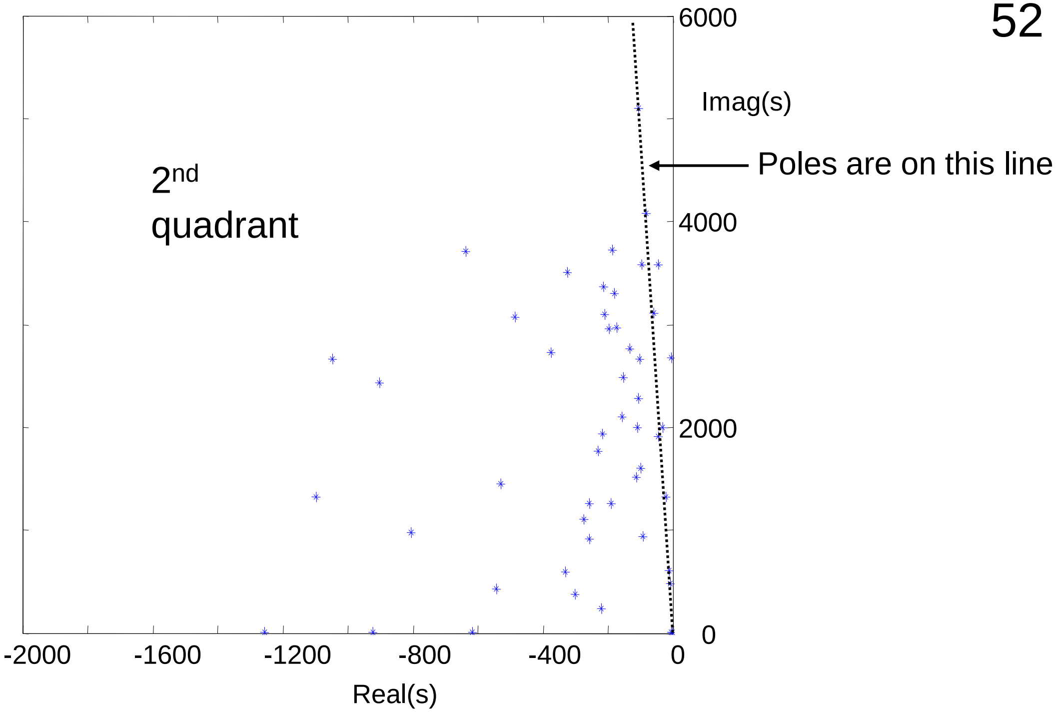


50 assumed identified

FEM – 356 modes

10 sensors $\rho(\Delta K)$ (for model) = 18 i.e., significant saturation

Transmission Zeros of Delta System



52 Transmission Zeros

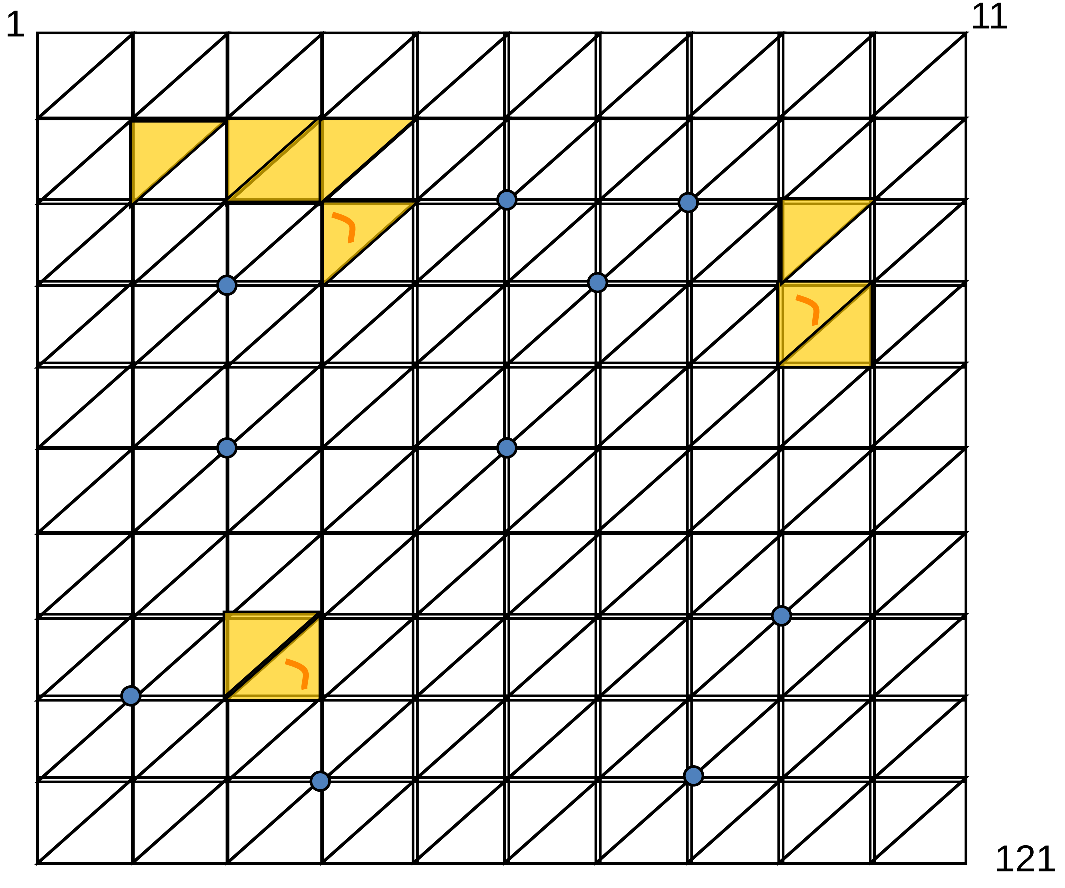
↓

52 Kernels

↓

52 Stress Fields

Damage Localization from Transmission Zeros of the Delta System



E #	< t
77	18
47	12
146	11
23	11
78	10
57	10
27	10
25	10
26	8
24	8
145	7

A Theorem Connecting Influence Lines to Damage Localization

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A Theorem Connecting Influence Lines to Damage Localization

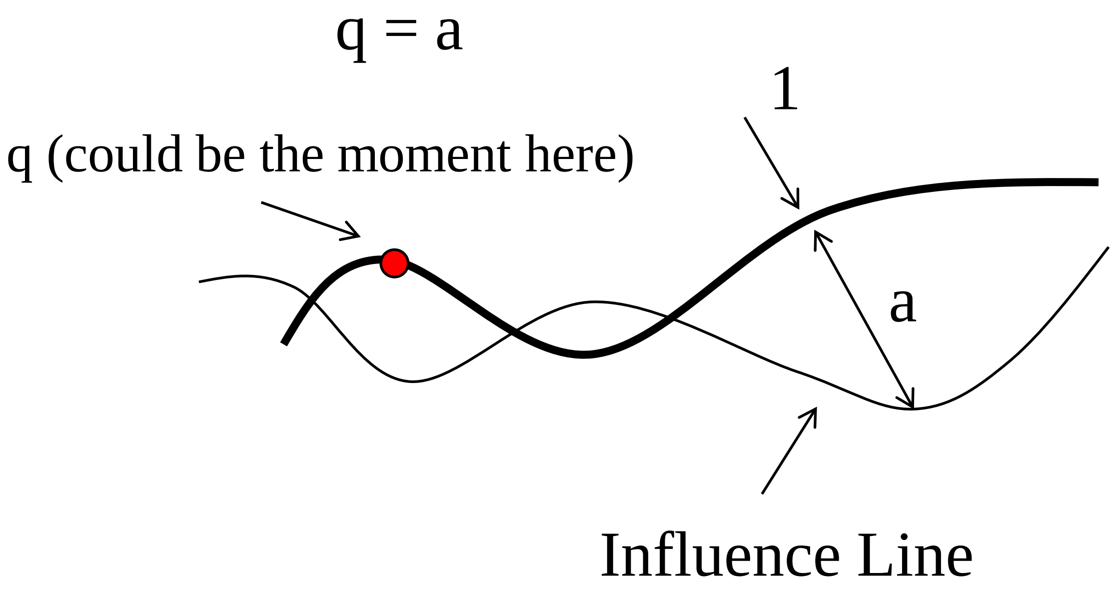
1) what is an influence line ?

2) what do we mean by damage ?

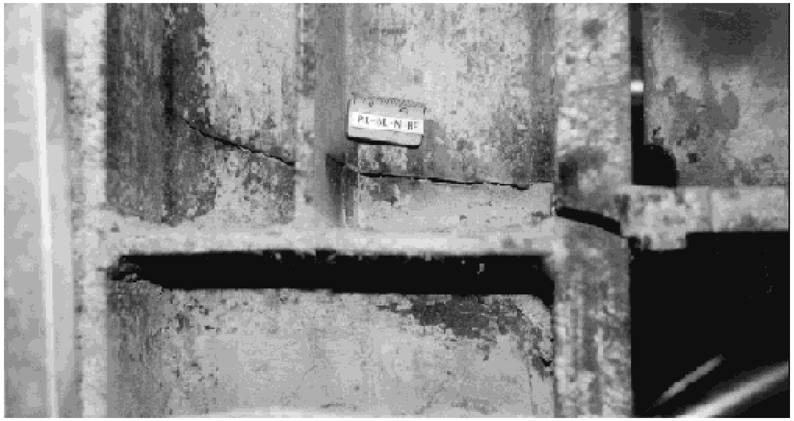
3) what do we mean by localization ?

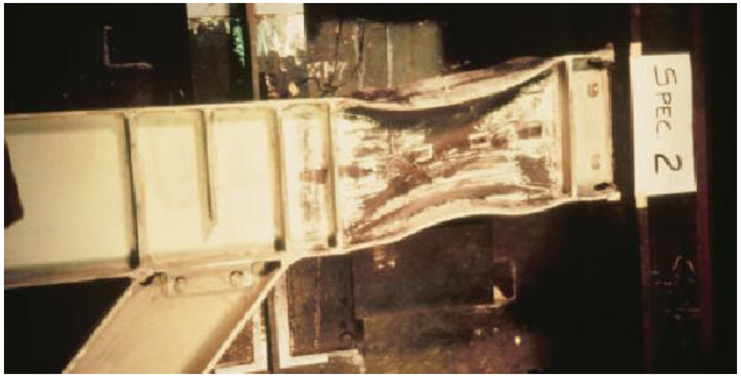
Influence Line

An influence line for quantity "q" is a plot of how quantity "q" varies as a unit source, acting along prescribed directions, moves along the structure.

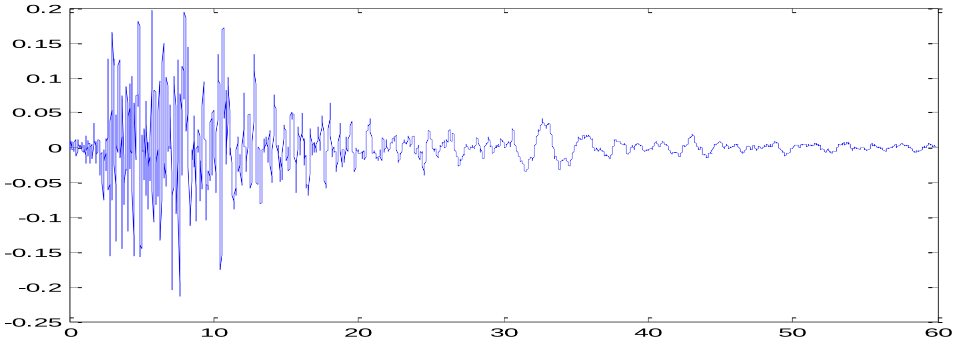


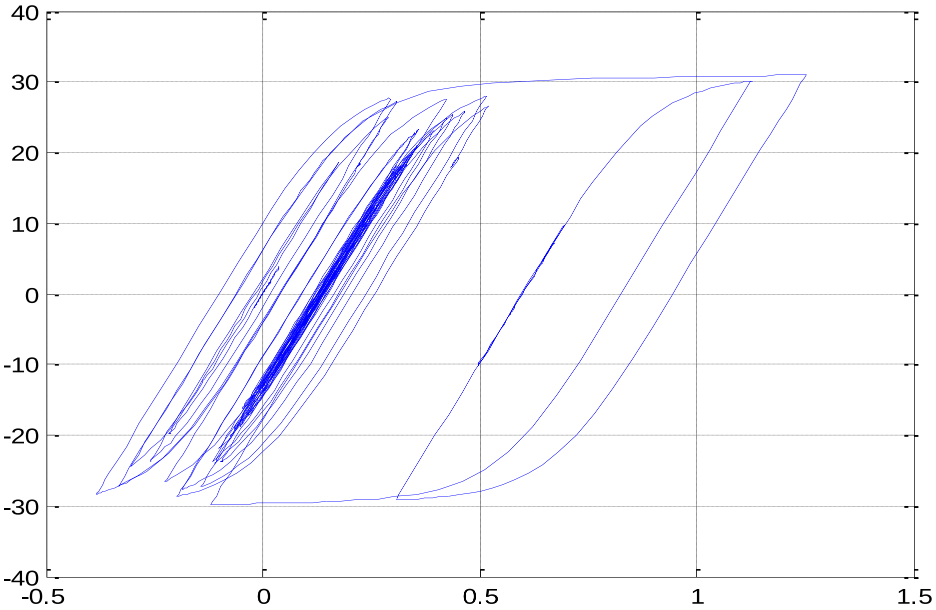
Damage





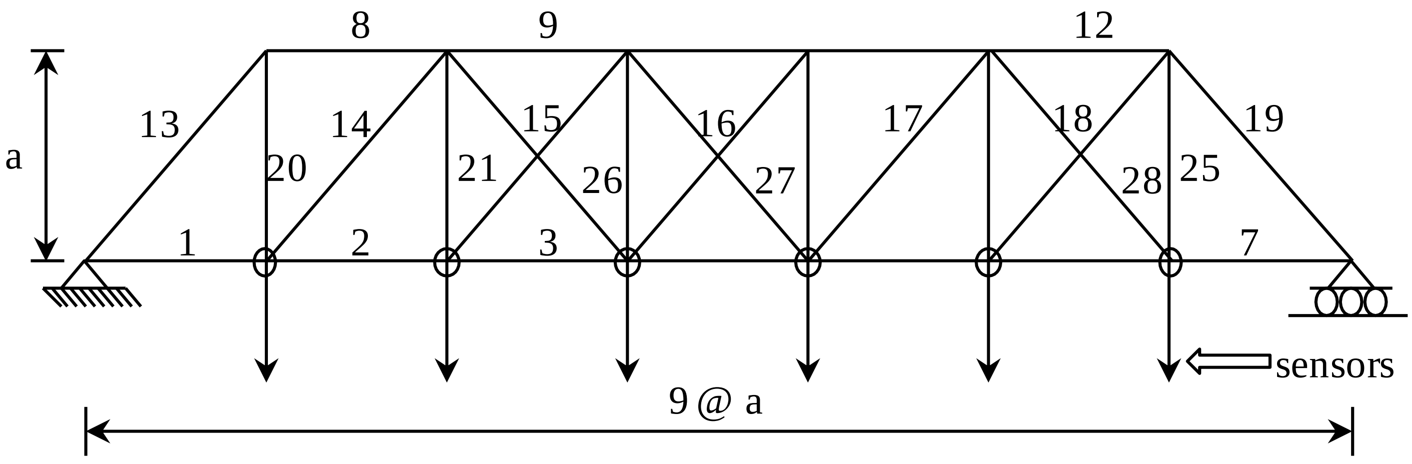
In our context it is a change in the stiffness properties of a system that is reflected in changes in the LOW AMPLITUDE VIBRATION characteristics (frequencies and mode shapes) “damage” that has no effect on the stiffness near the origin cannot be detected in a “before and after strategy”





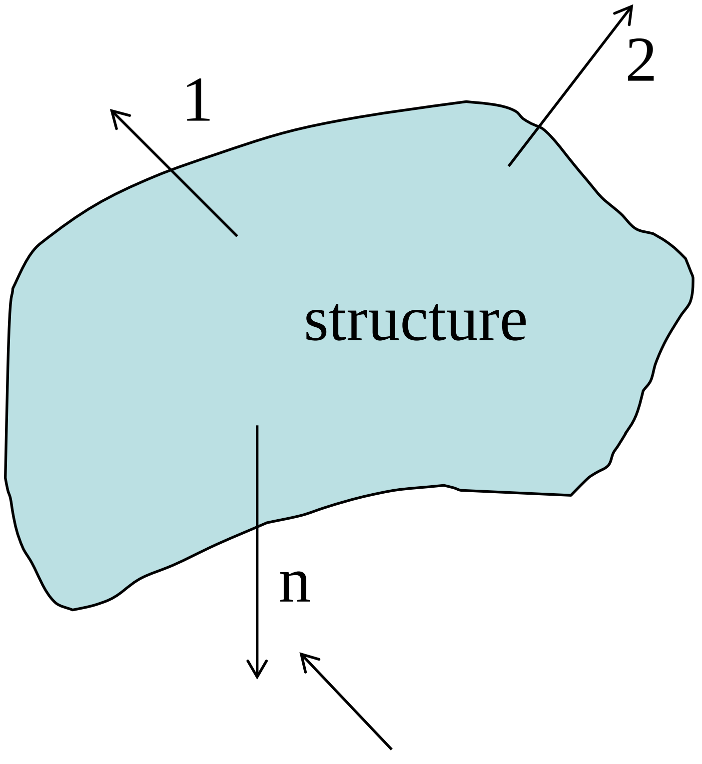
Localization

Finding WHERE is the damage.



We would say damage is on bar X (some bar) – but not where in bar X is the damage or what form it takes because it is not theoretically possible from changes in flexibility.

Flexibility Matrix

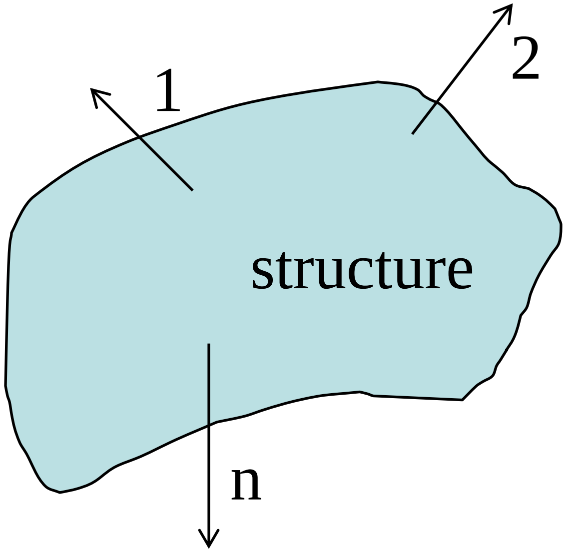


$$F = \begin{bmatrix} f_{11} & f_{12} & \\ & \text{etc} & \\ & & \end{bmatrix}$$

Coordinates defined by the sensors

$f_{i,j}$ = deformation at coordinate i due to a unit load at coordinate j.

Flexibility Matrix



$$F = \begin{bmatrix} f_{11} & f_{12} & \\ & \text{etc} & \\ & & \end{bmatrix}$$

In practice one cannot typically compute the flexibility from its definition (using static loads) so it is customary to estimate it using eigenvalues and eigenvectors - which can be extracted from vibration data.

Fundamental Subspaces of a Matrix

Let A $\in R^{rxc}$

The Column Space or the Image of A is a matrix U_1 such that, if A x = b there is always a vector y such that U_1 y = b. The minimum number of columns needed in U_1 is the rank of A.

The Null Space or the Kernel of A is a matrix U_2 such that, if A x = 0 then is always a vector y such that x=U_2 y. The number of columns in U_2 is the nullity of A.

Example

$$A = \begin{bmatrix} 17 & 22 & 39 \\ 22 & 29 & 51 \\ 39 & 51 & 90 \end{bmatrix}$$

$$U_1 = \begin{bmatrix} -0.3527 & 0.7364 \\ -0.4614 & -0.6736 \\ -0.8141 & 0.0628 \end{bmatrix}$$

Is the column space or image of A

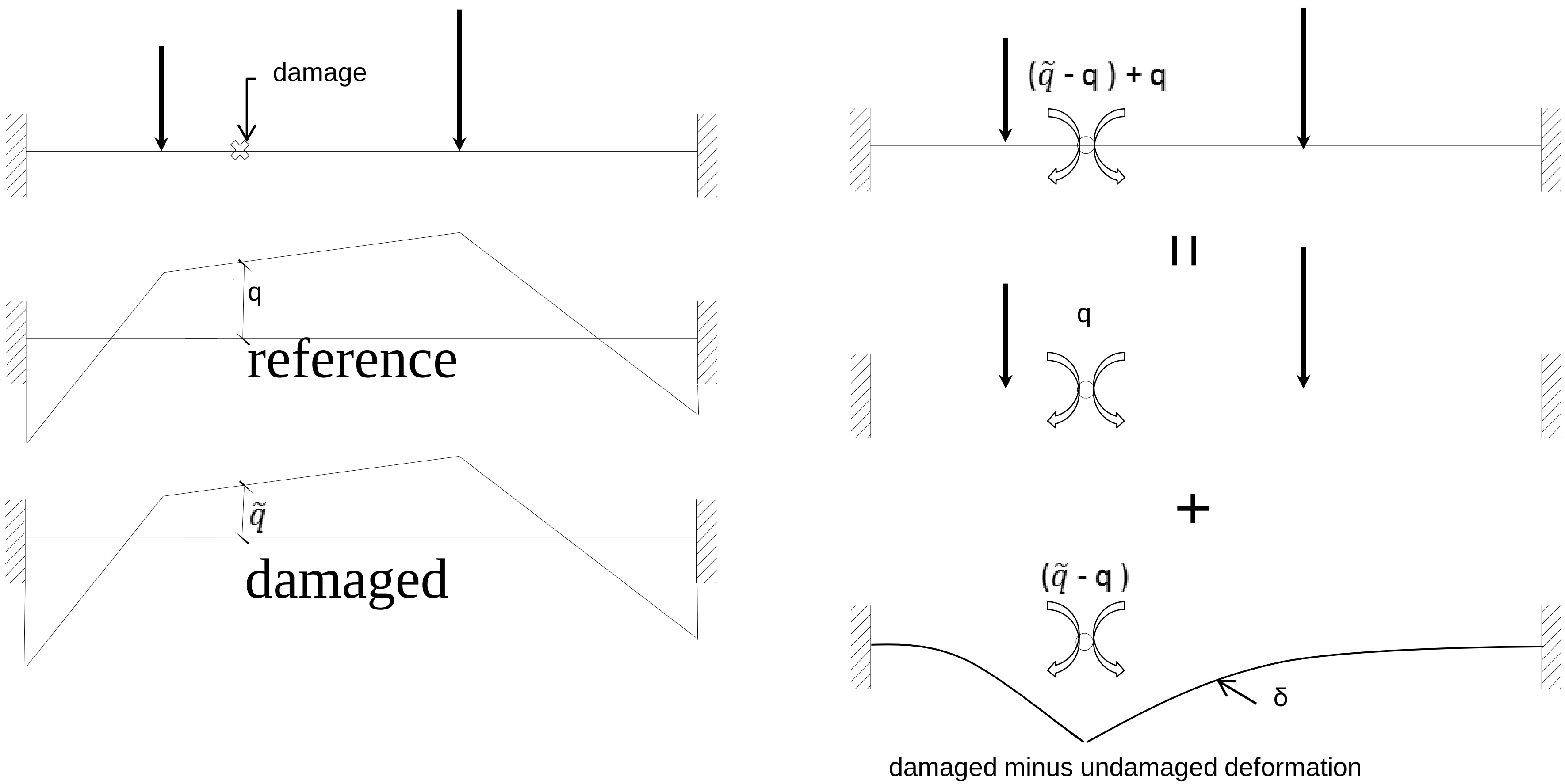
For example, if you multiply A by $[1\ 0\ 0]^T$ you get the first column. The same result is obtained by multiplying U_1 time [-47.8955 0.1464] and this holds for any multiplier of A.

Remark #1 – The difference between the damaged and the reference displacement fields (for any loading)

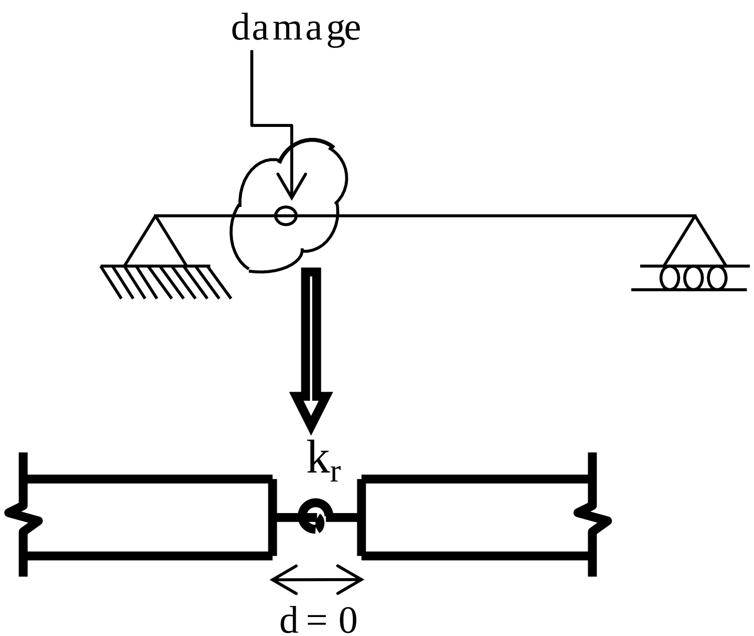
$$\delta = \tilde{y} - y$$

or its derivatives, depending on the type of member, can be viewed as having discontinuities at the damage locations.

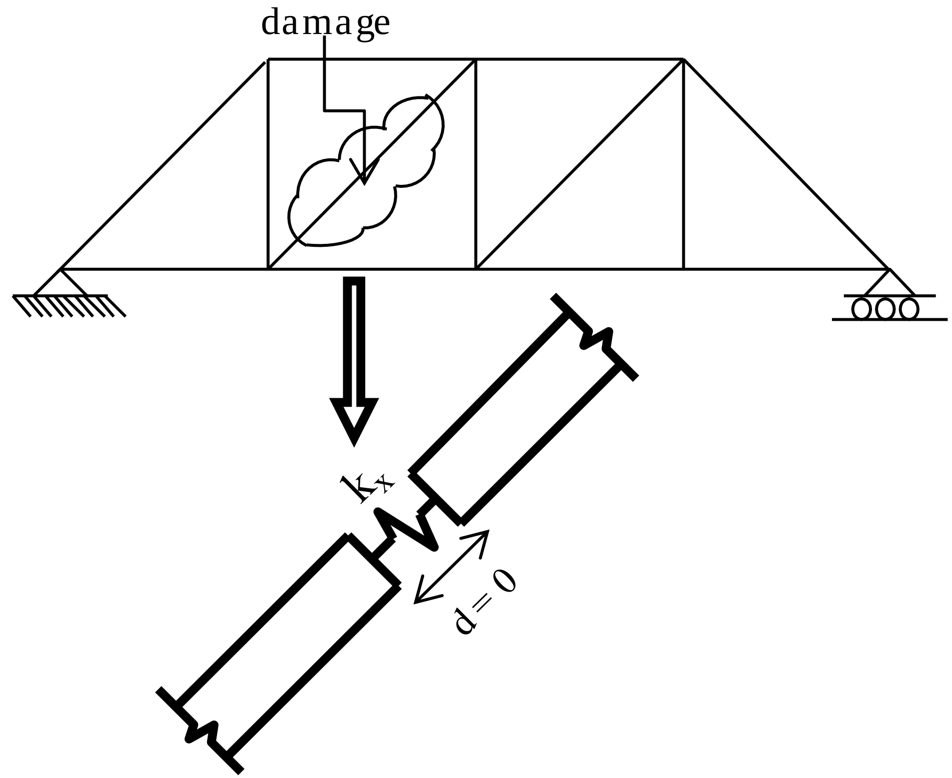
Illustration of Remark #1



Idealization (parameterization) of Damage

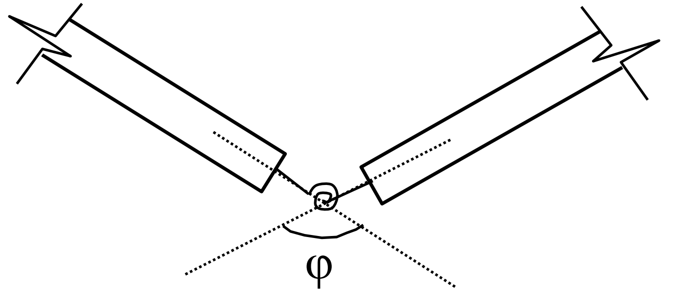


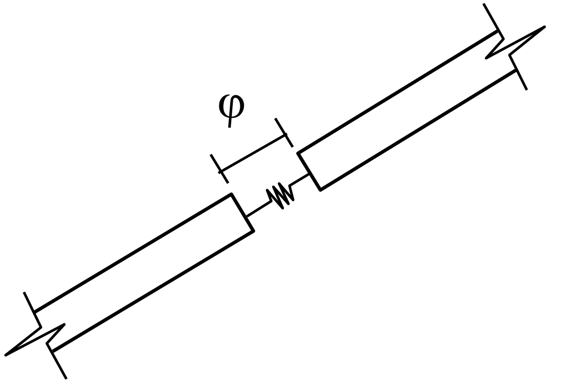
Ref. State $K_r = \infty$
Damaged State K_r = finite
(a)



Ref. State $K_x = \infty$
Damaged State K_x = finite
(b)

Damage Distortions (discontinuities)





Influence Lines and the Difference in the Deformation Fields

The difference in the deformation fields between the damage and the reference state for any load in sensor coordinates is a linear combination of the influence lines of the stress resultants at the damage locations and the scaling coefficients are the associated discontinuities.

Proof

The unit load is the "real case" – the difference in the deformation fields is the virtual deformation – it then follows that

$$1 \cdot \delta_k - \sum_{j=1}^{p} q_{j,k} \cdot \varphi_j = \int_{\Omega} \sigma_1^T \varepsilon_2 dV$$

$$0 = \int_{\Omega} \sigma_2^T \varepsilon_1 dV \qquad \delta_k = \sum_{j=1}^{p} q_{j,k} \cdot \varphi_j$$

$$q_j = \begin{bmatrix} q_{j,1} & q_{j,2} & . & q_{j,p} \end{bmatrix}^T \qquad \delta = \begin{Bmatrix} \delta_1 & \delta_2 & . & \delta_m \end{Bmatrix}^T$$

$$\delta = \begin{bmatrix} q_1 & q_2 & . & q_p \end{bmatrix} \cdot \begin{Bmatrix} \varphi_1 \\ \varphi_2 \\ . \\ \varphi_p \end{Bmatrix} = Q \cdot \varphi$$

$$\delta = \begin{bmatrix} q_1 & q_2 & . & q_p \end{bmatrix} \cdot \begin{Bmatrix} \varphi_1 \\ \varphi_2 \\ . \\ \varphi_p \end{Bmatrix} = Q \cdot \varphi$$

The difference in the deformation field is a linear combination of the influence lines of the stress resultants at the damaged locations.

Influence Line Damage Locating (ILDL) Theorem

The image of ΔF is a basis for influence lines of stress resultants at the damaged locations.

Proof

$$\delta = F_D \ell - F_U \ell = \Delta F \cdot \ell$$

$$\Delta F = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} s_1 & \\ & 0 \end{bmatrix} \begin{bmatrix} U_1 & U_2 \end{bmatrix}^T$$

$$\delta = U_1 \cdot v \qquad v = s_1 U_1^T \ell$$

$$U_1 \cdot v = Q \cdot \varphi$$

In words: anything that “fits” in U_1 “fits” in Q

Use of ILDL in Damage Localization

How can we locate the damage using this result? –

Simple:
The damage is at locations where the IL fits in the image of ΔF.

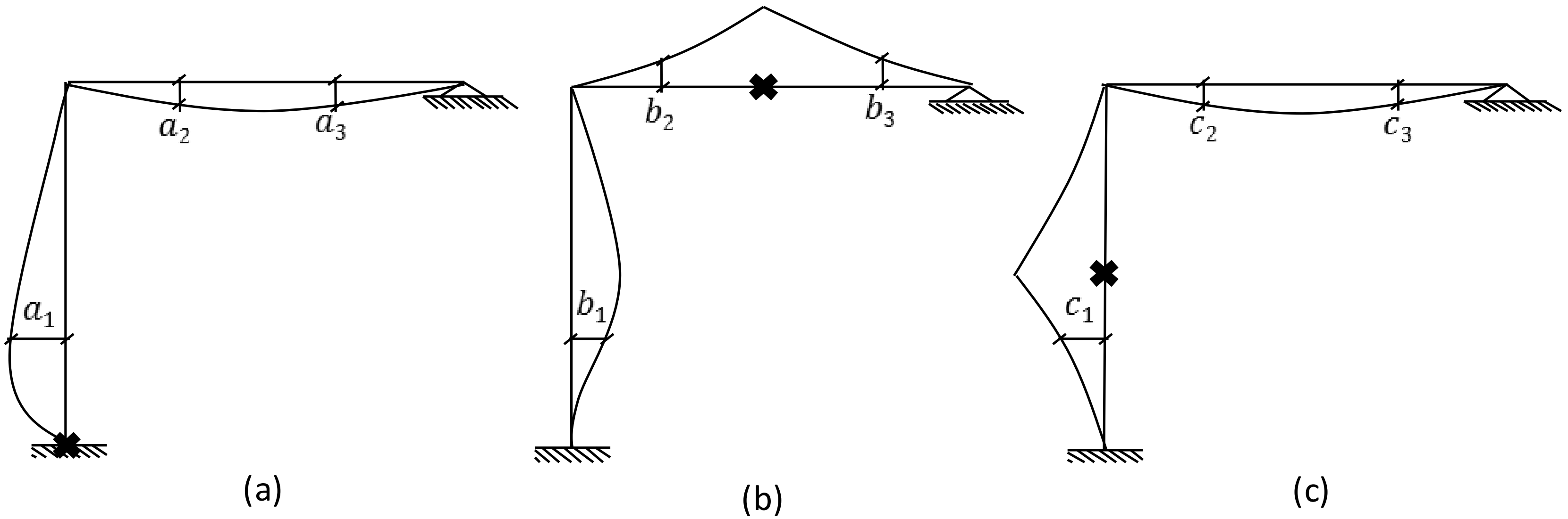
1)Find ΔF from test data
2) Postulate the possible locations of damage
3) Compute the IL for the positions
4) Check which ones fit in the image – the ones that do are damage locations

How does one check if it fits ?

In practice there are inevitable approximations so one needs a measure that is a continuous description of how well the IL fits – this is provided by the subspace angle between the image and the IL.

$$\theta_j = \cos^{-1} \left\| \left(\frac{U_1^T q_j}{\|q_j\|} \right) \right\|$$

Qualitative Illustration



$$\Delta F = \begin{bmatrix} x_1 & x_4 & x_7 \\ x_2 & x_5 & x_8 \\ x_3 & x_6 & x_9 \end{bmatrix} \begin{bmatrix} s_1 & & \\ & s_2 & \\ & & 0 \end{bmatrix} [V]$$

$\uparrow$

U_1

Since a and b are actual damage locations one would find that

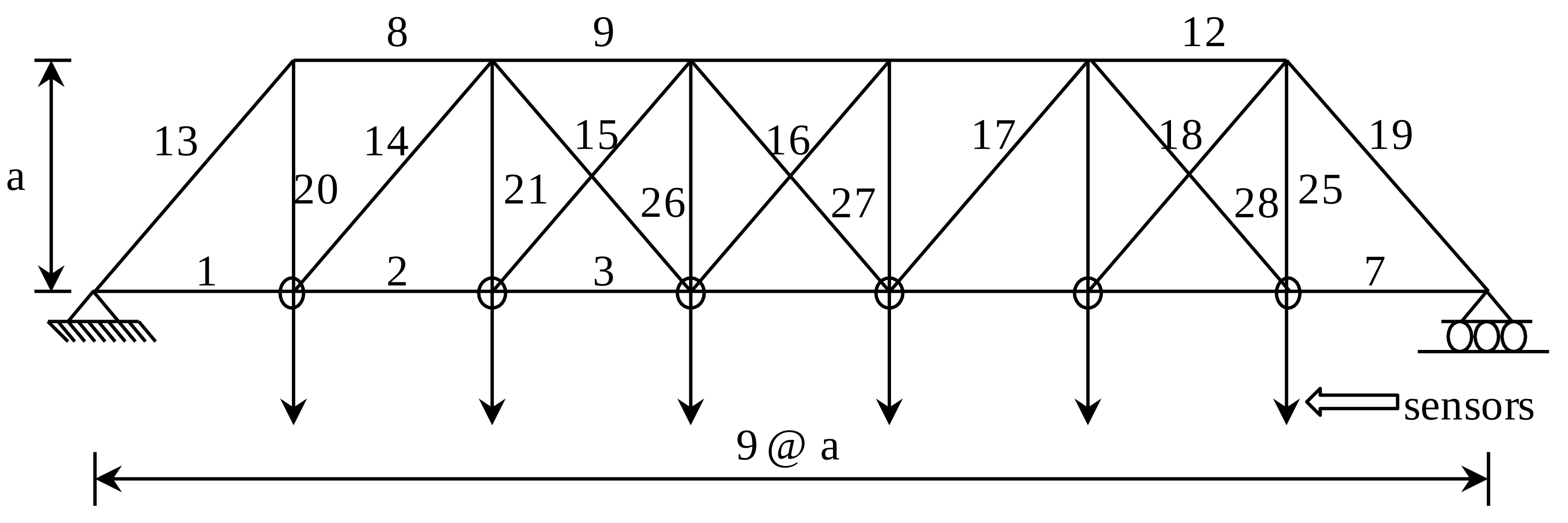
$$\left\| \begin{bmatrix} x_1 & x_2 & x_3 \\ x_4 & x_5 & x_6 \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix} \cdot \frac{1}{\left\| \begin{Bmatrix} a_1 & a_2 & a_3 \end{Bmatrix}^T \right\|} \right\| = 1 \quad \left\| \begin{bmatrix} x_1 & x_2 & x_3 \\ x_4 & x_5 & x_6 \end{bmatrix} \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \end{Bmatrix} \cdot \frac{1}{\left\| \begin{Bmatrix} b_1 & b_2 & b_3 \end{Bmatrix}^T \right\|} \right\| = 1$$

And given that c is not a damage location

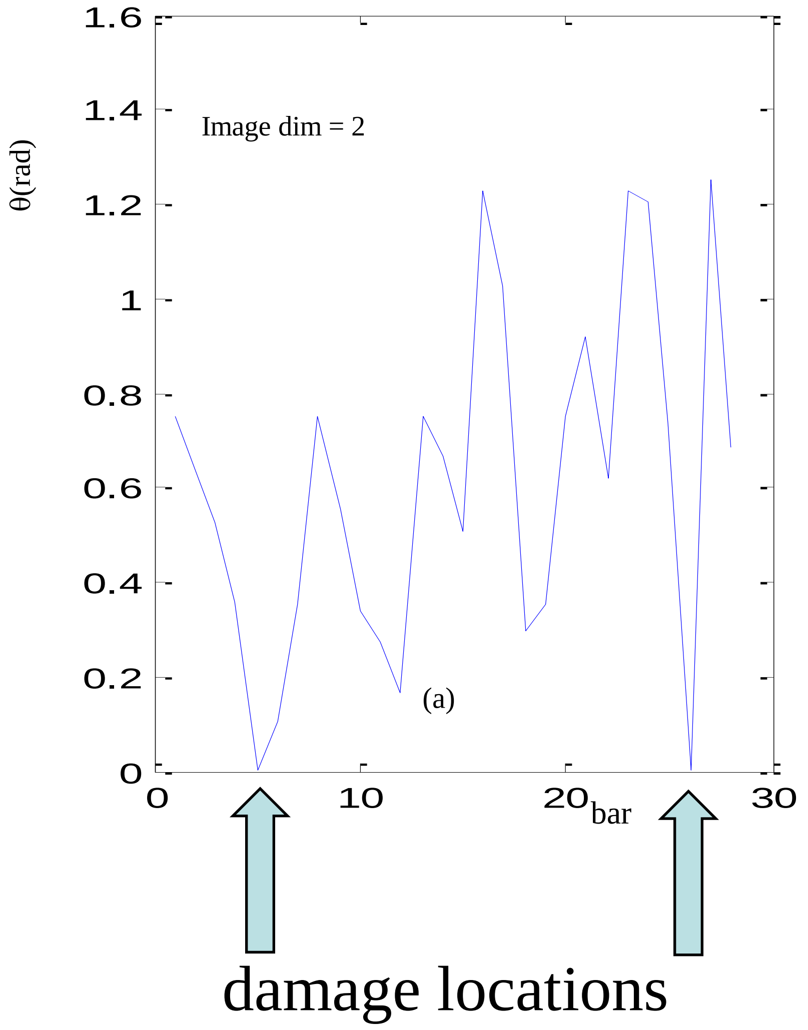
$$\left\| \begin{bmatrix} x_1 & x_2 & x_3 \\ x_4 & x_5 & x_6 \end{bmatrix} \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \end{Bmatrix} \cdot \frac{1}{\left\| \begin{Bmatrix} c_1 & c_2 & c_3 \end{Bmatrix}^T \right\|} \right\| < 1$$

EXAMPLE

Let there be damage (10%) in bars 5 and 26

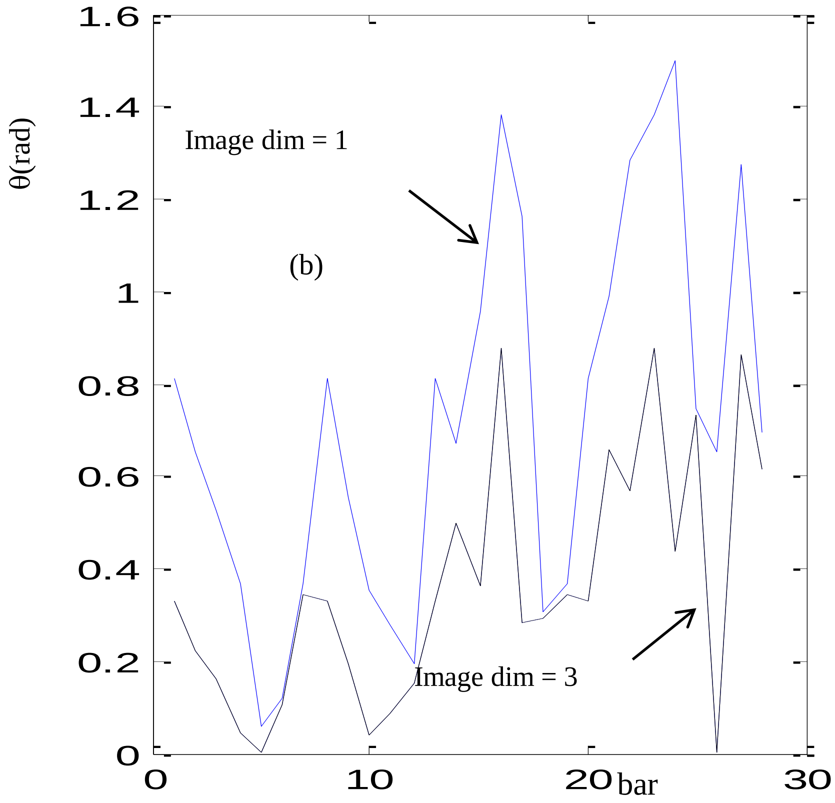


The subspace angle between all the IL and the image is in the next slide.



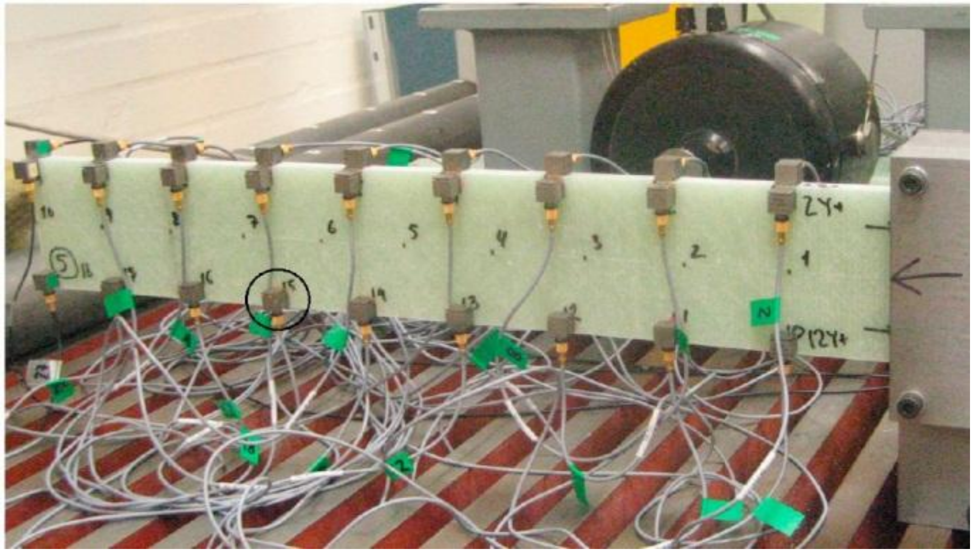
θ(rad)
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
10
20
30
bar
Image dim = 2
(a)
damage locations

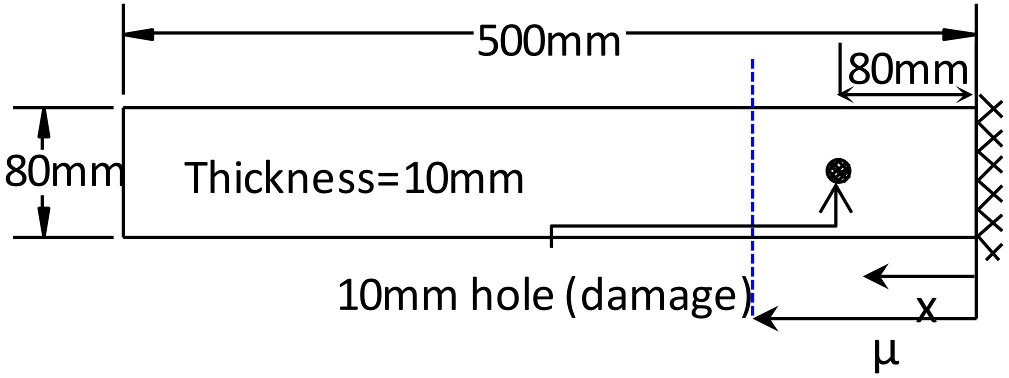
what if we make an error in estimating the dimension of the image?

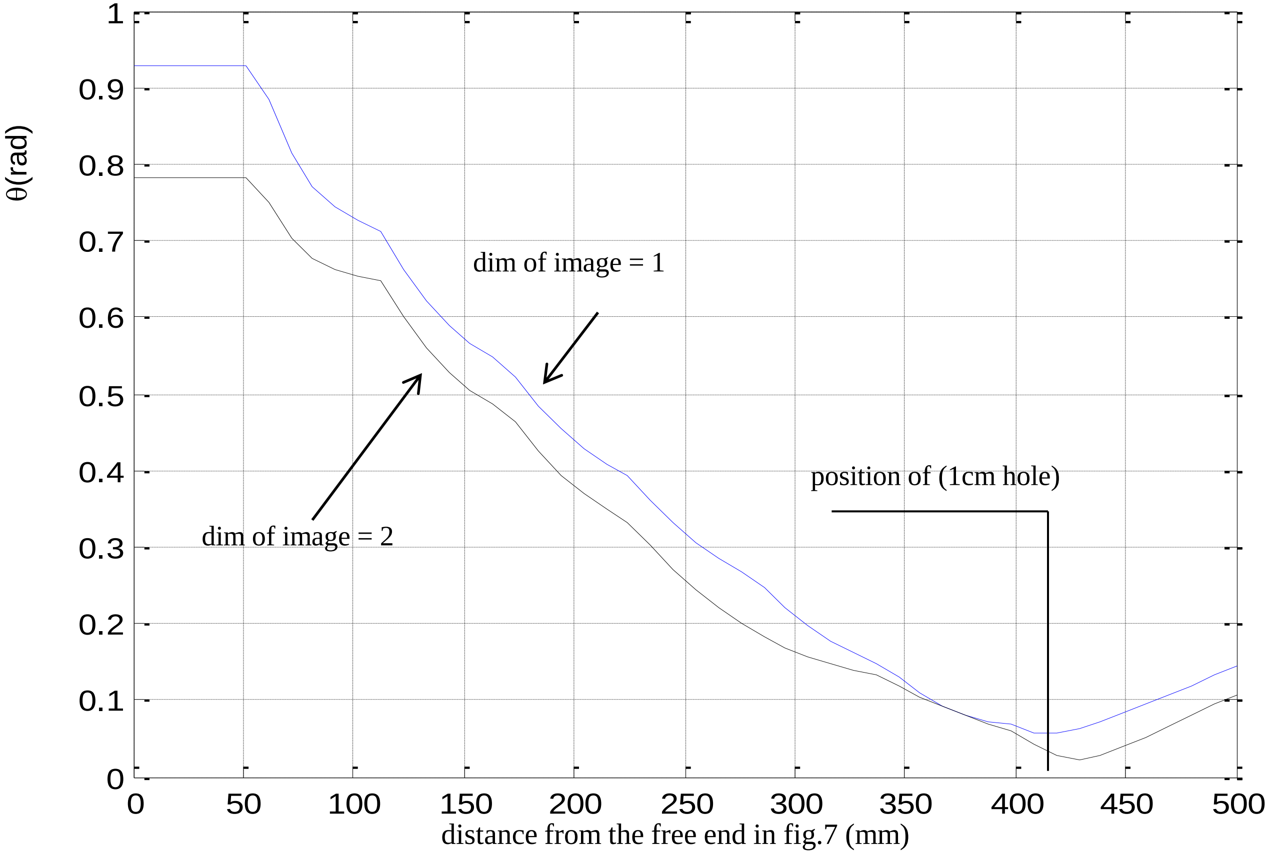


As the theory indicates – overestimation can reduce contrast but does no lead to false negatives

An Experimental Case







θ(rad)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
50
100
150
200
250
300
350
400
450
500
distance from the free end in fig.7 (mm)
dim of image = 1
dim of image = 2
position of (1cm hole)

SUMMARY:

Damage can be localized from changes in the flexibility by identifying the positions where the influence line of the associated stress resultant fits in the image.

The underlying assumptions are that the change in flexibility can be computed from the data and that the matrix is rank deficient (otherwise all influence lines fit in the image)

Quantification of Damage from the Image of the Change in Flexibility

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Center for Digital Signal Processing

What do we mean by Quantification?

We mean a description of the severity of the damage that has physical meaning - one that could be used by an engineer to decide whether the situation is minor or is serious.

Damage Quantification

From the properties of the SVD it follows that

Recall that $$U_1 \cdot v = \sum_{j=1}^{p} q_j \cdot \varphi_j$$

therefore

$$Q\varphi = u_j s_j$$

$$\varphi = Q^{-*} u_j s_j$$

Physically Meaningful Damage Severity Characterization

$$\frac{\tilde{z}_k^j \eta_k}{\tilde{R}_k} - \frac{\tilde{z}_k^j \eta_k}{R_k} = \varphi_k^j$$

R = EI or AE

$$\tilde{R}_k = \beta_k R_k$$

$$\beta_k = \frac{1}{1+\alpha_k^j} \qquad \alpha_k^j = \frac{\varphi_k^j R_k}{\tilde{z}_k^j \eta_k}$$

The solution is a fixed-point

(Iterations are needed in general)

∈

what is a fixed point?

Let X be any space and g a map of X into X.

A point x X is called a fixed point for g if x = g(x).

$$\beta_k = \frac{1}{1+\alpha_k^j} \qquad \alpha_k^j = \frac{\varphi_k^j R_k}{\tilde{z}_k^j \eta_k}$$

$$\beta = \frac{1}{1+\alpha(\beta)} \qquad \beta = g(\beta)$$

One is interested in Determining if the Fixed Point is Unique and if it is Unique whether it is Attractive.

If the Fixed Point is Unique and Attractive then an iterated solution is guaranteed to converge to the correct result.

Convergence

$$g(\tau) = \frac{1}{1 + \frac{c}{\tilde{z}(\tau)}} \quad = \frac{\tilde{z}(\tau)}{c + \tilde{z}(\tau)} \qquad c = \frac{\varphi R}{\eta} \qquad g'(\tau) = \frac{c\tilde{z}'}{(c + \tilde{z})^2}$$

Attractiveness is realized if

$$\left| c\tilde{z}' \right| < (c + \tilde{z})^2$$

Evaluating at the solution

$$\beta \quad = \frac{\tilde{z}(\tau)}{c + \tilde{z}(\tau)}$$

$$\left| \left(\frac{c}{\tilde{z}}\right)\left(\frac{\tilde{z}'}{\tilde{z}}\right) \right| < \frac{1}{\beta^2} \qquad \left| \frac{\tilde{z}'}{\tilde{z}} \right| < \frac{1}{\beta \; (1 - \beta \;\;)}$$

This is a requirement on the ratio of the derivative to the stress resultant value

Is this guaranteed?

For an arbitrary redundant

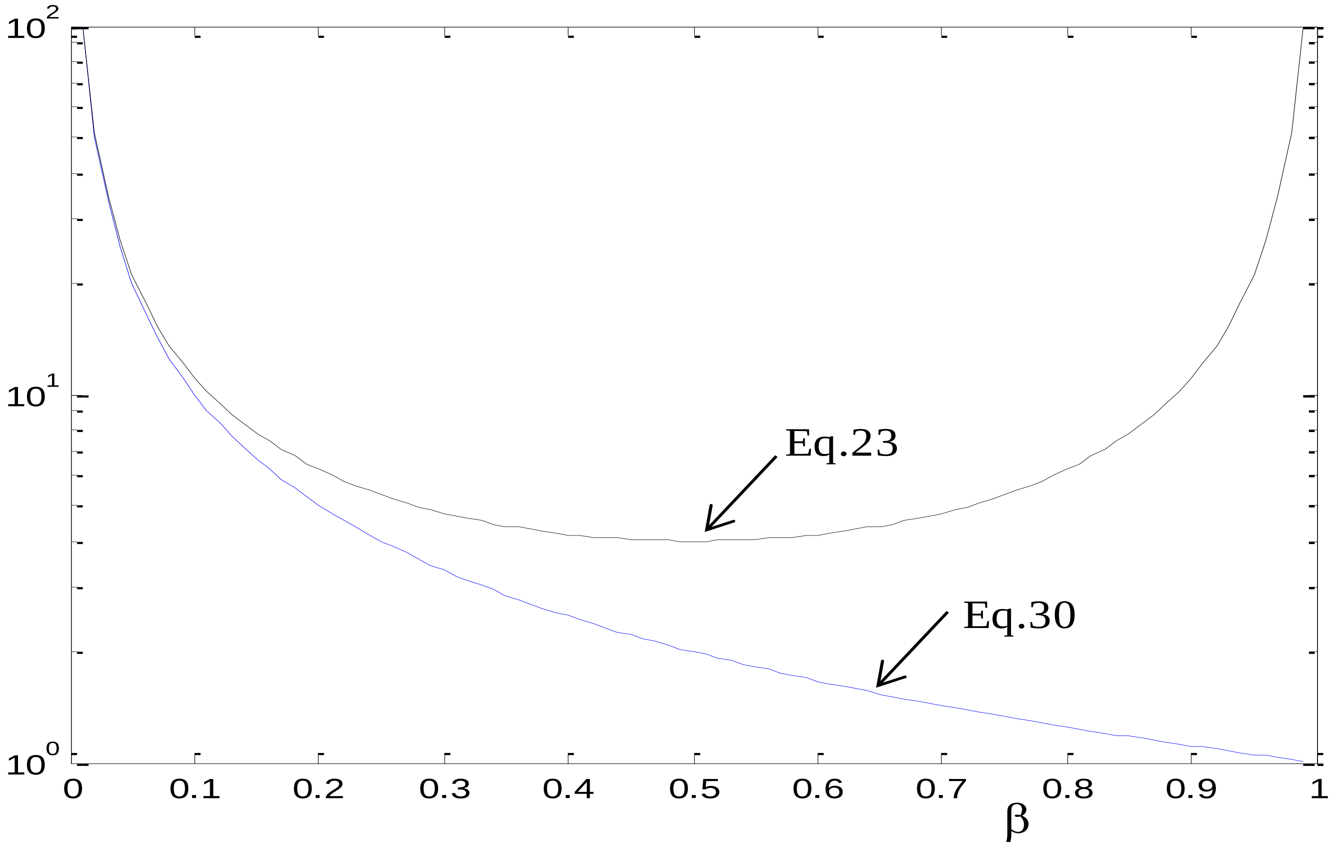
$$\left(d_0 + f_1\right)\tilde{z} = \Delta_0 \qquad f_1 = \frac{\eta}{\beta R}$$

$$d_0\tilde{z}' + f_1{}'\tilde{z} + f_1\tilde{z}' = 0 \qquad \frac{\tilde{z}'}{\tilde{z}} = -\frac{f_1{}'}{(d_0 + f_1)}$$

$$\left|\frac{\tilde{z}'}{\tilde{z}}\right| \le \frac{f_1{}'}{f_1} \qquad f_1{}' = -\frac{\eta}{\beta^2 R}$$

$$\left|\frac{\tilde{z}'}{\tilde{z}}\right| \le \frac{1}{\beta}$$

From before we had that the limit is $\left|\frac{\tilde{z}'}{\tilde{z}}\right| < \frac{1}{\beta\ (1-\beta\ )}$



It is guaranteed

The fixed point, therefore, is attractive so convergence is guaranteed.

Step by Step Summary:

$$\varphi = Q^{-*} u_1 s_1$$

$$\beta_k = \frac{1}{1+\alpha_k} \qquad \alpha_k = \frac{\varphi_k R_k}{\tilde{z}_k \eta_k}$$

1. Find the possible damage locations
2. Compute the distortions
3. Evaluate the stress resultants for the load u_1
4. Compute alpha
5. Compute the damage (beta)
6. Adjust the stiffness and repeat from 3.

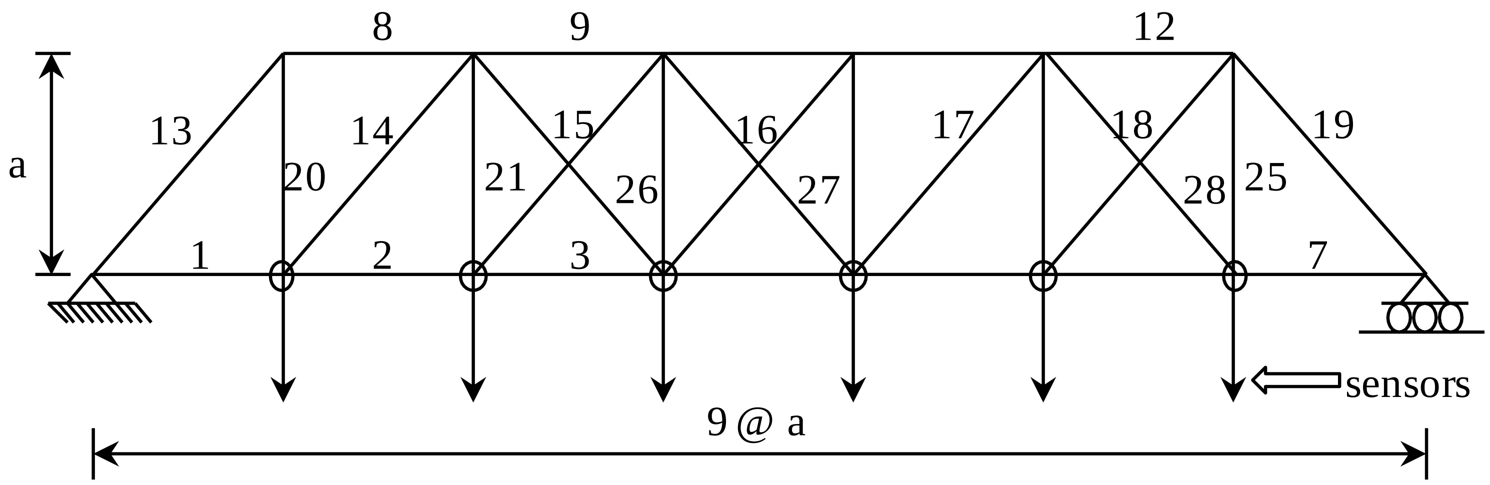
Do we really need Iterations?

$$\frac{d\beta_k}{\beta_k} = (\beta_k - 1)\frac{d\alpha_k}{\alpha_k}$$

e.g., if one has 20% damage $\beta = 0.8$ and 30% error in the stress resultant leads only to about 6% error in the estimation.

Iterations are typically not needed

Example #1



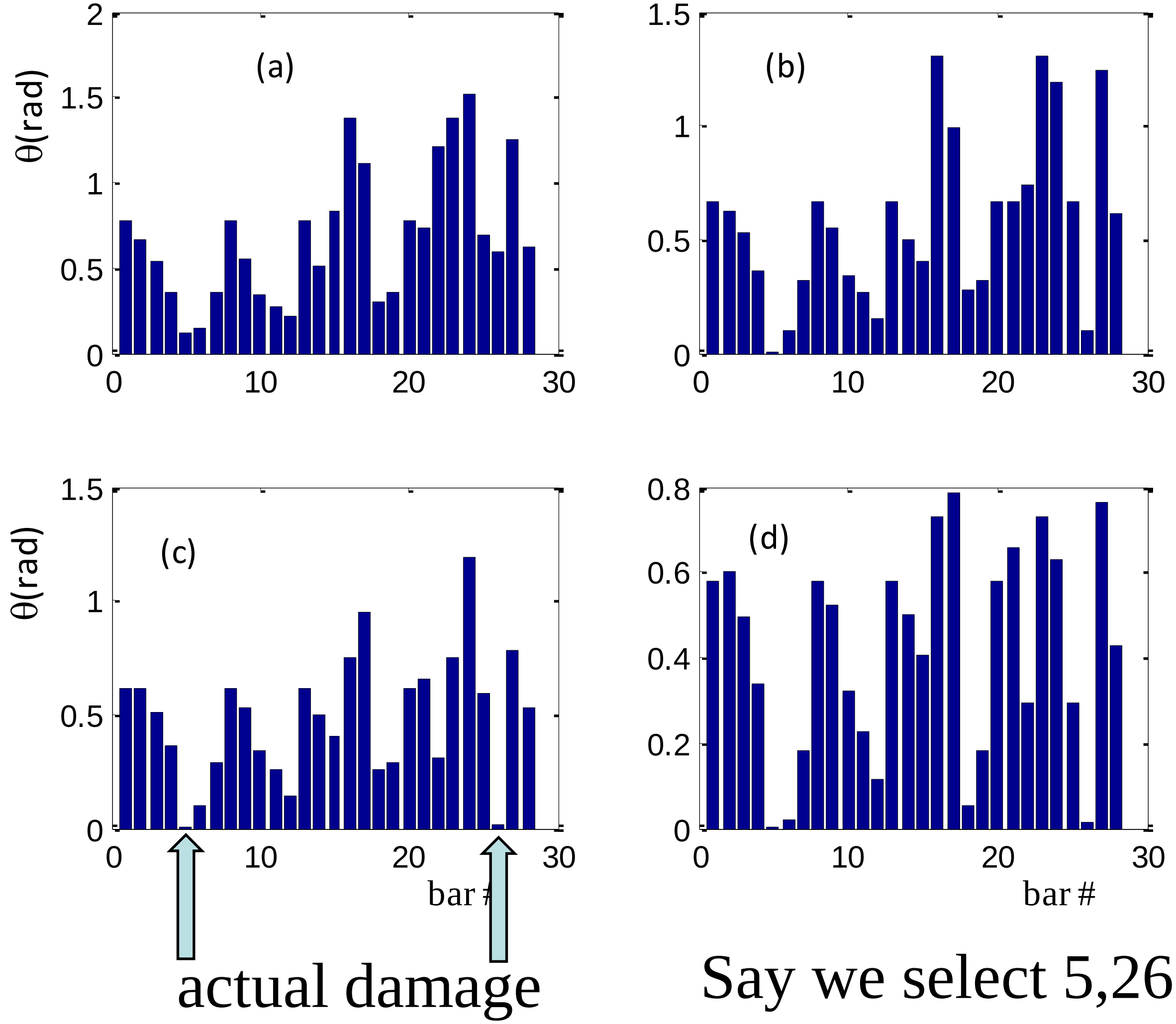
Damage in bars 5 and 26 (10% and 20%)

we begin by locating the damage
(previous lecture)

Assume that only 5 modes could be identified – so the flexibility is highly truncated.

Subspace angle for various estimates of the dimension of the image

S={1, 0.0783, 0.0771, 0.0271, 0.0067, 0.0003}.

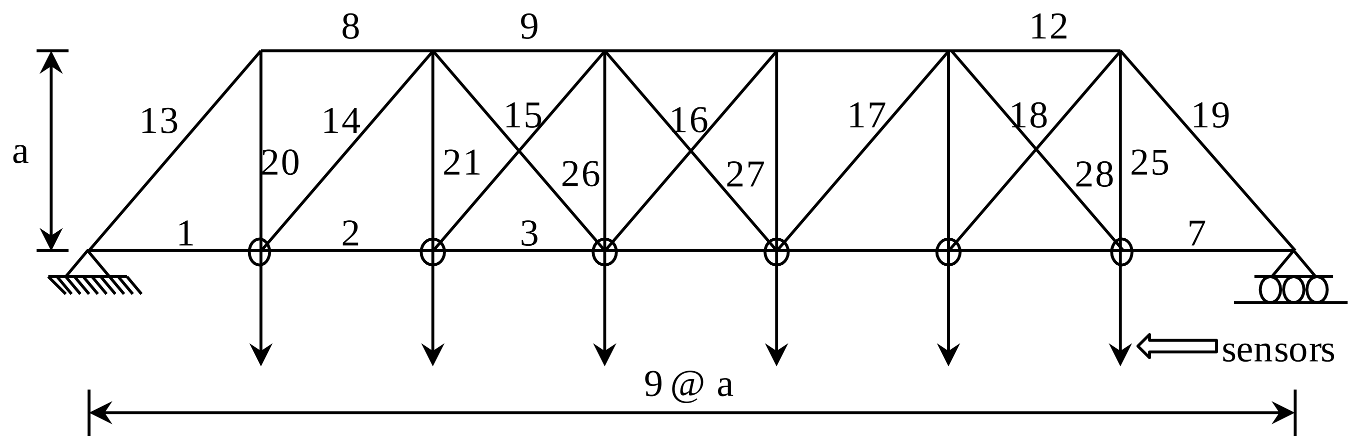


Say we select 5,26, 6 and 12

Table1. Iteration results in the quantification of damage for example#1

Cycle	z_5	z_6	z_{12}	z_{26}	β_5	β_6	β_{12}	β_{26}
0	-2.221	-1.646	1.483	-0.559	0.895	1.004	1.003	0.822
1	-2.221	-1.646	1.483	-0.525	0.895	1.004	1.003	0.813
2	-2.221	-1.646	1.483	-0.523	0.895	1.004	1.003	0.812
% damage (exact in parenthesis)	-	-	-	-	10.5 (10)	-	-	18.80 (20)

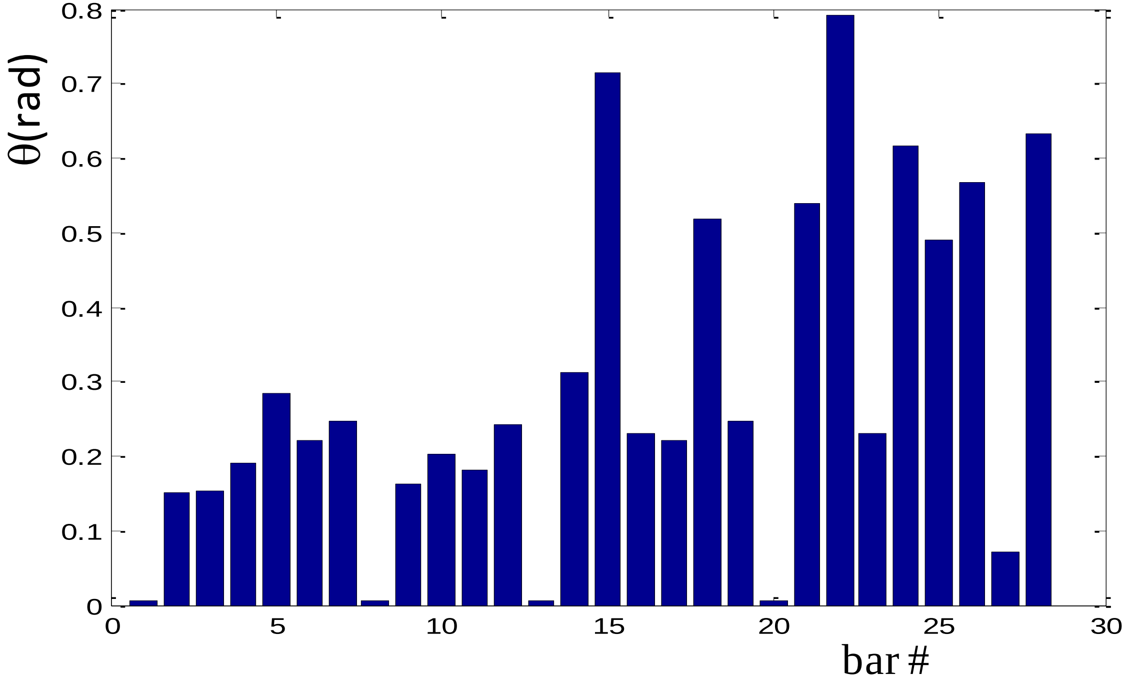
Inseparable Sets



Damage in bars 1 and 26 (10% and 10%)

Note that {1,13,8 and 20 } form an inseparable set.

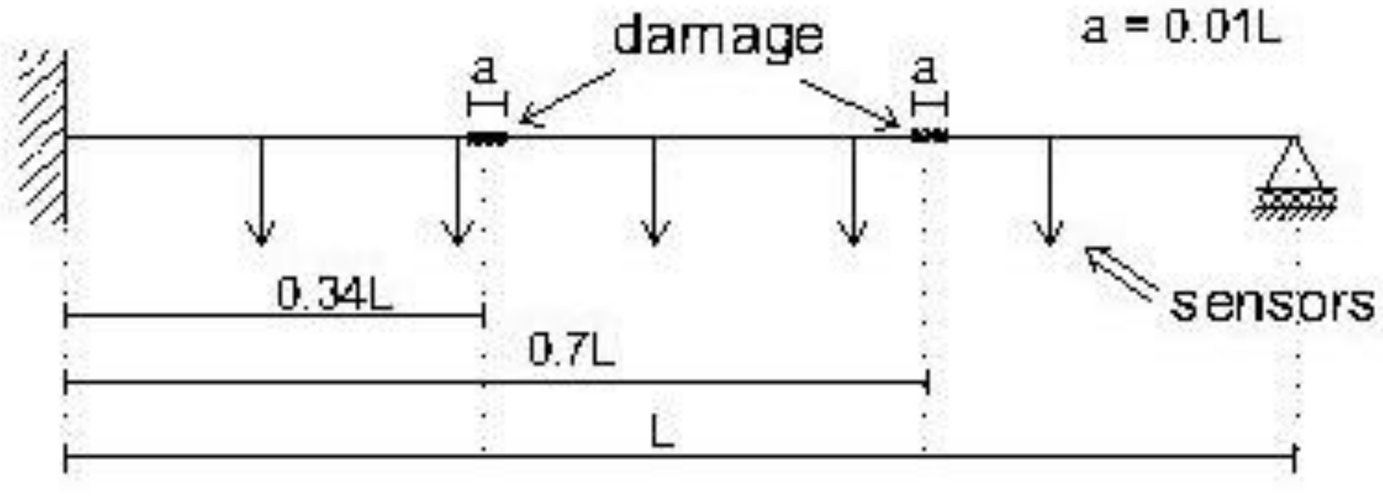
Subspace Angles for an Image of Dimension 3



Recognizing that there is an inseparable set the best that can be done is to obtain an upper bound for the damage in the bars of the set (the bars that are not part of the set are not a problem)

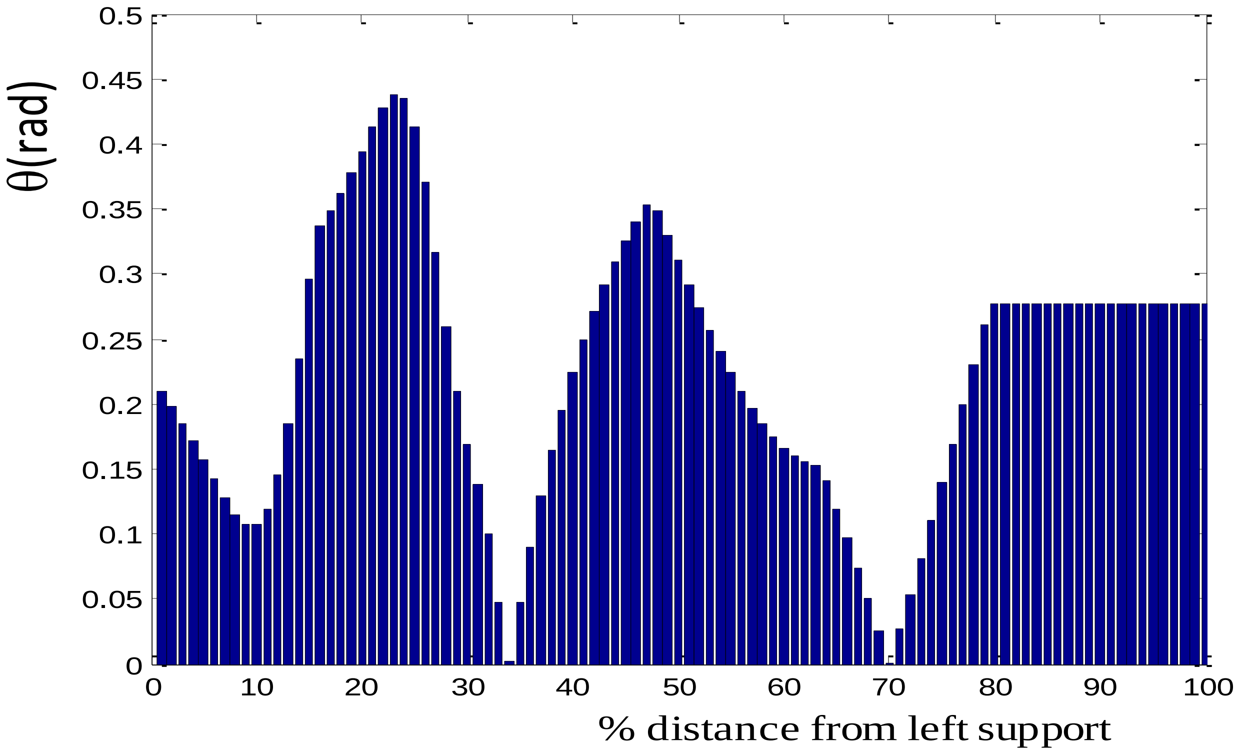
bar#	True Damage	Upper Bound
1	10%	30%
8	0%	30%
13	10%	13%
20	0%	30%

Another Example



S= {1, 0.0868, 0.0015, 0.0005, 0}

Subspace Angles with an Image of Dimension 2.



To illustrate we select 34, 70 (the true locations) plus 10.

Table3. Iteration results in the quantification of damage for the beam

Cycle	z_{34}	z_{70}	z_{10}	β_{34}	β_{70}	β_{10}
0	5.770	20.653	-19.018	0.763	0.602	0.994
1	5.743	20.640	-19.621	0.760	0.601	0.994
% damage (exact in parenthesis)	-	-	-	24 (25)	39.9 (40)	-

Observaciones de Cierre:

Detrás de todo lo que hemos hablado esta el echo de que las propiedades estructurales (para deformaciones pequeñas) se pueden estimar a partir de vibraciones medidas – en el caso de Ing. Civil es usual tener que trabajar con salidas solamente pero esto no causa grandes limitantes siempre y cuando la excitación ambiental sea suficientemente rica.

En estos dos días hablamos de ajuste de modelos, detección y localización de daño. Existen otras posibles aplicaciones de la tecnología que quizás encuentren aplicación comercial (high strength concrete unshoring, cable tension etc.).