# DAMAGE LOCALIZATION USING LOAD VECTORS

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**SUMMARY:** A technique to localize damage in structures that can be treated as linear in the pre and post-damage state is presented. Central to the approach is the computation of a set of vectors, designated as Damage Locating Vectors (DLVs) that have the property of inducing stress fields whose magnitude is zero in the damaged elements (small in the presence of truncations and approximations). The DLVs are associated with sensor coordinates and are computed systematically as the null space of the change in measured flexibility. The approach is not structure-type dependent and can be applied to single or multiple element damage scenarios. Knowledge about the system is restricted to that needed for a static analysis in the undamaged state, namely, the undamaged topology and, if the structure is indeterminate, the relative stiffness characteristics. Results from numerical simulations suggest that the method can operate successfully under realistic conditions.

**KEYWORDS**: Damage Localization, Changes in Flexibility, State-Space Realizations.

## INTRODUCTION

Research on the various aspects associated with the use of vibration data to detect locate and quantify damage in structures has increased notably in the past two decades. The situation most often contemplated is that where the system can be treated as linear in the pre and post damaged states, making damage tantamount to a shifting of values in a set of system parameters. Reduction in the effective area of a steel member from corrosion or the fracture of a weld in a moment-resistant connection are examples of situations for which the assumption of linearity in the pre and post-damage states may not be unreasonable. Linear damage characterization falls in the realm of model updating but, in contrast with the typical update problem where one searches for small adjustments to fit a base line model to measured data, in the damage detection case the adjustments need not be small.

Model update algorithms typically utilize identified modal parameters, instead of the physically measured response, as the targets to be matched in fitting a model to the data. In this regard, it is worth noting that much of the progress in the linear damage detection problem has been made possible by the development of robust and efficient algorithms that provide minimum order state-space realizations from measured input/output data (Juang and Papa, 1984). A fundamental difficulty in damage identification through a model update strategy, however, is found in the fact that the inverse problem posed, unless the number of free parameters can be made sufficiently small, is usually ill-conditioned and non-unique (Beck and Katafygiotis, 1998).

Since the likelihood of arriving at an accurate characterization of damage is intimately associated with the number parameters that need to be treated as free (potentially damaged) in the model update algorithm, methods that can narrow the parameter space are of outmost practical importance. Various techniques that attempt to extract information on the spatial distribution of damage without a detailed model of the structure have been examined (Doebling et.al., 1996). An examination shows, however, that very few are general and can operate consistently under the conditions anticipated in practical applications. Namely: a) in structures with many members, b) where the number of sensors is small in comparison to the number of significant DOF, c) where only a truncated modal basis is available and d) in an environment where the measured data is 'noisy'. Difficulties also arise due to dependence of the results on assumed knowledge about the system; i.e., on the need for the mass matrix or the stiffness matrix of the undamaged system.

An approach of general applicability that provides information on the localization of stiffness related damage is presented in this paper. The method uses changes in the computed flexibility as the basic source of information. Yet, in contrast with existing flexibility-based techniques [Pandey and Biswas (1994,1995); Toksoy and Aktan (1994)] it does not attempt to locate damage using pattern recognition or other system-dependent strategies. Instead, the method operates with results obtained from a singular value decomposition of the change in flexibility and computes a set of vectors that, when treated as load distributions at the sensor points, induce stress fields that by-pass the damaged elements. In other words, the technique identifies the elements of the structure that have suffered damage as belonging to the set of elements that have negligible internal forces under the action of the prescribed load vectors. These vectors, herein designated as Damage Locating Vectors (DLVs), are computed as the null space of the change in flexibility from the pre to the post-damage state. As will be illustrated, the technique is capable of considering single or multiple damage scenarios and can operate with a truncated modal basis and partial sensor data.

Implicit in the previous description is the fact that the DLV's are computed strictly from the measured data without introducing assumptions with respect to where damage is likely or on how many damaged locations there might be. Strict model independence, however, ends once the DLV's are computed since a structural analysis is required to evaluate the effect of these loads on the structure. Two observations in regards to this issue are appropriate. First, the fact that the model needed is only to be used for a static analysis implies that the uncertainties associated with inertial and damping characteristics (which in general may have changed without invalidating the approach) are de-coupled from the search for stiffness related damage. Second, dependence of the damage localization on knowledge of the undamaged stiffness properties is generally small, given that only the distribution of internal forces is relevant in the computations. We note, for clarity, that the model used to compute the effect of the DLVs need only describe the topology of the undamaged structure and that the loss of load paths due to complete member failures are accounted for automatically.

The remainder of the paper is organized as follows: the connection between the DLVs and the null space of the matrix given by the change in flexibility is presented first. The next section discusses how modal truncation and approximation in the identified eigenproperties affect the computation of the DLVs. An index to identify load vectors that belong to the null space but which, as a consequence of modal truncation, are not strong members of the DLV set is introduced. The theoretical section is followed by two examples where the DLV approach is tested using simulated data.

## THEORETICAL FORMULATION

Consider a system that can be treated as linear in the pre and post damage states, but which is otherwise arbitrary, having damaged and undamaged flexibility matrices $F_D$ and $F_U$ at *m* sensor locations. Assume there are a number of load distributions (defined in sensor coordinates) that produce identical deformations when applied to the undamaged and damaged systems. If all the distributions that satisfy this requirement are collected in the matrix L it is evident that one can write;

$$(F_D - F_U)L = 0 \tag{1}$$

Inspection of eq.1 shows that the relationship can be satisfied in two ways, either $(F_D - F_U) = 0$, in which case the damage can not be spatially located by changes in flexibility, or the matrix $(F_D - F_U)$ is not full rank and L contains the vectors that define the null space. To avoid the case of distributions that induce zero displacements in both $F_U$ and $F_D$ we assume, temporarily, that they are both full rank (or less restrictively, that the intersection of their null spaces is empty). The influence of modal truncation and potential rank deficiency in $F_U$ and $F_D$ is considered in a later section. Performing a Singular Value Decomposition one can write;

$$(F_D - F_U) = [q_1] \begin{bmatrix} s_r & 0 \\ 0 & s_n \approx 0 \end{bmatrix} \begin{bmatrix} \tilde{q}_2^T \\ L^T \end{bmatrix} \tag{2}$$

where the $\approx 0$ is introduced to emphasize that in actual applications the singular values associated with the 'null space' will not be exactly zero due to approximations in the identified eigenproperties and possible modal truncation. A discussion on the selection of the threshold used in separating the row space from the null space is presented later.

From a physical perspective one appreciates that the load distributions that induce no stress in the damaged elements are vectors that belong to *L*, these vectors are designated here as Damage Locating Vectors (DLV). While it is evident that all load vectors that bypass the damaged elements belong to *L*, whether or not all the vectors in *L* bypass the damaged elements is not immediately apparent. A proof asserting that, indeed, all vectors in *L* bypass the damage elements may be found in a paper to appear in the Journal of the Engineering Mechanics Division of ASCE.

**Number of Modes in the Flexibility Matrices**

One way to satisfy eq.1, previously obviated by the assumption that the flexibility matrices are full rank is;

$$F_U\, L = F_D\, L = 0 \tag{3}$$

For the situation in eq.3 to be satisfied exactly the flexibility matrices $F_U$ and $F_D$ need to be rank deficient and their null spaces must have a non-zero intersection. A sufficient condition for the flexibility matrices to be rank deficient is that the number of identified modes be less than the number of sensors. This situation is unusual in damage identification problems. Note, however, that having more modes than sensors is a necessary condition but not a sufficient one for ensuring full rank because some of the identified modes may prove linearly related over the limited number of coordinates associated with sensor locations.

In any event, the relevant issue when the modal basis is truncated is the need to discriminate between vectors in *L* that are DLVs and those that have a strong projection in the common null space of the individual flexibility matrices. An index that is useful for this purpose is derived next. We begin by noting that the singular values $s_n$ are related to the null space and the change in flexibility by;

$$s_n = L^T (F_D - F_U) L \tag{4}$$

or;

$$L^T F_D L - L^T F_U L = s_n \tag{5}$$

therefore, the relative size of the terms being subtracted, in comparison to their difference, is characterized by the 'normalized singular value' *lv*, where;

$$lv_i = \frac{s_{n,i}}{L_i^T \, F_U \, L_i} \tag{6}$$

Vectors with relatively large values of *lv* should not be treated as DLVs, even if they are associated with small singular values. A value of *lv* from 0.08 to 0.10 appears to be a reasonable cutoff. We note that while $s_n$ and *lv* provide guidance on selecting the DLVs, they do not eliminate all heuristics because the gap between $s_r$ and $s_n$ is not always evident by inspection.

## On the Number of DLVs and Other Implementation Issues

Before discussing the implementation of the DLV approach it is convenient to introduce a result that points to a limit on the number of independent damage locations that can be contemplated by a given sensor set. Namely, it can be shown that in the case of discrete systems the size of the null space of ($F_D$-$F_U$) is equal to the number of sensors minus the rank of a stress influence matrix, Q, whose entries are a function of the number and location of the damaged elements. In particular, if *z* is the vector of internal stress resultants that need to be zero for the undamaged and damaged systems to be indistinguishable, then, the *jth* column of Q lists the values *z* due to a unit load at sensor coordinate *j*. The previous result shows that there is a theoretical limit to the maximum number of independent damage locations that can be considered. Specifically, if the number of independent columns in Q is equal to or larger than the number of sensors then there is no null space in the change in flexibility and, therefore, no DLVs.

One concludes, therefore, that there are (at least) two conditions for which the DLV technique is not useful. The first is when the damage does not induce changes in the flexibility at the measured coordinates, and the second is when the number of independent damaged locations is too large for the number of sensors. Note that in the first case the damage is not "observable" and in case two the number of sensors is not sufficient to ensure "controlability".

### *Damage Localization using DLVs*

Central to the procedure outlined in this section is the fact that the information contained in the various DLVs is complementary and, therefore, identification of a 'precise' set of vectors is unnecessary. The basic approach is to compute the stress distribution using the DLV associated with the lowest singular value (assuming the *lv* index is acceptable) and from the results identify the set of elements that are stressed below a certain threshold. Additional DLVs (if available) can then be used to reduce the size of the identified set. To decide on what constitutes a low stress one can proceed in a number of ways. The one that we have explored is through the definition of a normalized stress index, *nsi*, which we define as the value of the stress resultant in an element divided by the largest value of that stress resultant over all the elements of its class. In the case of trusses, for example, axial force is the relevant stress resultant and all the elements belong to the same class. The same is true for beams or slender frames with axial force replaced by bending moment. A situation where there is more than one class is that of a moment-resisting frame where some bays are braced. In this instance the braces are treated in a set and the flexural elements in another.

To describe the approach for computing the normalized stress index assume the apparent size for the near null space, *L*, is $w > 1$. Designate the vector associated with *kth* singular value as DLV-*k* and the associated stress distribution in the elements of class j as qj*(k),* where $k = m, m\text{-}1, m\text{-}2 \ldots m\text{-}w\text{+}1$ (recall *m* is the number of sensors). The (cumulative) normalized stress index for class *j*, after adding the information from *i* DLVs, is defined as;

$$nsi = \sum_{k=m}^{m-i+1} \frac{|qj(k)|}{|qj(k)|_{\max}} \tag{7}$$

Although the number of load vectors selected from an initial inspection of the singular values may be *w,* one can always opt to select the 'potentially damaged elements' at any stage before the last vector is added in eq.7. In fact, a good approach is to stop the process if an element that is not below the nsi threshold in step *i* shows up below the threshold in step *i+1*. The selection of an appropriate *nsi* cutoff depends on the accuracy with which the flexibility matrices have been obtained. Numerical results appear to indicate that values ranging from 0.05 to 0.1 may be realistic. Localization of damage using the DLV technique is illustrated in the numerical examples presented next.

## NUMERICAL EXAMPLES

**Example #1**

The first example is a fixed-fixed beam with 8 lumped masses. Independent white noise signals applied at masses 5 and 7 (see Fig.1) are used as excitation and sensors measuring acceleration in the vertical direction are assumed present at each of the lumped mass locations. Sensor noise is contemplated in the excitation and the computed response. The output noise is prescribed to have an RMS equal to 10 % of the RMS of the response measured on the sensor located at the first mass and the input noise RMS is 5% of the excitation level. Viscous dissipation is included in the form of Rayleigh damping with a magnitude of 5% of critical in the first two modes. Damage is simulated as a reduction of 25% in the flexural stiffness of segment 3-4 and 50 % in segment 7-8.

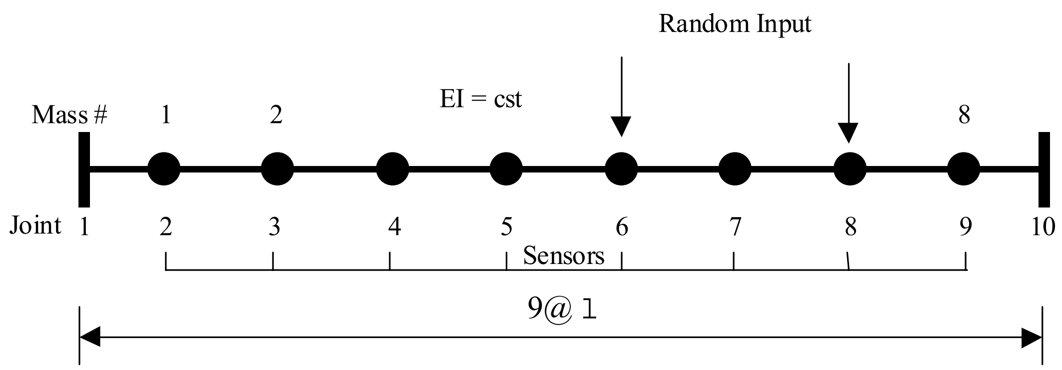

Fig.1 Structure of example #1 (EI and $\ell$ such that $f_1$ = 1 Hz).

As noted previously, the capacity of the DLV technique to identify damage hinges on the quality of the identified flexibility matrices. Extraction of the flexibility matrices from the recorded data depends, in turn, on the quality of the data, on the sophistication of the modal identification algorithm employed and the technique used to normalize the mode shapes with respect to mass. In this example we use the ERA-OKID algorithm to perform the modal identification (Juang and Papa, 1994) and normalize the modes with respect to mass, without introducing assumptions about the inertial characteristics, using an approach reported in Bernal (2000).

*Modal Identification*

Application of the ERA-OKID algorithm to the data identified 7 modes having MAC values in excess of 0.9 in the undamaged state and all 8 modes in the damaged condition. A comparison of the identified results with the exact values, as depicted in Table 1, shows that the accuracy of the identification is excellent.

*Table 1. Frequencies for Structure of Example #1 (in Hz)*

| Mode | Undamaged | | Damaged | |
|---|---|---|---|---|
| | Exact | Identified | Exact | Identified |
| 1 | 1.000 | 1.000 | 0.938 | 0.938 |
| 2 | 2.755 | 2.757 | 2.244 | 2.244 |
| 3 | 5.389 | 5.386 | 4.878 | 4.872 |
| 4 | 8.847 | 8.848 | 7.730 | 7.738 |
| 5 | 12.983 | 13.022 | 11.350 | 11.340 |
| 6 | 17.425 | 17.582 | 15.773 | 16.077 |
| 7 | 21.446 | 21.657 | 18.026 | 17.918 |
| 8 | 24.161 | ** | 21.329 | 22.125 |

** not identified

*Flexibility Matrices*

Fig.2 shows the percent error in the identified flexibility coefficients for the damaged and undamaged case. The errors are computed as the deviation between the computed flexibility coefficient and the exact value, normalized by the largest value in the associated column of the exact matrix. In this figure, the relationship between the index and the location of the coefficient in the matrix is given by counting from the main diagonal downward, from column to column, sequentially. As can be seen from the figure, the identified flexibility is very accurate (the RMS of the error in flexibility coefficients is less than 1%). We note that the relatively larger error associated with the first and the last coefficient arises from the fact that the deformations associated with loads at nodes 2 and 9 are very small.

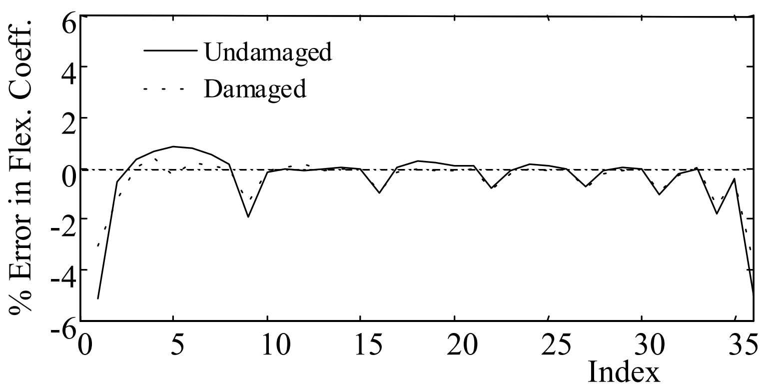

*Fig.2 Error in the Coefficients of the Identified Flexibility Matrices*

<u>*Computation of DLVs*</u>

The singular values and the *lv* indices are listed in Table2:

*Table 2. Singular values and lv indices for the structure of example #1*

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| sv | 3.434E-03 | 1.838E-03 | 1.681E-04 | 6.187E-05 | 1.962E-05 | 1.275E-05 | 4.625E-06 | 2.574E-06 |
| lv | 1.179E+00 | 9.891E-01 | 1.511E-02 | 1.391E-02 | 6.029E-03 | 2.067E-02 | 4.885E-03 | 5.606E-04 |

Inspecting the values in Table2 one concludes that there may be as many as 5 DLVs.

<u>*Normalized Stress Indices and Localization*</u>

The governing stress resultant in this problem is bending moment. The plot of the accumulated, normalized bending moment (nsi) from eq.7 is depicted in Fig.3. An inspection of the figure shows that the damage in segments 3-4 and 6-7 is clearly identified. Note that in this case the first DLV already contains all the localization information.

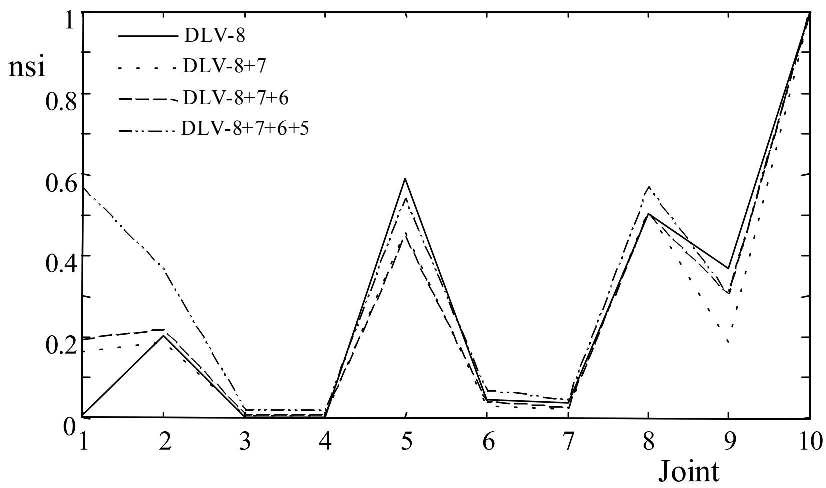


Fig.3. Normalized stress index for the beam of example #1.

**Example #2**

The structure is a 10-story shear building with an irregular distribution of mass and stiffness as shown in Fig.4. Damage is simulated as a loss of stiffness of 25 % in level-2 and 50% in level-6. Five accelerometers are used to record the output (one every other floor starting in level-2). The system is excited with white noise acting at the roof and at level-7 (note there is only one co-located input/output sensor pair). Noise in the input and the output is considered as described in example#1 (with the sensor at level-2 as the reference for the output noise RMS). The natural frequencies for the undamaged and damaged systems are listed in Table3.

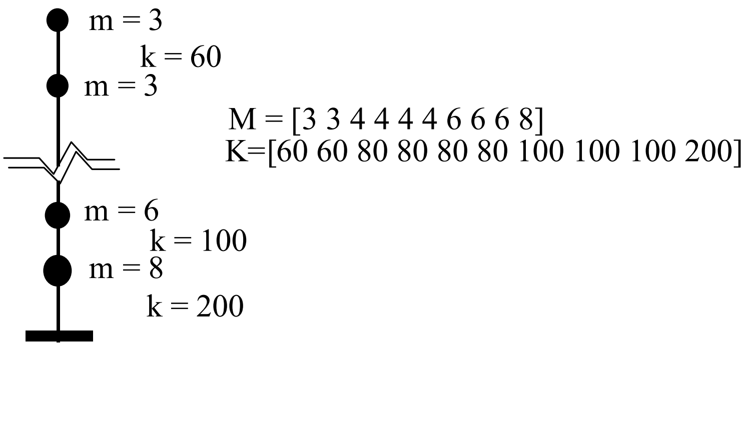

*Fig.4 Shear building of example #2*

*Table 3. Frequencies for Structure of Example #2 (in Hz)*

| Mode | Undamaged | | Damaged | |
|---|---|---|---|---|
| | Exact | Identified | Exact | Identified |
| 1 | 0.388 | 0.387 | 0.348 | 0.349 |
| 2 | 1.002 | 1.002 | 0.912 | 0.912 |
| 3 | 1.623 | 1.623 | 1.572 | 1.572 |
| 4 | 2.196 | 2.198 | 2.139 | 2.140 |
| 5 | 2.694 | 2.677 | 2.677 | 2.703 |
| 6 | 3.196 | 3.358 | 2.931 | 3.271 |
| 7 | 3.538 | 3.748 | 3.342 | 3.732 |
| 8 | 3.848 | 3.984 | 3.723 | 3.998 |
| 9 | 4.023 | 4.326 | 3.922 | 4.223 |

The singular values and the lv indices are depicted in Table 4. An inspection of this table indicates two or three DLVs.

*Table 4. Singular values and lv indices for the structure of example #2*

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| sv | 5.772E-03 | 4.489E-04 | 1.070E-04 | 9.189E-05 | 3.058E-05 |
| lv | 7.844E-01 | 1.217E-01 | 5.045E-02 | 6.001E-03 | 1.243E-02 |

The relevant stress resultant in this example is the inter-story shear. Fig.5 plots the normalized inter-story shear (nsi) corresponding to DLV-5+4. A cursory inspection shows that the damage is clearly identified within the spatial resolution of the available sensors. It is opportune to note that since the relevant stress resultant is a statically determinate quantity, the localization of the damage is done in this case without any reference to the stiffness characteristics of the undamaged structure.

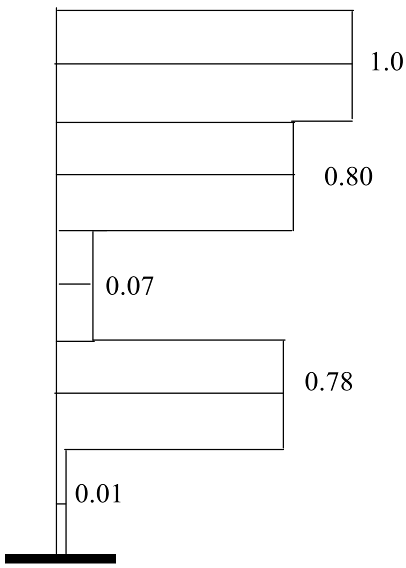


Fig.5 (a) Shear building of example#2 (b) nsi (for interstory shear).

## CONCLUSIONS

Methods for damage localization have traditionally focused on finding *differences* between the undamaged and the damaged structure, i.e., differences between mode shapes, differences in deformed shapes due to an applied load, etc. The Damage Locating Vector technique presented here, however, identifies load distributions where the static response of the structure is *the same* in the undamaged and the damaged systems. The approach, therefore, represents the complement of the traditional strategy. Implicit in this complementary perspective is the fact that the DLV based technique does not actually search for damaged elements but rather identifies undamaged elements as those that have significant stresses when the structure is loaded with the DLVs. The set of elements which are dormant under the action of the DLVs include the damaged elements but may also include some elements that are undamaged. This result, however, is entirely consistent with the theoretical basis of the approach and must not, therefore, be interpreted as false detection.

An attractive feature of the approach is the fact that the DLVs are computed systematically in a structure-type independent fashion and strictly from the measured data. Furthermore, model dependence is typically small since only the undamaged topology - in statically determinate structures, plus the relative values of the stiffness characteristics, in indeterminate systems, enter into the computations. The fact that the method operates with all the available sensors and with all the identified modes without recourse to DOF expansion or reduction strategies is worth restating.

As with any other damage identification approach, the real test is whether or not the DLV technique can operate successfully under realistic conditions. Although the method has been found to perform well with noise-contaminated simulated data, further examination using experimentally measured data is required to gain an understanding of its true capabilities.

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