Modeling, Simulation, and Testing of the Nonlinear Oscillations of a MEMS torsional mirror With Mechanical-Electrostatic Coupling

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MEMS micromirrors can be used to create miniature scanning displays in, for example, head mounted computer displays. They also have uses in communication and sensor applications. In order to obtain high angular displacement, the mirrors may be driven electrostatically at resonance. Although the resonance frequency of the torsional MEMS oscillators is adequately predicted by a linear mechanical model of the system, prediction of the details of the motion at the design displacements requires attention to non-linear effects. This paper discusses both experimental results and the modeling and dynamic simulations of this non-linear, coupled mechanical-electrostatic system.

Resonant MEMS torsion mirrors, 0.5 - 1mm on a side, were fabricated of bulk micromachined silicon and electrodeposited metal springs in one of several geometries. A typical mirror is shown in Figure 1, with a close-up of the spring in Figure 2. The mirrors are designed to be driven electrostatically at their resonant frequency, in the range of 3-15 kHz. The experimentally obtained frequency dependence of the amplitude shows significant non-linear effects, which include both spring softening and spring hardening type behaviors, depending on the spring material, the geometry, and the amplitude of the motion. Figures 3 and 4 present some experimental results for similar devices made with standard electroplated nickel and nanocrystalline nickel. As the driving force increases, the force-displacement curves exhibit significant non-linear stiffening, limiting the displacement achieved for both types of nickel springs (Figure 3). Furthermore (figure 4), with increased motion amplitude, the resonance frequency shifts to lower frequencies (spring softening) for nickel springs whereas for nanocrystalline springs of the same geometry there is a stiffening effect.

Several mechanisms contribute to the non-linearity, including non-linear deformation, non-linear drive force, and non-linear material properties. At the design displacement angles, the torsion beams deform sufficiently to contribute a non-linear term to the restoring torque. The magnitude of this term may be a significant fraction of the total restoring torque. In addition, in electrostatically driven mirrors the drive capacitor gap varies with mirror position, so that the torque depends on the displacement angle and phase. Quasistatically, the effect is to amplify the torque; more pronounced non-linear effects are seen at higher displacements, where latching can occur, and nearer resonance, as the phase of the motion relative to the drive signal changes. Non-linear mechanical material properties also contribute, especially at large displacements where localized stresses may approach the yield strength of the electroplated metal torsion beams.

In order to understand the non-linear mechanisms important in this system, we developed a coupled dynamic mechanical-electrostatic model of the torsionally supported micromirrors. The differential equation of motion is

$$J_m\ddot{\varphi} + 2\zeta\omega_n\dot{\phi} + \frac{2\alpha}{L}\phi = \frac{V_1^2 L\varepsilon_0}{4\delta\phi^2}\left[(d-\frac{w}{2})\phi + \delta\ln\left(\frac{\delta - d\phi}{\delta - \frac{w}{2}\phi}\right)\right] - \frac{V_2^2 L\varepsilon_0}{4\delta\phi^2}\left[(\frac{w}{2}-d)\phi + \delta\ln\left(\frac{\delta - d\phi}{\delta - \frac{w}{2}\phi}\right)\right]$$

in which the displacement ϕ depends on the applied voltages V_1 and V_2, the damping ζ, and various geometric dimensions and material constants. This equation was solved numerically, and the results of the simulation showed both the expected spring softening at low frequencies and spring stiffening near the resonant frequency. The nonlinear mechanical behavior of the torsion springs at the design displacements was also incorporated into the model but gave a relatively small effect.

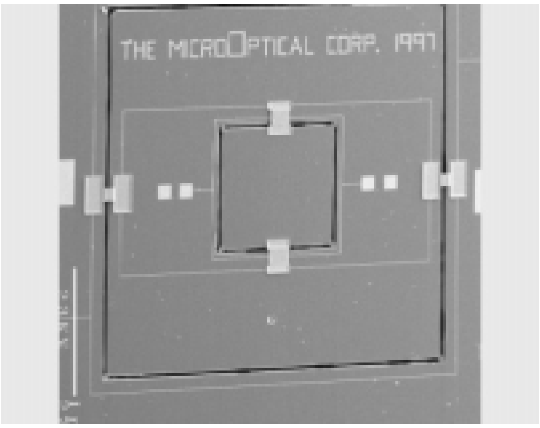

Figure 1. Photograph of biaxial MEMS torsion mirror. The center mirror is 1 mm on a side and is driven electrostatically at resonance.

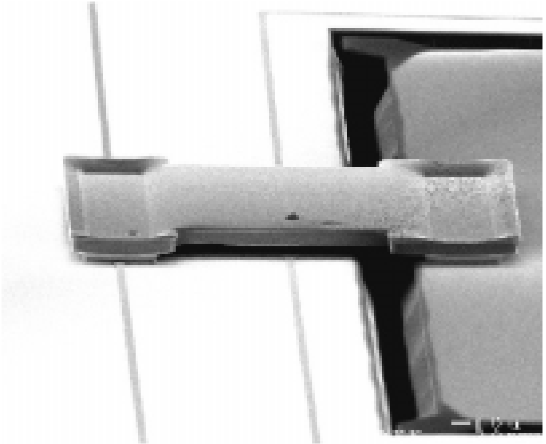

Figure 2. Close-up of torsion spring supporting resonant torsion mirror.

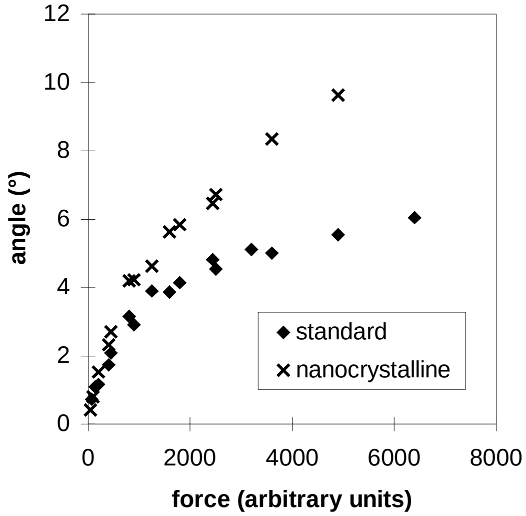

Figure 3. The displacement vs. driving force is non-linear: for increased force, the displacement is less than expected for both standard and nanocrystalline nickel (spring hardening).

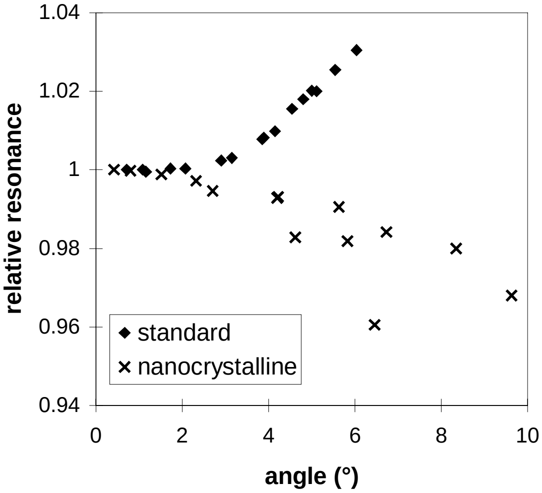

Figure 4: For increasing displacements, the resonance shifts to higher frequencies (hardening) for standard nickel and to lower frequencies (softening) for nanocrystalline nickel.