Short Turning on Transit Routes

PETER G. FURTH

It is common to have certain trips short turn—begin or end partway along the route rather than at the route terminus)—in recognition of the characteristic drop in passenger volume at the end of a transit route. Because many passenger demands can be met by a trip following either a full-length or short-turn pattern, schedule coordination between the patterns is essential. Possible schedule coordination modes are described. Algorithms are presented for finding the schedule offset between the patterns that will balance loads and minimize overall cost. It is shown that even when overall capacity exceeds volume on every link, there may still be no schedule for which the trips of one or more patterns are not systematically overcrowded.

On heavy-demand transit routes the conventional strategy of operating all trips locally from one end of the line to the other can often be far less efficient than more complex strategies that are better tailored to a route's particular passenger origin-destination (O-D) distribution. When the O-D pattern shows a gradual drop in volume from the peak volume point to either end of the route, a commonly used strategy is short turning—having some trips cover only the more heavily used part of the route. Although the short-turn strategy is commonly used in the transit industry, a formal analysis of the strategy is lacking in the literature. The strategy is described by Furth and Day (*1*) who cite some examples of its use and compare it with other strategies, such as zoning, restricted and semirestricted service, and express service. The only known analysis of the strategy is Ceder's (*2*). However, only aggregate volumes and capacities are considered in this approach, and passenger behavior in the case of overlapping service patterns is not addressed. As will be seen, this approach can easily lead to systematic overcrowding on some trips, while excess capacity exists on others. In the absence of a satisfactory published analysis, one might expect that this strategy is not applied as often as it could be, and that when applied, it is not always designed with maximum efficiency.

As defined in this paper, all trips in a short-turn system operate locally with no boarding or alighting restrictions. The short-turn system includes one full-length pattern and one or more short-turn patterns. Each pattern operates at regular intervals, and each short-turn pattern is entirely overlapped by the next longer pattern. All patterns are assumed to cover the peak volume segment of the route. A common configuration is for all the patterns to have the same central business district (CBD) terminus and to have turnback points at different distances from the CBD. However, our framework permits turnbacks at both ends of the corridor, and is therefore applicable to crosstown and through routes, as well as radial routes. A top priority is to minimize fleet size to serve a given demand. Level of service also plays a role at a higher level of design. In this paper terminology appropriate to bus systems will be used; however, the analysis can be applied to rail transit systems as well because the strategy does not rely on overtaking.

Northeastern University, 420 Snell Engineering Center, Boston, Mass. 02115.

SCHEDULE COORDINATION MODES FOR SHORT TURNING

Consider a route with a full-length pattern and a single short-turn pattern. Passengers whose trips lie entirely within the inner zone of the corridor (the portion served by the short-turn pattern) can use either pattern and constitute the choice market. Other passengers, who have at least one trip end outside the inner zone, can use only the full-length pattern and constitute the full-length market. The two patterns compete for choice market patrons who will use the first bus that comes along unless it is overcrowded, a condition that the design described here aims to avoid. Thus, within the inner zone, the load on any trip (of either pattern) will depend on the elapsed time since the previous trip (of either pattern), therefore, the schedules of the two patterns must be coordinated in order for the loads to be regular.

When there is a single short-turn pattern, this need for coordination can most easily be accomplished if the two service patterns operate with the same frequency. The two patterns will then alternate in serving the inner zone, and the schedule offset between the patterns will determine how much of the choice market will be carried on each pattern. For example, if the offset is one-half of the headway, then each pattern will carry one-half of the choice market. However, such a schedule will lead to unequal loads. This is most easily seen in the inbound direction: vehicles serving the full-length pattern will arrive at the turnback point already partially loaded, while vehicles serving the short-turn pattern will begin there empty. Unless the full-length pattern uses larger vehicles than the short-turn pattern, balancing loads between the two patterns requires that a short-turn trip lead each full-length trip by a small fraction of the full-length pattern's headway, so that the short-turn trips capture most of the choice market. An example of such a schedule (inbound) would be for full-length trips to pass the turnback point at times 8, 16, 24 min, and so on, while the short-turn trips leave the turnback point at times 6, 14, 22 min, and so on.

Schedule coordination is especially needed in the outbound direction on radial routes. Without proper schedule coordination, choice market passengers can overcrowd the full-length trips. Almost all passengers desiring to board after the point of overcrowding will be forced to use the short-turn pattern. Those traveling to the outer zone will then have to transfer at the turnback point to the next full-length trip. This problem of "induced transfers" is discussed by Wilson et al. (*3*). Without proper schedule coordination in the inbound direction, choice passengers will again overcrowd the full-length trips, but this will not occur until the bus reaches the inner zone; therefore, passengers who are turned away can use the next short-turn trip without difficulty. However, if the full-length pattern extends beyond the short-turn pattern on both sides, then induced transfers can occur in both directions. Although some transit systems, notably subway systems, may tolerate such unbalanced loads and induced transfers, most transit systems want to avoid them by designing the schedule so that passengers can board the first bus serving their destination without causing overcrowding.

Effective schedule coordination is achieved in a multipattern system when each trip (save trips on the shortest pattern) follows a trip of the next shorter pattern, so that each pattern's frequency is a multiple of the next longer pattern's frequency. Let pattern p be defined as the pth longest pattern ($p = 1,...,P$), and let zone p be the portion of the route covered by patterns $1,...,p$ only. The "scheduling mode" may then be expressed as $1{:}r_2{:}...{:}r_P$, where r_p is the relative frequency of pattern p, and where r_p is a multiple of r_{p-1} for $p > 1$. An example of a 1:2 schedule is for full-length trips to pass the turnback point every 7 min, while short-turn trips leave the turnback point at 3 and 6 min after each full-length trip. Then each full-length trip will carry 7 min of the full-length market and 1 min of the choice market, while each short-turn trip carries 3 min of the choice market only.

An important characteristic of a mode is the relative (vehicular) trip volume in each zone, given by

$$T(p) = \sum_{i=1}^{p} r_i \tag{1}$$

CAPACITY

The operation of competing patterns within the same corridor suggests a special treatment of capacity constraints. In the customary style of deterministic analysis, randomness in both passenger and vehicle arrival patterns is accounted for through the use of design load factors (maximum allowable expected occupancy at any point as a fraction of a vehicle's nominal capacity), which are set low enough to prevent overcrowding most of the time. In short-turn systems, different parts of the system are affected by randomness in different ways; therefore, it seems reasonable to use different load factors accordingly. For example, in a 1:1 mode, overcrowding of full-length trips has more severe consequences for excluded passengers than overcrowding of the short-turn pattern, especially in the outbound direction. This suggests that the design load factor of the full-length pattern should be less than that of the short-turn pattern.

Given the scheduling mode and vehicle design capacities of each pattern, the aggregate passenger carrying capacity per full length pattern headway in zone p is given by

$$C(p) = \sum_{i=1}^{p} r_i k_i \tag{2}$$

where k_i is the design capacity of vehicles on pattern i, reflecting both vehicle size and the design load factor.

DESIGN FRAMEWORK

The decisions to be made in finding the best short-turn pattern include the schedule coordination mode, location of turnback points, the vehicle sizes, full-length pattern headway (h), and the peak direction offsets for the $T(P) - 1$ short-turn trips that repeat every interval h. (Reverse direction offsets are ignored because they can usually be scheduled independently of the peak direction offsets by appropriate distribution of layover time between the route endpoints.) Because of the limited number, feasible coordination modes, turnback points, and vehicle sizes will be considered as exogenous parameters in this paper, leaving h and the offsets as the decision variables in the optimization problem. Before formulating the optimization problem, the next section offers guidance on the choice of the exogenous parameters. By varying these parameters, a full range of designs can be generated and compared to complete the design process.

The primary objective is to minimize fleet size. Given the schedule coordination mode, the location of the turnback points, and the vehicle capacities, this means h has to be maximized. The main passenger impact of short-turn design is on waiting time because in-vehicle time is only slightly affected, if at all. Therefore, the secondary objective is to minimize wait time (equivalent to minimizing h) for a given fleet size. If a combined objective of operator and passenger cost is desired, it is a simple matter to parametrically vary h (as well as the other parameters) and to calculate the cost and travel time impacts to obtain an optimal trade-off.

SCREENING FOR DESIGN PARAMETERS

The ideal behind short turning is to match the provided capacity with the demand. An obvious constraint is that the total provided capacity in any zone must exceed the peak passenger volume in that zone; that is,

$$C(p)/h \geq V(p) \qquad \text{for } p = 1,...,P \tag{3}$$

where $Vs(p)$ equals peak passenger volume in zone p.

Given a choice of scheduling mode, vehicle capacities, and location of turnback points, Equation 3 provides an upper bound on h as shown by

$$h \leq \min_{p} \frac{C(p)}{V(p)} \tag{4}$$

Choice of Schedule Coordination Mode

One indication of the efficiency of a design is E, which is the relative excess capacity at the peak point as shown by

$$E = \frac{C(P)}{hV^*} - 1 \tag{5}$$

where V^* is the peak point volume. Large values of E are inefficient, indicating wasted capacity at the peak point. Transit agencies usually have a maximum allowable headway, h_{max}, that must be maintained by the full length pattern (because it alone serves the outermost zone). Substituting h_{max} for h in Equation 5 yields the minimum attainable value of E for a particular choice of mode and vehicle capacity. Configurations with a minimum attainable E that is above some threshold E_{max} can be screened out as inefficient. This device can be used to limit the number of modes to be considered. If $\bar{k} = C(P)/T(P)$ equals the average vehicle design capacity, then requiring that $E \leq E_{max}$ implies that

$$T(P) \leq \frac{(1+E_{max})\ V^*\ h_{max}}{\bar{k}} \tag{6}$$

In a heavy demand corridor, h_{max} is typically 12 min maximum, and peak volume is rarely more than 1 vehicle-load per 2.5 min. Considering these worst case values with $E_{max} = 0.15$, Equation 6 yields $T(P) < 5.5$. There are only 11 scheduling modes that meet this requirement: 1:1, 1:2, 1:3, 1:4, 1:1:1, 1:1:2, 1:1:3, 1:2:2, 1:1:1:1, 1:1:1:2, and 1:1:1:1:1. With lighter passenger volumes, the upper limit for $T(P)$ will often be 2, 3, or 4, for which there are only 1, 3, and 6 possible modes, respectively. Thus, the choice of scheduling mode is quite restricted.

Choice of Turnback Point

Equation 3 can serve as a guideline for selecting turnback points, given the scheduling mode and vehicle capacities. By first applying Equation 3 to the (still undefined) innermost zone (zone P), the right hand side is V^*, and an upper bound on h is obtained. If the headway must be in whole minutes, h should be rounded down to the next whole minute. Then, given h, Equation 3 provides upper bounds on the peak volume of each zone. The outermost stop j at which the volume profile in either direction exceeds the volume upper bound for zone p is an inner bound location for the turnback point of pattern p, in the sense that the turnback point may be no closer to the peak volume point. As a first guess for an efficient design, the turnback points at their respective inner bounds are located. If analysis of the resulting configuration (described in later sections) proves it to be infeasible with the given h, turnback point locations can be moved farther out, or h can be lowered, resulting in a new set of inner bound locations.

The process of choosing turnback inner bound locations can be compared to choosing the locations where a freeway should add and drop lanes in response to volume changes. This approach is satisfactory with freeways because of the ability of vehicles on a multilane freeway to transfer without penalty between lanes. However, on a multipattern transit route, transfers between patterns are highly undesirable. Therefore, simply comparing overall capacity with overall volume is inadequate for transit design; instead, each pattern must be analyzed individually in the light of passenger behavior.

ANALYSIS OF 1:1 SCHEDULING MODE

A complete analysis will be given for the simplest short-turn system, one with a 1:1 schedule coordination mode. The turnback point is given. The analysis relies on a stop-level O-D matrix that can be measured directly using a survey, or that can be estimated using methods described by Simon and Furth (*4*) and Ben-Akiva et al. (*5*).

From the O-D matrix, the volume profile can be constructed for each direction. The peak volume, V^*, occurs at the point PVP^*. The O-D matrix is then partitioned into Market 1, the full-length market that contains O-D pairs whose outermost zone is zone 1, and Market 2, the choice market; volume profiles of each market are constructed separately. The variables used in this analysis are given as follows:

- Pattern p = pth longest pattern;
- Zone p = portion of route served only by patterns $1,\ldots,p$;
- Market p = portion of the route O-D matrix served by patterns $1,\ldots,p$;
- c_p = cycle time for pattern p (including recovery time);
- $C(p)$ = aggregate capacity per interval h in zone p;
- f = offset = interval between a full-length trip and the preceding short-turn trip;
- h = full-length pattern headway;
- J_p = set of (stop-to-stop) segments in zone p;
- k_i = design vehicle capacity for pattern i;
- $\bar{k}$ = mean design vehicle capacity;
- M_p = size of market p (peak direction);
- P = number of patterns, index of innermost (shortest) pattern;
- PVP_p = peak volume point for market p (peak direction);
- PVP^* = peak volume point for combined markets;
- q = frequency of full-length pattern = $1/h$;
- r_p = number of pattern p trips per interval h;
- R = capacity ratio = k_2/k_1;
- $T(p)$ = number of trips in zone p per interval h;
- v_{pj} = market p volume in segment j (peak direction);
- v_p = market p peak volume;
- $V(p)$ = peak volume in zone p (combined markets);
- V^* = volume at peak point (combined markets);
- w = overall average wait time, peak direction; and
- z = relative offset = f/h.

There are two trips that repeat every interval h: a full-length trip and a short-turn trip. In the outer zone, only the full-length trip operates, and its loading constraint is

$$v_1 h \leq k_1 \tag{7}$$

In the inner zone, loading constraints apply to both trips. It is assumed that on average, both patterns travel at the same speed over the same segment, which implies that the schedule offset is the same at every stop in the inner zone. (See the section on other practical considerations.) Because the short-turn trip carries only choice passengers, its peak load point will be PVP_2 regardless of the offset, therefore, the loading constraint is

$$v_2(1-z)h \leq k_2 \tag{8}$$

However, the peak volume point of full-length trips is a function of z because z determines the share of choice market using that trip. For example, if $z = 1$, its peak volume point will not be in the inner zone; if z=1, its peak volume point will be *PVP**. Therefore every stop in the inner zone must be considered:

$$(v_{1j} + zv_{2j})h \le k_1 \qquad \text{for all } j \in J_2 \tag{9}$$

By replacing h with its reciprocal q, the problem may be stated thus:

$$\min q$$

$$\text{s.t. } q \ge v_1/k_1 \tag{10}$$

$$q \ge (1-z)v_2/k_2 \tag{11}$$

$$q \ge (v_{1j} + zv_{2j})/k_1 \qquad \text{for all } j \in J_2 \tag{12}$$

$$0 \le z \le 1 \tag{13}$$

The constraints are shown in Figure 1. This linear optimization in q and z is easily solved. For a particular segment j, let $z(j)$ denote the value of z at the intersection of Equation 11 and

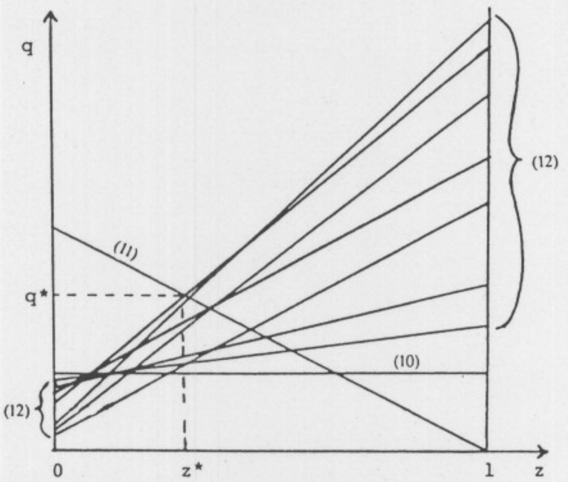

FIGURE 1 Constraints for 1:1 mode.

Equation 12; $z(j)$ is thus the offset that balances the load in segment j:

$$z(j) = \frac{Rv_2 - v_{1j}}{Rv_2 + v_{2j}} \tag{14}$$

where the capacity ratio $R = k_2/k_1$. Let j^* be the segment with the smallest $z(j)$. Then the optimal z is $z^* = max[z(j^*), 0]$. The optimal q, q^*, is then the smallest bound given by Equations 10 and 11. However, when Equation 10 is binding, the design will generally prove inefficient, and the short-turn pattern should probably be extended.

The average wait time in the peak direction, w, is given by

$$w = \frac{h}{2}\left\{M_1 + M_2\,[z^2 + (1-z)^2]\right\} \Big/ (M_1 + M_2) \tag{15}$$

INCORPORATING WHOLE-MINUTE SCHEDULING CONSTRAINTS

Thus far, the analysis has treated frequency and offset as continuous variables. However, in practical terms departures usually must be scheduled in minutes, or, in a few systems, in one-half minutes, yielding a discrete set of acceptable headways and offsets. If $1/q^*$ does not belong to the set of acceptable headways, it must be rounded down. Let h be the rounded headway, and then let $q = 1/h$. [It is also possible to begin the design procedure here with h as the (rounded down) upper bound provided by Equation 4.] Now the problem is to find, for the given q, an acceptable offset that is feasible with respect to the constraints represented in Equations 11 and 12. Equation 11 yields a lower bound on the relative offset, z_{low}, and Equation 12 yields a family of upper bounds, z_{up} (j):

$$z_{low} = 1 - \frac{k_2 q}{v_2} \tag{16}$$

$$z_{up}(j) = \frac{qk_1 - v_{1j}}{v_{2j}} \tag{17}$$

Let $\hat{j}$ be the segment with the smallest $z_{up}(j)$ (it is likely, but not necessary, that j^* and $\hat{j}$ will be the same), then the range of feasible offsets is

$$z_{low}h \le f \le z_{up}(\hat{j})h \tag{18}$$

If there is more than one acceptable offset in this range, the value closest to z^*h will best balance the loads. However, if there is no acceptable offset in this range, then h must be lowered to its next acceptable value, possibly increasing the fleet size. This will in turn enlarge the range for the offset, making it very likely that a feasible offset can be found without lowering the headway a second time. Alternatively, the turnback point or points can be moved farther out, widening the range for f; this may or may not increase operating cost, depending on whether additional vehicles are needed to cover the extra distance.

When a solution is found, the next lower value of h should be examined to see whether it leaves the fleet size unchanged and yields an offset range that contains a feasible offset. If so, passenger waiting time can be lowered at very little cost. Also, as pointed out by Ceder (2), consideration should be given as to whether the short-turn pattern can be extended without increasing the fleet size. Extending the pattern will tend to make the offset range broader, but will also tend to make it rise; therefore, it is necessary to check that it still contains an acceptable offset.

DEADHEADING AND INTERLINING TO REDUCE FLEET SIZE

During peak periods, the reverse direction passenger volumes are often small enough that they can be served by the full-length trips only or by some other subset of the trips. In such a case, the short-turn patterns not needed can deadhead. Deadheading reduces the cycle time but otherwise leaves the analysis unchanged.

Another way to reduce operating costs when the patterns share a common terminus is to interline patterns. (The interlining analysis is the same whether the short-turn cycle involves deadheading or not. Interlining with routes outside the short-turn system can also be done, but this is beyond the scope of the paper.) If the two patterns are operated without interlining, the required fleet size is $<c_1/h>^+ + <c_2/h>^+$, which can be expressed as

$$Int(c_1/h) + Int(c_2/h) + <m_1>^+ + <m_2>^+ \tag{19}$$

where $m_p = mod(c_p/h)$, $Int(x)$ and $mod(x)$ are the integer and fraction portions of x, respectively; and $<>^+$ indicates rounding up to the next whole number.

If two routes or patterns are interlined with no restriction on the offset, then the minimum length of the composite cycle is $(c_1 + c_2)$. Therefore, the required fleet size, when the offset is unrestricted, is $<(c_1 + c_2)/h>^+$, which can be expressed as

$$Int(c_1/h) + Int(c_2/h) + <m_1 + m_2>^+ \tag{20}$$

However, the need to balance loads in a short-turn system restricts the offset to a single value or a narrow range of values. An interlined schedule with a fixed offset is shown by the time-space diagram in Figure 2. Both the full-length and the short-turn pattern have departures at every headway h. The time period illustrated is the evening peak; accordingly, short-turn departures from a common CBD terminus lead full-length departures by the offset f.

As the diagram shows, the minimum allowed cycle time is $(c_1 + c_2 + s_2)$, where s_2 equals the wait time required at the common terminus after completion of a short-turn cycle. Therefore, the fleet size required when interlining with a fixed offset equals the following

$$Int(c_1/h) + Int(c_2/h) + <m_1 + m_2 + s_2>^+ \tag{21}$$

As shown in Figure 2, s_2 must be between 0 and h, and must satisfy the equation $c_2 + s_2 = f + Ih$, where I is an integer. The solution is

$$s_2 = def_h(c_2 - f) \tag{22}$$

where $def_h(y)$ is the amount needed to round up y to a multiple of h. (For example, $def_4(17) = 3$.)

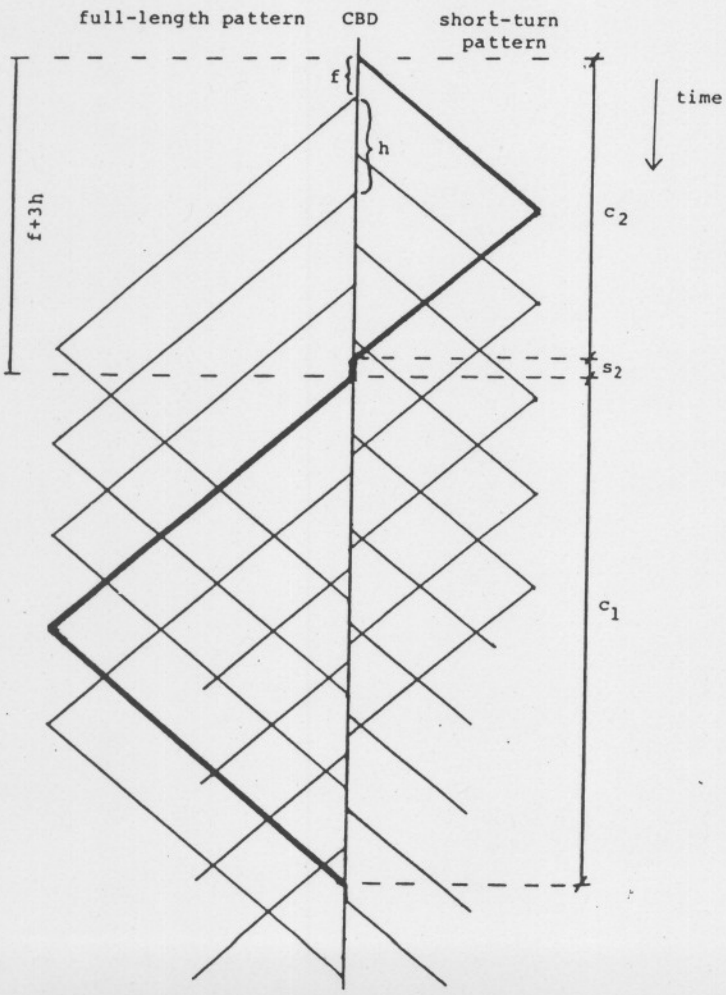

FIGURE 2 Interlining cycle for 1 : 1 mode.

Because it requires the insertion of slack time s_2 into the composite cycle, fixing the offset reduces the potential of interlining for saving vehicles. Consider the case of two randomly chosen routes with a common terminus and headway and independent cycle times. If the cycle times are considered as continuous random variables, then m_1 and m_2 are independent and uniformly distributed on the interval (0,1). If the offset is unrestricted, then, comparing Equations 19 and 20, interlining can save a bus if

$$<m_1 + m_2>^+ < <m_1>^+ + <m_2>^+ = 2 \tag{23}$$

This condition is met if $(m_1 + m_2) \leq 1$; the probability is

$$\int_0^1 \int_0^{1-x} dydx = ½ \tag{24}$$

However, if the routes must maintain an exogenously determined offset *f*, which will also be treated as a continuous and independent random variable, then s_2 is also uniformly distributed between 0 and 1 and is independent of m_1 and m_2. In this case, a bus can be saved by interlining if $<m_1 + m_2 + s_2>^+ \leq 1$; the probability is

$$P[m_1 + m_2 + s_2 < 1] = \int_0^1 \int_0^{1-x-y} dzdydx = 1/6 \tag{25}$$

Thus, the presence of an exogenously determined offset greatly reduces the chances that interlining will save a bus. In fact, interlining can require an extra bus if $[<m_1 + m_2 + s_2^+] \geq 2$. (The probability of this occurrence is also 1/6.) If there is some flexibility in the choice of offset, the probability of saving a bus will increase.

OTHER PRACTICAL CONSIDERATIONS

A special case that can emerge from the design is for the relative offset to be zero, implying that the full-length pattern should not carry any of the choice market. This would theoretically be accomplished by having full-length trips immediately follow short-turn trips. This arrangement is impractical for many reasons that include the need for full-length trips to make all the stops that the short-turn trips make in order to avoid overtaking them. A more practical way of keeping choice passengers off full-length trips is for full-length trips to simply prohibit boarding within the inner zone in the inbound direction, and similarly to prohibit alighting within this zone in the outbound direction. With this policy, described by Furth and Day (*1*) as "restricted zonal service," passengers no longer have a choice of patterns, eliminating the need for coordinating schedules; therefore, the two patterns may be scheduled with different headways, which can lead to further efficiencies. Design methods for this strategy are discussed by Furth (*6*).

One of the assumptions of this analysis is that the offset will be the same throughout the inner zone. (Of course, there will be random variations, which are accounted for in deterministic analysis by the design load factor. This paragraph is concerned with systematic changes in offset.) However, in the inbound direction, because the offset is generally smaller than one-half of the headway, full-length trips will generally make fewer stops than short-turn trips in much of the inner zone; therefore, full-length trips will tend to catch up with the leading short-turn trip. This can be modeled by treating the expected offset at each stop as a function of both the initial offset and the demand profile; however, such precision seems unwarranted. A sufficient adjustment for the inbound schedule might be to consider *f* as the average desired offset, and to make the initial offset slightly longer. In the outbound direction, there is less of a tendency for short-turning to cause bunching because the schedule will be constructed in such a way that all vehicles will pick up approximately one busload of passengers in the CBD, the primary collection area.

Experience in the transit industry indicates that proper supervision is necessary for the successful implementation of a short-turn strategy. Without supervision, the driver of a short-turn trip, who is scheduled to lead a full-length trip, might purposefully follow the full-length trip instead and carry a very light load while causing overcrowding on the full-length trip. Offsets in the outbound direction are the most critical, for reasons discussed earlier; fortunately, these are usually the easiest to enforce because there are usually dispatchers at the downtown terminus. However, if street traffic is so heavy and headways so small that bunching cannot be prevented, a routing strategy that does not depend on the schedule offset should be used, such as restricted zonal service.

The short-turn strategy lends itself well to a distance-based fare structure. Because people making interzonal trips must use the full-length pattern, a higher fare can be charged on the longer pattern. To avoid penalizing those whose entire trip lies in the outer zone, fares for outbound boardings in the outer zone could be reduced. Such a policy will certainly affect the choice of inner-zone passengers. Those who would prefer to wait for a short-turn trip rather than pay the fare differential effectively leave the choice market and become a third market called the short-turn market. If the fraction diverted to the short-turn market is a constant, *d*, for all O-D pairs in the inner zone, then the variable *z* in Equations 8, 9, 14, 16, and 17 should be replaced with $z(1 - d)$. If different O-D pairs have different diversion factors, the loading constraints should be modified to explicitly account for three different markets (the full-length market, the short-turn market, and the net choice market), each with its own volume profile.

This analysis assumes demand rates and run times constant over a period of time of roughly 90 min or more. If, instead, they are variable, the closed-form solutions no longer apply, and fleet size must be determined from a more general approach, such as Salzborn's (*7*) or Ceder's and Stern's (*8*). However, the need for schedule coordination remains, and this need, along with the restriction (to avoid passenger confusion) that turnback locations remain constant, greatly restricts the search for an optimal schedule.

DESIGN FOR OTHER SCHEDULE COORDINATION MODES

Design for other scheduling modes follows the same procedures as for the design of the 1:1 mode. Steps for this design are summarized as follows:

1. The material on screening provides guidance on the initial choice of turnback points and coordination mode. An upper bound on h is also given.

2. Partition the route O-D matrix into P markets, where market p is that portion of the O-D matrix with an outermost zone p.

3. There are T^* trips that repeat every interval h. Their sequence is determined by the basic strategy of short turning: for $p<P$, a pattern p trip must follow a pattern $p+1$ trip. Index the trips according to the order they pass the peak point, with the full-length (Pattern 1) trip as trip T^*. Trip 1 will always be a pattern P trip. Then define the relative offset for trip t as z_t, which equals the fraction of h by which trip t follows the preceding trip in the peak direction. Construct the loading constraint or family of loading constraints for each trip in each zone that the trip operates, considering the reverse direction as well. These constraints include as unknowns h and z_t for $t=1,\ldots,T^*$. (z in the fifth section of the paper is what is referred to as z_2 in this section.) These constraints are easily constructed because the sequence of trips is known. Replace h with its reciprocal q, and add the constraints $\sum z_t = 1$ and $z_t \geq 0$ for all t.

4. Solve the problem (linear in q and z_t) of minimizing q subject to the constraints of Step 3, yielding an upper bound for $h = 1/q$. (Alternatively, fix h at the upper bound resulting from screening.) Round h down, if required, by a whole minute constraint, and solve the loading constraints for offset upper bounds. Round down these upper bounds in accordance with any integer constraints, and then sum them. If the sum equals or exceeds h, there is a feasible solution. If not, lower h and repeat, or change modes or turnback points.

5. For complex modes, the number of theoretically possible deadheading and interlining options can be very large. However, the number of interesting options will usually be small enough for each to be analyzed.

This approach will be illustrated with the 1:3 mode. As with the 1:1 mode, there are 2 patterns, therefore, the O-D matrix is partitioned into two markets. There are four trips that repeat every interval h. Trips 1, 2, and 3 are short-turn trips, and Trip 4 is the full-length trip. The loading constraints are as follows. For Trip 4 in the outer zone

$$v_1 h \leq k_F \tag{26}$$

For Trips 1,..,3 in the inner zone

$$v_2 h z_t \leq k_2 \qquad \text{for } t = 1,\ldots,3 \tag{27}$$

For Trip 4 in the inner zone

$$(v_{1j} + z_4 v_{2j}) h \leq k_1 \qquad \text{for all } j \in J_2 \tag{28}$$

The direct constraints on the offsets are

$$z_1 + z_2 + z_3 + z_4 = 1 \tag{29}$$

$$z_t \geq 0 \qquad \text{for } t = 1,\ldots,4 \tag{30}$$

With a little manipulation, these constraints are all linear in $q(=1/h)$ and z_t.

Due to symmetry, in the unrounded solution: $z_1 = z_2 = z_3 = (1-z_4)/3$; therefore, the problem may be cast in terms of two variables, z_4 and q. The reason for distinguishing the offsets of multiple trips of the same pattern is that they may differ in the rounded solution. For example, suppose that after solving for h and rounding down, h equals 12 min, the upper bound for z_1, z_2, and z_3 equals 4 min and the upper bound for z_4 equals 2 min. Because the sum of these upper bounds exceeds 12, there are several solutions including those in which z_1, z_2 and z_3 are not all equal.

EXAMPLE

The inbound O-D matrix for a hypothetical 20-stop route is given in Figure 3. Figure 4a shows the volume profile derived from this O-D matrix. The minimum cycle times for routing patterns beginning at selected points and ending at the CBD (Stop 20) are shown as follows:

Turnback Point	*Cycle Time (min)*
1	84
2	76
3	72
4	66
5	62
6	58
7	54
8	50
9	46

The nominal vehicle capacity is 60 for all patterns, and headways must be in whole minutes. Without short turning, the peak volume of 580 passengers per hour requires 6-min headways and a fleet of $\langle 84/6 \rangle^+ = 14$ buses.

In this example, only the 1:1 mode will be analyzed. The screening process for choosing a turnback point will be demonstrated first. Given the equal vehicle design capacities of 60, the zonal capacities are $C(1) = 60$ and $C(2) = 120$. Applying Equation 3 to the Zone 2, the upper bound for h is 120 passengers divided by 580 passengers per hour, equalling 12.4 min, which rounds down to $h = 12$ min. Next, application of Equation 3 to Zone 1 yields the upper bound $V(j) \leq$ (60 passengers)/(12 min) = 300 passengers/hr. The outermost stop where the volume exceeds this limit is Stop 9; therefore, Stop 9 is the innermost stop that will be considered as a turnback point.

This simple solution provides enough aggregate capacity to meet the aggregate demand at every point and requires only $\langle 84/12 \rangle + \langle 46/12 \rangle^+ = 7 + 4 = 11$ buses. However, as this example will demonstrate, this solution is not feasible when loading constraints on the individual patterns are considered.

The O-D matrix is partitioned into the choice and full-length markets, as shown in Figure 3. From the row and column totals of the two resulting submatrices, volume profiles for the two markets are constructed in Table 1. The computation of $z(j)$ is illustrated in Table 1. The minimum value is $z(j^*) = 0.226$, where j^* is Stop 11. The meaning of $z(j^*)$ is shown by Figure 4. In Figure 4a the volume profiles for the two markets are shown. The peak volume of the short-turn market (420 per hour) is

TABLE 1 VOLUME PROFILES AND RELATIVE OFFSETS

	Stop																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Full-Length Market																				
On	70	30	40	40	30	40	30	50	–	–	–	–	–	–	–	–	–	–	–	–
Off	–	–	–	–	10	10	–	20	10	–	10	25	25	20	20	20	20	30	20	90
v_{1j}	70	100	140	180	200	230	260	290	280	280	270	245	220	200	180	160	140	110	90	0
Choice Market																				
On									100	70	80	60	70	70	70	50	60	30	–	–
Off									–	–	5	15	30	30	40	40	50	80	70	300
v_{2j}									100	170	245	290	330	370	400	410	420[a]	370	300	0
Total volume	70	100	140	180	200	230	260	290	380	450	515	535	550	570	580	570	560	480	390	0
$z(j)$									0.269	0.237	0.226	0.246	0.267	0.278	0.292	0.313	0.333			
$h = 11$																				
$z_{up}(j)$									–[b]	–[b]	0.234[b]	–[b]	–[b]	–[b]	–[b]	–[b]	–[b]			
$h = 10$																				
$z_{up}(j)$									0.80	0.47	0.367	0.397	0.42	0.43	0.45	0.49	0.52			

[a] $420 = v_2^*$.
[b] Superscript $^b \geq z(j)$ and therefore > 0.234.

From \ To	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	TOTAL ON
1	6	5	-	4	3	-	2	7	6	5	3	3	4	5	3	14	70
2	4	5	-	6	1	-	1	2	2	-	1	1	-	1	1	5	30
3				5	2	-	2	3	3	3	3	3	2	3	2	9	40
4				5	2	-	2	3	3	2	3	2	3	3	3	9	40
5					2	-	1	2	3	2	2	2	2	4	2	8	30
6							2	3	3	3	3	3	2	3	2	16	40
7							-	2	2	2	2	2	2	4	3	11	30
8							-	3	3	3	3	4	5	7	4	18	50
Subtotal off stops 1-8	10	10	-	20	10	-	10	25	25	20	20	20	20	30	20	90	330
From																	TOTAL ON
9							5	15	11	8	7	6	6	9	8	25	100
10									10	8	8	7	5	6	5	21	70
11									9	7	9	7	5	9	7	27	80
12										7	7	5	5	7	5	24	60
13											9	7	10	9	7	28	70
14												8	10	15	7	30	70
15													9	15	10	36	70
16														10	10	30	50
17															11	49	60
18																30	30
Subtotal off stops 9-18	-	-	-	-	-	-	5	15	30	30	40	40	50	80	70	300	660
TOTAL OFF	10	10	-	20	10	-	15	40	55	50	60	60	70	110	90	390	990

FIGURE 3 Example of an O-D matrix.

much higher than that of the full-length market (290 per hour). In Figure 4b, 22.6 percent of the choice market is added to the full-length market to become the volume profile for the full-length pattern, while the short-turn pattern's profile represents 77.4 percent of the choice market. Both profiles now have the same peak volume, albeit at different points. This peak volume of 325 per hour calls for a headway of 11.07 min, which rounds down to $h = 11$ min.

Because the headway was rounded down by such a small amount, a small offset range is expected. z_{low} is found to be 0.221. In Table 1, $z_{up}(j)$ is calculated for every inner zone stop; the lowest value, 0.234, governs. (Because $z_{up}(j)$ is known to be greater or equal to $z(j)$ some of these calculations become unnecessary.) By multiplying these bounds by $h = 11$, the offset range is 2.43 to 2.57 min.

Suppose that a half-minute offset is acceptable. Then $f = 2.5$ min is chosen as the offset, and fleet size can now be calculated. Without interlining, the fleet size is $<84/11>^+ + <46/11>^+ = <7.64>^+ + <4.18>^+ = 8 + 5 = 13$. Next, considering interlining, calculate $s_1 = def_{11}(46 - 2.5) = 0.5$ so the fleet size needed with interlining is $<(84 + 46 + 0.5)/11>^+ = <11.86>^+ = 12$.

However, if whole-minute offsets are required, there are two choices: lower the headway to 10 min, or move the turnback point farther out. If $h = 10$ (with the same turnback point), $z_{low} = 0.143$ and the lowest $z_{up}(j)$, calculated in Table 1, is 0.367. Multiplying by h, the offset range is 1.43 to 3.67 min, with two integer solutions possible: f = 2 or 3 min. The number of buses needed without interlining is $<84/10>^+ + <46/10>^+ = 14$. Taking $f = 2$, $s_1 = 0$ is obtained, and the number of buses needed with interlining is $<(84 + 46 + 0)/10>^+ = 13$.

An alternative to lowering the headway is to move the turnback point farther back. Using an electronic spreadsheet

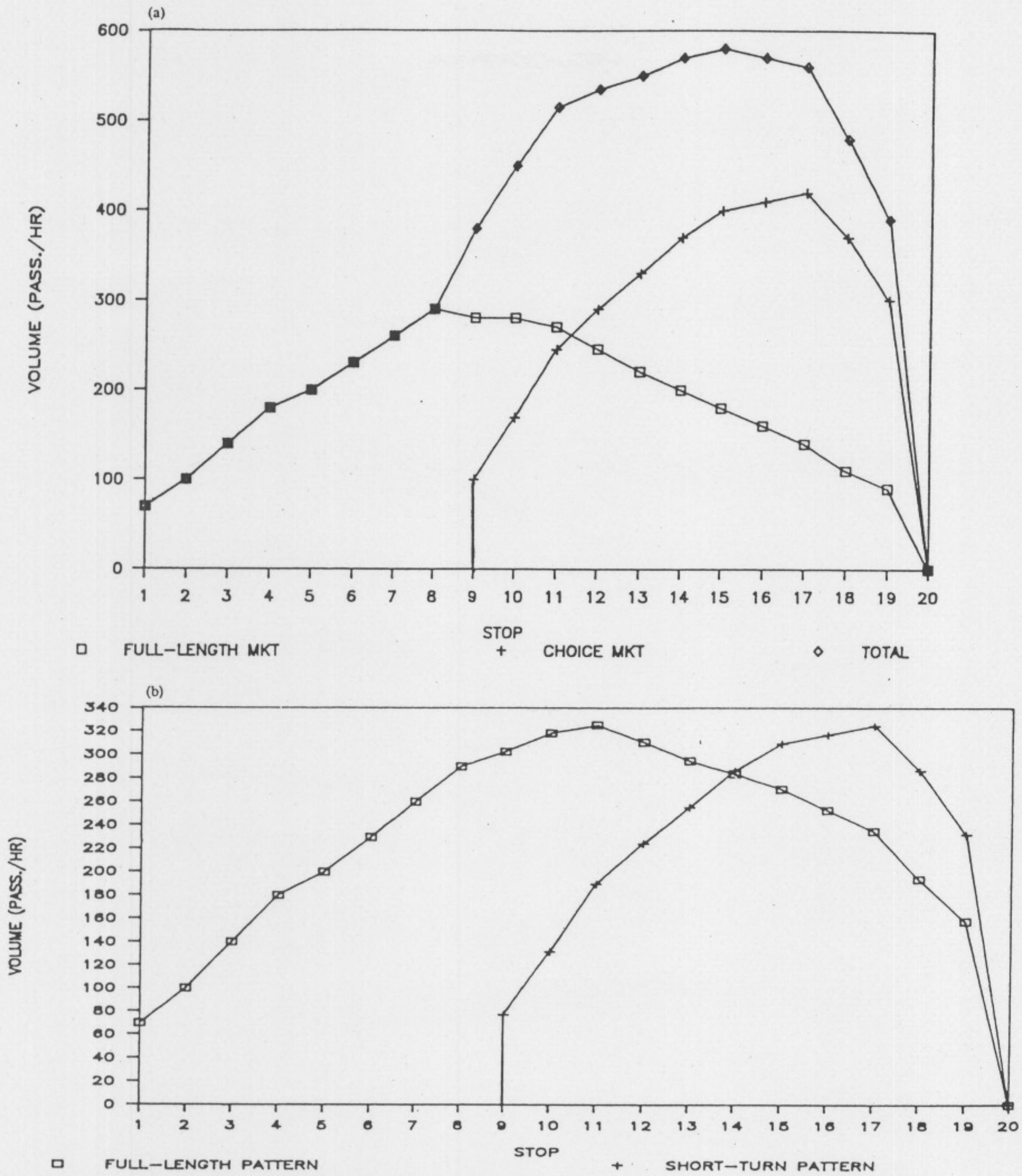

FIGURE 4 Volume profiles: (a) market profiles, and (b) pattern profiles with 22.6 percent offset.

makes the calculations for alternative turnback points easy to perform. As stated earlier, as the turnback point is moved outward, the relative offset range both rises and widens. In addition, a greater headway may become feasible (as it is in this case because a 12-min headway is feasible if the turnback is extended to Stop 6).

In Table 2 the offset range, fleet size, and average waiting time for $h = 10$, 11, and 12 min are shown as the turnback point is extended back from Stop 9. Where the offset range includes one or more whole-minute offsets, the required fleet size is shown. An (*i*) next to the given fleet size indicates where an interlining has saved a bus. Observe that not only is the "naive solution" with its 11-bus fleet infeasible, but the 12-bus solution is also infeasible when whole-minute offsets are required. Among the many solutions requiring 13 buses, the lowest wait-time solution has a turnback at Stop 6 with $h = 11$ min. For an examination of a trade-off between fleet size and wait time, the superior solutions (minimum wait time for a given fleet) are indicated. It is also interesting to note that of the 16 short-turning solutions, six show a savings through interlining, and of these, only 3 would show a savings (if there were no flexibility in the choice of offset), in close agreement with the predicted

TABLE 2 OFFSET RANGE AND FLEET SIZE FOR DIFFERENT HEADWAYS AND TURNBACK POINTS

Turn-back Stop	z	$h = 10$ min				$h = 11$ min				$h = 12$ min			
		f_{low}	f_{up}	N	$\bar{w}$	f_{low}	f_{up}	N	$\bar{w}$	f_{low}	f_{up}	N	$\bar{w}$
9	0.226	1.5	3.6	13(i)	3.8	2.3	2.5	—[a]	4.2	—	—	—	—
8	0.308	2.0	4.7	14	3.5	3.0	4.0	13	3.8	—	—	—	—
7	0.350	2.3	5.1	14(i)	3.3	3.3	4.6	13	3.6	—	—	—	—
6	0.395	2.7	5.4	15	3.1	3.7	5.2	13(i)	3.4[b]	4.7	4.9	—[a]	3.7
5	0.423	2.9	5.6	15(i)	3.0	3.9	5.4	14	3.3	4.9	5.3	13	3.6
4	0.447	3.2	5.8	16	2.9	4.2	5.6	14	3.2	5.2	5.5	—	—
3	0.466	3.4	5.9	16(i)	2.8	4.4	5.8	15	3.0	5.4	5.8	—	—
2	0.474	3.5	6.0	16(i)	2.7[b]	4.5	5.9	15	3.0	5.5	5.9	—	—
1	0.5	3.8	6.2	17(i)	2.5[b]	4.8	6.2	16	2.8	5.8	6.2	14	3.0[b]

Note: h = headway; f_{low}, f_{up} = lower, upper bound of offset range (min); N = fleet size required; z = balancing relative offset; and $\bar{w}$ = average wait time (min). Also (i) means interlining saved a vehicle.

[a]N=12 if noninteger offset is allowed.

[b]Superior solutions (minimum wait time for a given fleet size are given for an examination of trade off between fleet size and wait time.

1-in-6 average. This example illustrates how the flexibility afforded by the offset range increases the chances that interlining will save a bus.

The final bus savings in this example was not very large, as the example was meant to illustrate some of the concepts behind short-turn design rather than demonstrate the strategy's value for saving vehicles. Examples showing remarkable vehicle savings can easily be constructed and are probably unnecessary because the value of the strategy is well proven in transit systems across the nation. In a study of a Los Angeles short-turning bus route described by Furth et al. (*9*), the route would need 35 vehicles without short-turning, but need only 26 in the 1:1 mode configuration currently operated. Application of the procedures described in this paper yielded a more efficient 1:1 configuration requiring only 24 buses. The 1:2 mode was also examined, and the best configuration with that mode required 27 buses.

ACKNOWLEDGMENTS

This research was supported in part by the U.S. Department of Transportation.

REFERENCES

1. P. G. Furth and F. B. Day. Transit Routing and Scheduling Strategies for Heavy Demand Corridors. In *Transportation Research Record 1011*, TRB, National Research Council, Washington, D.C., 1985, pp. 23–26.
2. A. Ceder. *Methods for Setting Bus Timetables, Volume II: Optimal Design of Transit Short-Turn Trips*. Multisystems, Inc., Cambridge, Mass., 1985.
3. N. H. Wilson, P. G. Furth, and A. Han. Procedure for the Allocation of a Bus Fleet Among a Set of Routes. Presented at Fourth European Congress of Operations Research, Cambridge, England, 1980.
4. J. Simon and P. G. Furth. Generating a Bus Route O-D Matrix from On-Off Data. *Transportation Engineering Journal, ASCE*, Vol. 111, No. 6, 1985, pp. 583–593.
5. M. A. Ben-Akiva, P. P. Macke, and P. S. Hsu. Alternative Methods to Estimate Route Level Trip Tables and Expand On-Board Surveys. In *Transportation Research Record 1037*, TRB, National Research Council, Washington, D.C., 1986, pp. 1–11.
6. P. G. Furth. Zonal Route Design for Transit Corridors. *Transportation Science*, Vol. 20, No. 1, 1986, pp. 1–12.
7. F. J. Salzborn. Optimum Bus Scheduling. *Transportation Science*, Vol. 6, 1972, pp. 137–148.
8. A. Ceder and H. I. Stern. Deficit Function Bus Scheduling with Deadheading Trip Insertions for Fleet Size Reduction. *Transportation Science*, Vol. 15, 1981, pp. 338–363.
9. P. G. Furth, F. B. Day, and J. P. Attanucci. Bus Route and Service Design: Application of Methods and Procedures. Multisystems, Inc., Cambridge, Mass., 1983.

Publication of this paper sponsored by Committee on Bus Transit Systems.