Lyapunov-based Model Predictive Control of Nonlinear Systems Subject to Time-Varying Measurement Delay

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The problem of designing feedback control systems for nonlinear systems subject to time-varying measurement delays is a fundamental one and its solution can find significant application in a number of control engineering problems including, for example, design of networked control systems (NCS). The importance of time delays in the context of networked control systems has motivated significant research effort in modeling such delays and designing control systems to deal with them, primarily in the context of linear systems (e.g., [1, 2, 3, 4, 5, 6]). This work deals with the design of predictive controllers for nonlinear system subject to time-varying measurement delay in the feedback loop. The available results on Lyapunov-based MPC [7, 8, 9, 10] do not account for the effect of time-varying measurement delays, and when time-varying measurement delays are taken into account, these schemes are not guaranteed to maintain the desired closed-loop stability properties. We also note that most of the available results on MPC of systems with delays deal with linear systems (e.g., [11, 12]). In this work, we modify the Lyapunov-based MPC schemes developed previously by our group to take into account time-varying measurement delay, both in the optimization problem formulation and in the controller implementation. The proposed LMPC scheme allows for an explicit characterization of the stability region, guarantees practical stability in the absence of delay, and guarantees that the stability region is an invariant set for the closed-loop system if the maximum delay is shorter than a given constant that depends on the parameters of the system and the Lyapunov-based controller that is used to formulate the optimization problem.

We consider a nonlinear system subject to disturbances with the following state-space description

$$\dot{x}(t) = f(x(t), u(t), w(t)) \tag{1}$$

where $x(t) \in R^{n_x}$ denotes the vector of state variables, $u(t) \in R^{n_u}$ denotes the vector of manipulated input variables, $w(t) \in R^{n_x}$ denotes the vector of disturbance variables, and f is a locally Lipschitz vector function on $R^{n_x} \times R^{n_u} \times R^{n_w}$. The disturbance vector is bounded, i.e., $w(t) \in W$. We assume that the nominal closed-loop system (system (1) with $w(t) \equiv 0$ for all t) has an asymptotically stable equilibrium at the origin $x = 0$ for a given feedback control $h : R^{n_x} \to R^{n_u}$ which satisfies $h(0) = 0$.

Although system (1) is defined in continuous time, we focus on a sample and hold implementation of the controller subject to time-varying measurement delay because we work within a model predictive control framework. State measurements are obtained with a sampling time Δ at times $t_k = t_0 + k\Delta$ where t_0 is the initial time and $k = 0, 1 \ldots$. To model measurement delay, an auxiliary random variable $d(t_k)$ is introduced to indicate the number of sampling times that the measurement received at time t_k is delayed. That is, at sampling time t_k, the controller receives the delayed measurement $x(t_{k-d(t_k)})$. In order to study the stability properties in a deterministic framework, in this paper we consider systems where there exists an upper bound D on the delay of the state measurement that we receive at each sampling time.

A controller for a system subject to time-varying measurement delays must take into account two important issues. First, when a new measurement is received, this measurement may not correspond to the current state of the system. This implies that in this case, the controller has to take a decision using an estimated state. Second, because the delays are time-varying, the controller may not receive new information every sample time. This implies that in this case, the controller has to operate in open-loop using the last received measurements. In order to deal with these two issues, we propose to take advantage of the model predictive control scheme to decide the control input based on a prediction obtained using the model. This prediction is used both for estimating the current state from previous measurements and for deciding the input when the controller does not receive new information. This is achieved using the following scheme: At each sampling time t_k if the controller receives a new measurement $x(t_{k-d(t_k)})$, then an estimate of the current state $\hat{x}(t_k)$ is obtained using the nominal model of the system (system (1) with $w(t) \equiv 0$ for all t) and the control inputs applied from $t_{k-d(t_k)}$ to t_k. Note that this implies that the controller has to store the past control input trajectory. The estimated state $\hat{x}(t_k)$ is used to obtain the optimal future control input trajectory solving a finite horizon constrained optimal control problem. The Lyapunov-based MPC controller uses the nominal model to predict the future trajectory $\tilde{x}(t)$ for a given input trajectory $u(t) \in S(\Delta)$ with $t \in [t_k, t_{k+N}]$ where N is the prediction horizon. A cost function is minimized, while assuring that the value of the Lyapunov function along the predicted trajectory $\tilde{x}(t)$ satisfies a Lyapunov-based contractive constraint. The proposed Lyapunov-based MPC that takes into account time-varying measurement delay in an explicit way is based on the following finite horizon constrained optimal control problem

$$\begin{aligned}
u_k^*(t) = \quad & \arg\min_{u_k \in S(\Delta)} \int_{t_{k-d(t_k)}}^{t_{k+N}} [\tilde{x}(\tau)^T Q_c \tilde{x}(\tau) + u_k(\tau)^T R_c u_k(\tau)] d\tau && \text{(2a)}\\
& \text{s.t.} \quad \dot{\tilde{x}}(t) = f(\tilde{x}(t), u_k(t), 0) && \text{(2b)}\\
& \tilde{x}(t_{k-d(t_k)}) = x(t_{k-d(t_k)}) && \text{(2c)}\\
& \dot{\hat{x}}(t) = f(\hat{x}(t), h(\hat{x}(t_j)), 0),\ t \in [t_j, t_{j+1}],\ j = k, \ldots, k+N-1 && \text{(2d)}\\
& \hat{x}(t_{k)}) = \tilde{x}(t_k) && \text{(2e)}\\
& u_k(t) = u_{k-1}^*(t), \forall t \in [t_{k-d(t_k)}, t_k] && \text{(2f)}\\
& V(\tilde{x}(t)) \leq V(\hat{x}(t)),\ \forall t \in [t_k, t_{k+D+1-d(t_k)}] && \text{(2g)}
\end{aligned}$$

where $S(\Delta)$ is the family of piece-wise constant functions with sampling period Δ, $\tilde{x}(t)$ is the predicted sampled trajectory of the nominal system for the input trajectory computed by the LMPC (2), $x(t_{k-d(t_k)})$ is the delayed measurement that is received at t_k, $u_{k-1}^*(t)$ is the optimal control input trajectory computed at time t_{k-1}, $\hat{x}(t)$ is the nominal sampled trajectory under the Lyapunov-based controller $u = h(\hat{x}(t))$ along the prediction horizon with initial state the estimated state $\tilde{x}(t_k)$, and Q_c, R_c are weight matrices that define the cost. The receding horizon scheme is modified as follows to take into account time-varying delays: If a new measurement is received, then solve (2) and obtain $u_k^*(t)$, else $u_k^*(t) = u_{k-1}^*(t)$. Apply $u(t) = u_k^*(t)$ for all $t \in [t_k, t_{k+1}]$. Obtain a new sample and repeat. The proposed LMPC controller allows for an explicit characterization of the stability region, guarantees practical stability in the presence of measurement delay, and guarantees that the stability region is an invariant set if the maximum delay is shorter than a constant that depends on the parameters of the system and the controller used to design the LMPC.

The proposed controller has been applied to a well mixed, non-isothermal continuous stirred tank reactor where three parallel irreversible elementary exothermic reactions take place used in previous works, see [10] for a complete description of the dynamic model of the process. The CSTR has three steady-states (two locally asymptotically stable and one unstable). The control objective is to stabilize the system at the open-loop unstable steady state. In order to simulate the process in the presence of measurement delay, we use a random process to generate the delay sequence $d(t_k)$, and the delay sequence $d(t_k)$ in which the control system is subjected to is shown in Figure 1. When measurement delay is present, the proposed LMPC is more robust. The stability region is invariant for the closed-loop system if $D + 1 \leq N$. This is not the case with the original LMPC scheme of [7, 8, 9]. In Figure 2, the trajectories of the closed-loop system under both controllers are shown in the presence of measurement delay with $D = 6$. It can be seen that the original LMPC controller can not stabilize the system at the desired open-loop unstable steady-state and the trajectories leave the stability region, while the proposed LMPC scheme keeps the trajectories inside the stability region.

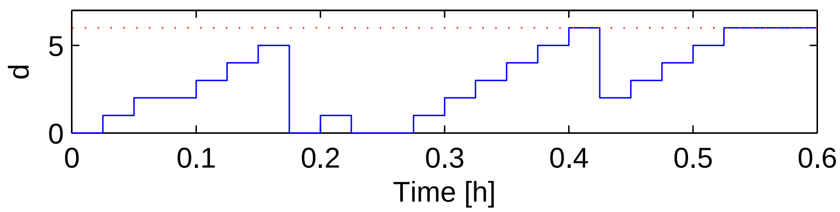
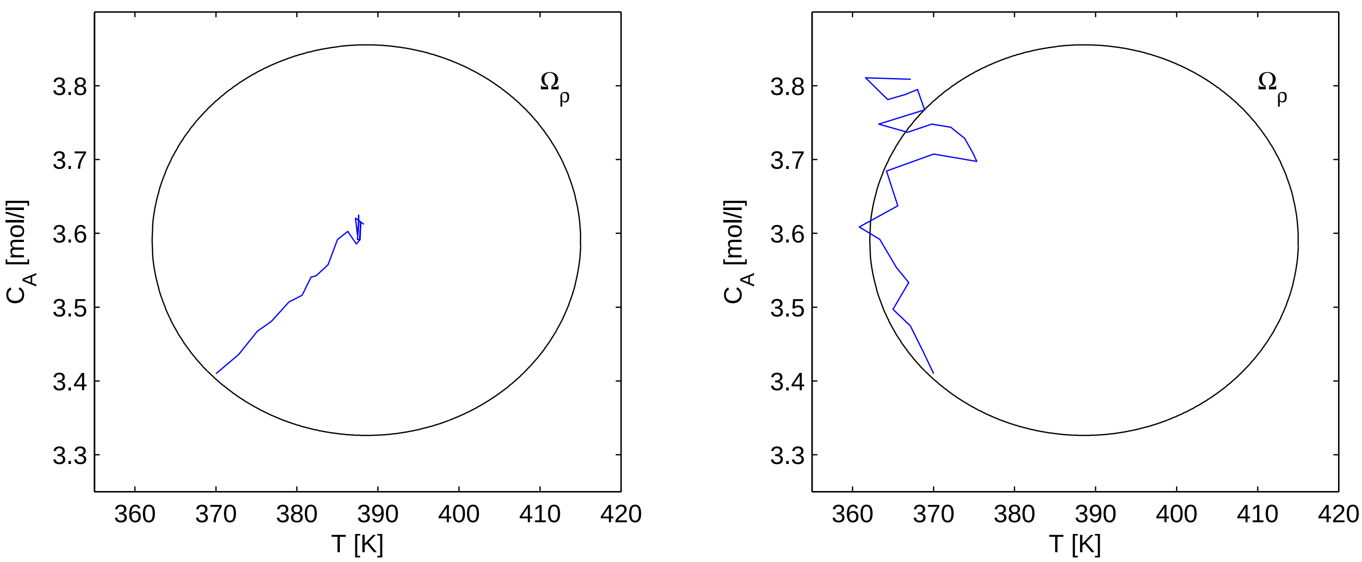


Figure 1: Delay sequence $d(t_k)$ used in the simulation shown in Figure 2.



(a) (b)

Figure 2: Trajectories trajectories of the CSTR with the original LMPC scheme when the maximum allowable measurement delay D is 6. Set Ω_ρ is the stability region of the proposed LMPC controller.

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