**Fall 2006** **Name:** Master

**Northeastern University**
**Introduction to Microelectromechanical Systems (MEMS)**
**ECE G244 / MIM G260**
**Exam #2**

1) (10 pts) A silicon chip has a very thin deposited layer of silicon nitride which is stress-free at $950^0$ C. Determine the stress in the nitride if it is cooled to room temperature. Is the stress tensile or compressive?

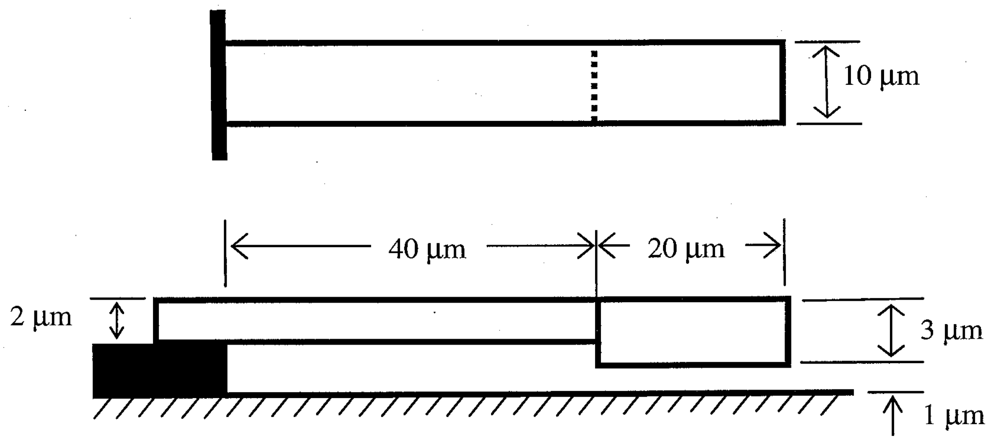
$\alpha_{Si} = 2.8\times10^{-6}/K^o$

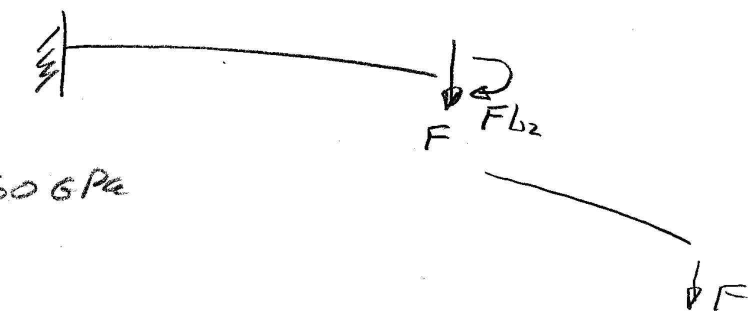
$\alpha_{SiN} = 2.3\times10^{-6}/K^o$

$\epsilon = (\alpha_{SiN} - \alpha_{Si})\Delta T = 0.5\times10^{-6}(950-25) = 462.5\times10^{-6}$

$\sigma = \frac{E_{SiN}}{1-\nu}\epsilon = \frac{270\times10^{9}\times462.5\times10^{-6}}{1-0.27} = 171\ MPa\ (C)$

2) (20 pts) Determine the release force for the polysilicon beam shown.




$E = 160\,GPa$

$$\delta = \frac{FL_1^3}{3EI_1} + \frac{(FL_2)L_1^2}{2EI_1} + \frac{FL_2^3}{3EI_2} + \theta L_2$$

$I_1 = \frac{1}{12}(10)(2)^3 = 6.667\,\mu m^4$, $L_1 = 40\,\mu m$

$I_2 = \frac{1}{12}(10)(3)^3 = 22.50\,\mu m^4$, $L_2 = 20\,\mu m$

$$\theta = \frac{FL_1^2}{2EI_1} + \frac{FL_2L_1}{EI_1} = 1500F$$

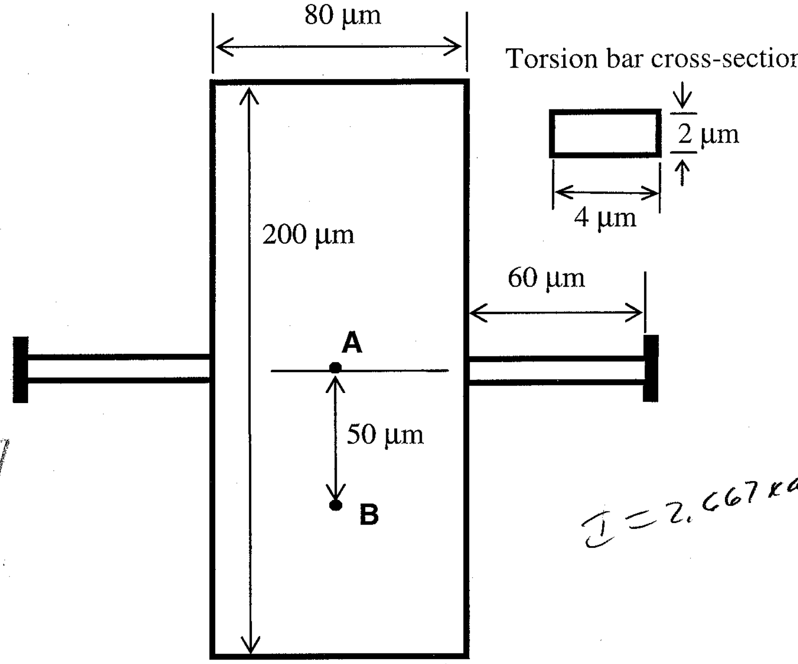
$$\delta = 20\times10^{-3}F + 15\times10^{-3}F + 0.741\times10^{-3}F + 1500(20)\times10^{-6}F = 10^{-6}$$

$30\times10^{-3}F$

$F = 15.21\times10^{-6}$

$F = 15.2\,\mu N$

3) (30 pts) The micromirror is supported by two identical nickel torsion bars as shown. The Young's modulus of the nickel is 210 GPa and the Poisson's ratio is 0.3. Assume that the mirror is rigid.
a) If a probe is applied at pt. **A** which causes that pt to ~~move down~~ displace 0.6 μm, determine the force exerted by the probe.
b) Suppose that that same force is applied at pt. B. Determine the displacement of that point.




$$k = \frac{12EI}{L^3}$$

$$= Ew\left(\frac{H}{L}\right)^3$$

$$= 31.111 \text{ N/m}$$

$$\boxed{F = (2k)\delta = 37.3 \ \mu N}$$

$$I = 2.667 \times 10^{-24}$$

b) $\delta_A = 0.6 \ \mu m$

$$\delta_{B/A} = \frac{(Fd)}{2k_t} d \qquad G = \frac{E}{2(1+\nu)} = 80.77 \text{ GPa}$$

$$k_t = \frac{k_1 G a^3 b}{L} = \frac{0.229 (80.77) \times 10^9 (2)^3 \times 10^{-18} (4 \times 10^{-6})}{60 \times 10^{-6}}$$

$$= 9.865 \times 10^{-9}$$

$$\left(\theta = \frac{Fd}{2k_t} = 94.53 \times 10^{-3}\right)$$

$$\delta_{B/A} = \frac{37.3 \times 10^{-6} (50 \times 10^{-6})^2}{2 (9.865 \times 10^{-9})} = 4.73 \ \mu m$$

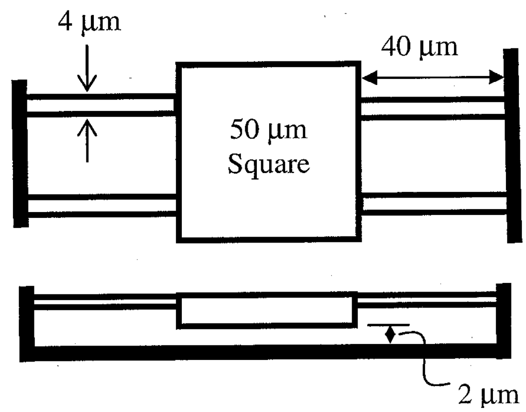
$$\boxed{\delta_B = 5.33 \ \mu m}$$

$$M_t = 933.3 \times 10^{-12} \text{ N·m}$$

4) (30 pts) The square mass is 20 µm thick silicon (which you may consider rigid) and the four identical silicon (E = 130 GPa) supporting beams each of which are 2 µm thick.

a) Determine the natural frequency of the structure shown.

b) Determine the damping force as the mirror passes through its equilibrium position with a velocity of 5 µm/µs downward. Is incompressible squeeze film justified in this case?




c) Damp[illegible]

$$k = \frac{12EI}{L^3}$$

$$= Ew\left(\frac{H}{L}\right)^3$$

$m_B = 0.384(4)\,4\times2\times40\times2331\times10^{-18} \ll m$

$$= 130\times10^9\,(4\times10^{-6})\left(\frac{2}{40}\right)^3 = 65\ N/m$$

$$m = \rho V = 2331\,(50)^2(20)\times10^{-18} = 116.55\times10^{-12}$$

$$\omega = \sqrt{\frac{4k}{m}} = 1.494\times10^6$$

$$\boxed{f = 238\ kHz}$$

$$F = K_s \eta \frac{LW^3}{h^3}\frac{dh}{dt}$$

$m_B = 2331\,(2)\times4\times40\times10^{-18} = 0.746\times10^{-12}$

$m_{eff} = 0.384(4)\,0.746\times10^{-12} \cong 1.14\times10^{-12}$

$$= 0.425\times1.86\times10^{-5}\;50\times10^{-6}\,\frac{(50\times10^{-6})^3}{(2\times10^{-6})^3}$$

$F = 30.88\ \mu N$

$$\boxed{F = 30.9\ \mu N}$$

$P_{AV} = \frac{F}{A} = 12.35\ kPa$, $P_{Atm} = 101.3\ kPa$

$\boxed{P_{AV} \ll P_{Atm}}$ Justifying incompressible theory

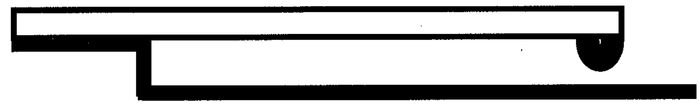
$\zeta = \frac{c}{c_c}$, $c_c = 2\sqrt{m4k} = 0.348\times10^{-3}\ N.s/m$

$c = \frac{F}{\dot{h}} = 6.18\times10^{-6}\ N.s/m$ $\boxed{\zeta = 0.0178}$

5) (10 pts) The ruthenium cantilever beam has a 4 μm radius hemispherical tip ($E = 410$ GPa, $\nu = 0.3$) at its end and is pressed elastically against a flat gold surface ($E = 80$ GPa, $\nu = 0.3$). The work of adhesion for this interface is 400 mJ/m$^2$ and $z_0 = 0.2$ nm.

a) Estimate the pull-off force due to dry adhesion.

b) Suppose there is a water meniscus which forms with a contact angle of 300 and a surface tension of 72 mN/m.



$$\frac{1}{E^*} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}, \quad E^* = 73.56 \text{ GPa}$$

$$\mu = \left(\frac{\Delta\gamma^2 R}{E^{*2} z_0^3}\right)^{1/3} = 2.45, \quad \lambda = 2.7$$

$$F \cong 1.5\pi\Delta\gamma R = 7.54\ \mu N$$

$$F = 4\pi R \Gamma \cos\theta = 3.13\ \mu N$$