An Examination of Instantaneous Frequency as a Damage Detection Tool

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Introduction

Research on vibration based damage identification has been expanding rapidly over the last decade. Much of the focus of this research has been placed on damage detection based on comparison of system properties 'before' and 'after' damage under the premise that the system can be treated as linear in both states. It is worth noting that in this approach the type of 'damage' that can be identified is restricted to that which has an influence in the small amplitude vibration characteristics of the system. Severe inelastic response during an extreme event in a system idealized as elasto-plastic, for example, is not detectable in this approach to the problem. Furthermore, even for the type of damage that can be handled by the before and after strategy, separation of structural from non-structural damage may be very difficult from examination of the small amplitude signals that can be obtained for Civil Engineering Structures in service.

An alternative for damage detection associated with response to extreme events is to use signals measured *during* the event. An attractive feature of this alternative is that the influence of non-structural elements, at the large amplitudes, is likely to be much less than at the ambient vibration level. Of lesser importance, but also worth noting as a positive feature, is the fact that the signal-to-noise ratio is much larger for the response to the damaging event than for ambient vibration signals. On the negative side, however, detection and localization of damage by the examination of signals recorded during a damaging event require consideration of nonlinear behavior, a condition that restricts the number of available analysis techniques severely.

In this paper we examine the concept of Instantaneous Frequency (*IF*) as a potential candidate for damage detection purposes. The *IF* of a signal is, loosely speaking, the frequency of a sine curve that locally fits the signal. Since inelastic behavior and other types of damage can be expected to affect the frequency composition of the response, the *IF* may prove useful in damage characterization.

Instantaneous Frequency and Empirical Mode Decomposition Method

As a generalization of the definition of frequency, *IF* is defined as the rate of change of the phase angle at time t of the analytic version of the signal (Ville, 1948). Given a real signal $s(t)$, the analytic signal $z(t)$ is a complex signal having the actual signal as the real part and the Hilbert transform of the signal as the imaginary component, namely;

$$z(t) = s(t) + j\mathrm{H}[s(t)] = a(t)e^{j\phi(t)} \tag{1}$$

where the amplitude $a(t)$ and the phase $\phi(t)$ are clearly given by:

$$a(t) = \sqrt{(s(t))^2 + (H[s(t)])^2} \quad \text{and} \quad \phi(t) = \tan^{-1}\left(\frac{\mathrm{H}[s(t)]}{s(t)}\right) \tag{2a,b}$$

and the Hilbert transform is given by the principal value of the integral in eq.3

$$\mathrm{H}[s(t)] = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{s(t)}{t-\tau}dt \tag{3}$$

The instantaneous frequency is, by definition;

$$f_i(t) = \frac{1}{2\pi}\frac{d\phi(t)}{dt} \tag{4}$$

One can easily confirm from the previous definitions that the *IF* of a harmonic function is constant and coincides with the frequency of the function. One can gain intuitive appreciation for the concept of *IF* by examining a chirp signal. A linear chirp is defined as *y(t) = cos(at)t* from where the interpretation of a frequency varying linearly with time is evident. A plot of a linear chirp is shown in fig.1(a) with the phase angle of the analytic signal and the instantaneous frequency computed from eq.4 plotted in fig.1(b). As one can see, the *IF* definition captures the time variation of the frequency accurately. Note that when the chirp is represented in the Fourier domain the result contains a large number of components with different frequencies and the simple nature of the signal is lost.

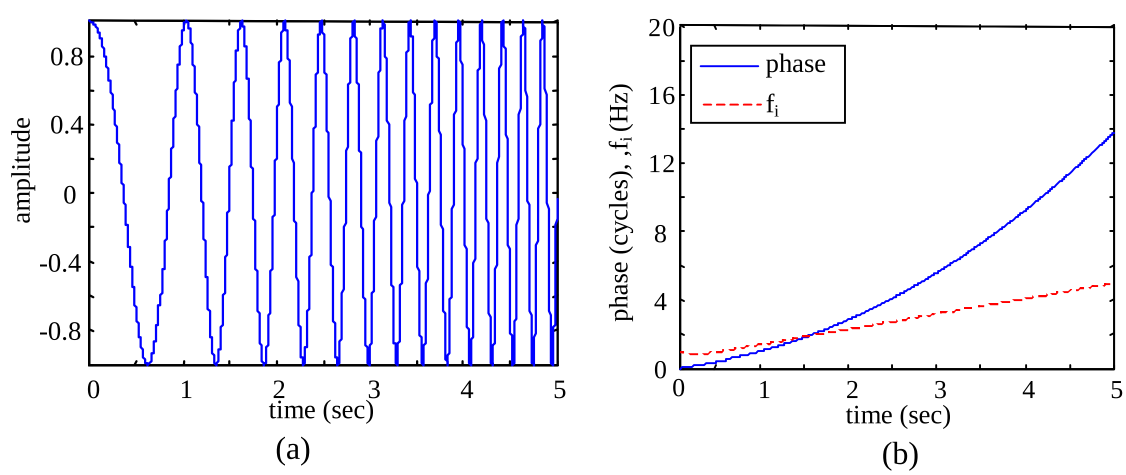


Figure 1 **(a)** Linear chirp, **(b)** Phase angle and Instantaneous Frequency

The definition of *IF* presented in eq.4 is the most basic but is by no means unique. Ville (1948) formulated a distribution in time and frequency known as the Wigner-Ville Distribution, *WVD*, and defined another estimator for the instantaneous frequency as the first moment of the distribution with respect to frequency. Cohen (1988) developed a generalized formulation for the distribution of energy in time and in frequency and defined the instantaneous frequency to be the average of the frequencies that exist in the time-frequency plane at a given time. A comprehensive discussion on the various proposed formulations may be found in Boashash (1992).

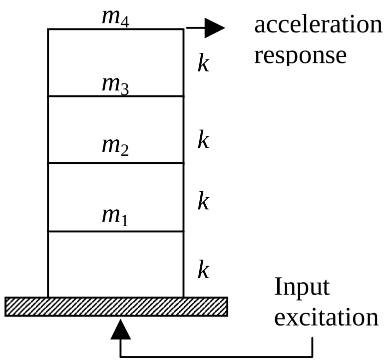
The search for alternative definitions of *IF* has been motivated by the fact that in many cases the variability of the phase angle is large and, as a consequence, there are large fluctuations of the *IF* about the mean value. In any case, there has long been consensus on the fact that the concept of *IF* is physically meaningful only when applied to *mono-component* signals, which have been loosely defined as narrow band. To apply the concept of *IF* to arbitrary signals (with any hope of extracting physically meaningful information) it is necessary to first decompose the signal into a series of mono-component contributions. A recent approach to carry out this decomposition in a systematic manner has been presented by Huang et al. (1996) and is designated as the Empirical Mode Decomposition (EMD) technique. The EMD technique decomposes the signal, $f(t)$, into a series of mono-component contributions designated as intrinsic mode function (IMF), namely;

$$f(t) = \sum_{j=1}^{n} c_j(t) + r_n(t) \tag{7}$$

where r_n is the residue after the n IMFs have been extracted. An IMF is a function that satisfies two conditions: (1) in the data set the number of extrema and the number of zero crossings must either equal zero or differ at most by one; and (2) at any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero (Huang 1998). In this study we use the basic definition of instantaneous frequency which is given in eq.4. To reduce the variance associated with differentiation of the phase one can either filter the signal outside the expected bandwidth or smooth the phase difference estimator (Boashash et al., 1990). In this study we have provided simple piece-wise linear fits to the computed phase variations.

Example#1

A 4-story shear building is considered. The mass, the initial stiffness corresponding to each floor and the system frequencies prior to any damage are shown in fig.2. The structure has 5% damping in all modes. In the first case considered the system is subjected to El Centro (1940) ground motion and at t = 10 secs the first floor is assumed to suffer a sudden 80% loss in stiffness. The system frequencies after the damage are 5.47, 21.70, 38.83 and 48.83 Hz. Once the empirical mode decomposition is carried out and the IMF components are obtained, the Hilbert transform of the first IMF component is calculated and used to evaluate the analytic signal (using the roof acceleration as the output).



Mode	f (Hz)
1	9.67
2	26.37
3	39.81
4	49.03

k=7.5×10^{7}N/m
m_1=3600 kg, m_2=m_3=2850 kg,
m_4=1800 kg

Figure 2 Structural model for numerical study

The phase angle and the instantaneous frequency plots are shown in Figure 3. One can easily see the indication of damage as the change in the slope of the phase angle and the drop of the instantaneous frequency around t=10 sec.

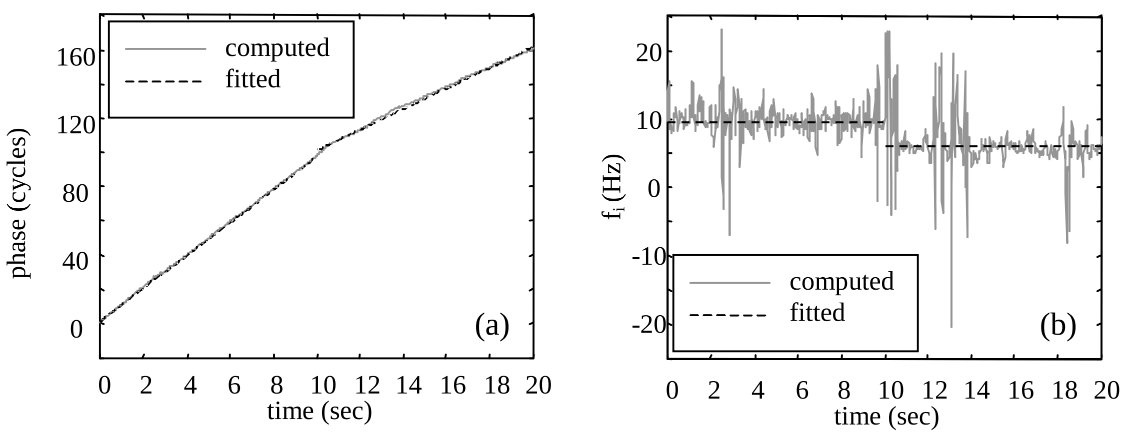


Figure 3 (a) Phase angle and (b) Instantaneous frequency of IMF 1

Example #2

We consider the same structure of example#1 but in this case the focus is placed on the effect of hysteretic action. In particular, the restoring force relationship for the first floor is assumed to be of the Bouc-Wen (1967, 1976) type with parameters that result in a smooth transition from elastic to plastic behavior. To examine the influence of the extent of inelastic response on the *IF* two levels of excitation are contemplated, the first floor shear vs. drift for both excitation levels are shown in Figure 4. Other floors are assumed to remain linearly elastic during the entire monitoring period. In both cases the intensity

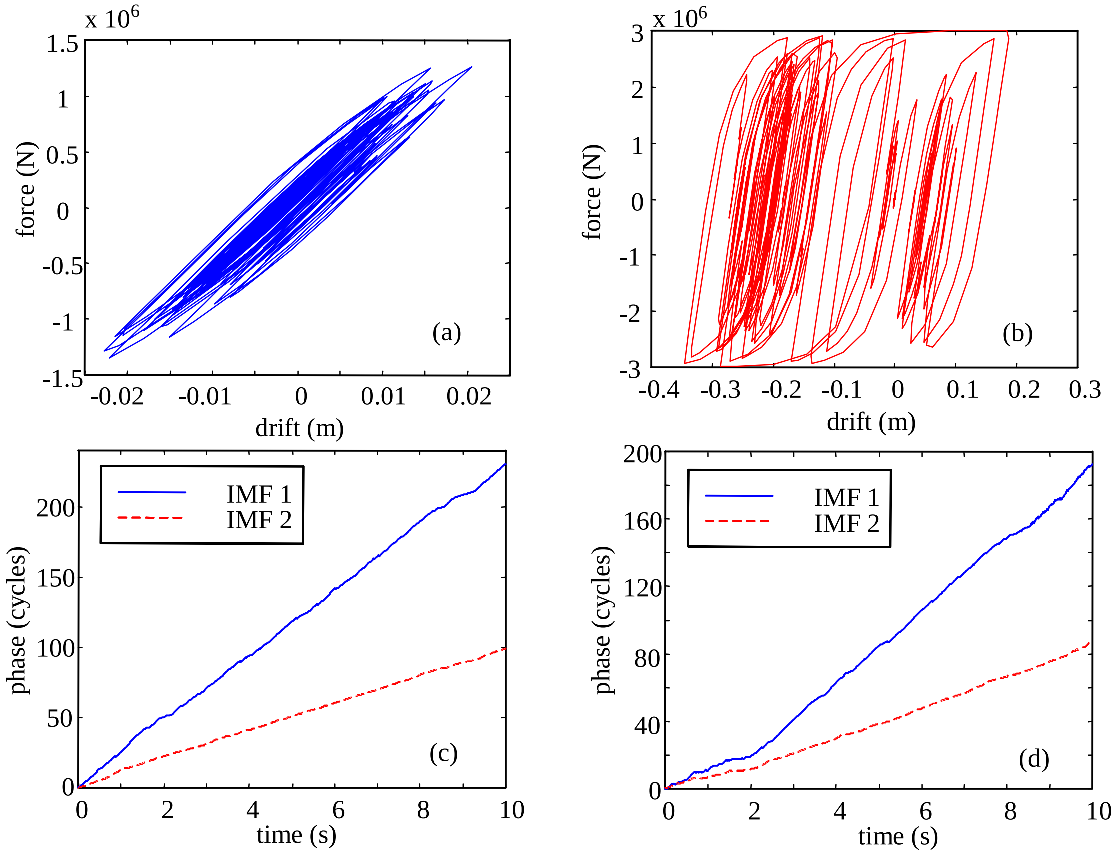


Figure 4 (a, b) Force-drift relations for the 1st floor, (c, d) Phase angle plots

of the excitation is small in the first two and the last two seconds. The phase angle plots for the first two IMFs are also depicted in fig.4 for the moderate and the strong excitation cases. The following observations can be made:

1) In the case of mild nonlinearity the two IMFs display *IFs* that approximate those of the second and the first mode of the elastic structure (the 3rd and 4th modes are not captured). Although some reduction in the average slope of the phase can be detected in the region when the response is inelastic, the indication is not sufficiently strong (given the variability of the parameters) to be of practical value as an indicator of inelasticity.
2) In the strongly nonlinear case the first IMF displays the frequency of the first mode for the first two seconds (when the response is elastic) and then increases sharply. This behavior is interesting since at first glance one may expect the frequency to decrease as inelasticity ensues. This result, however, may be rationalized by noting that when the first floor yields extensively the first mode associated with the tangent stiffness properties has a very low frequency and is unlikely to contribute significantly to the measured response. What happens, it seems, is that the second mode of the structure with a yielded first level becomes dominant and the frequency of this mode (one can easily confirm) is larger than the first mode of the elastic structure (and close to the computed value of the second slope). We conclude, therefore, that while modest inelasticity may be reflected in reductions of the *IF*,

extensive inelastic behavior can lead to increases in the *IF* by elimination of the initially dominant mode.

Concluding Remarks

This paper represents an initial effort to examine the potential merits of instantaneous frequency as a damage indicator. The empirical mode decomposition method was utilized to decompose the signal into several monocomponent signals to improve the likelihood that the *IF* concept will prove physical meaningful. In the case of a sudden severe damage in which the structure remains linear after the damage, the technique was capable of identifying the time and extent of the damage. Nevertheless, the computations here were carried out for noiseless conditions and it remains to be seen if the approach can give useful information under realistic conditions (large systems with noisy measurements). For the case of hysteretic response the instantaneous frequency was found inadequate as a robust indicator of modest nonlinearity. For the case of severe nonlinear behavior the *IF* showed some clear trends although these were opposite to what one would have perhaps anticipated since the onset of inelasticity was reflected in an increase in the *IF*. It was argued, however, that this trend is consistent with an interpretation where the inelastic behavior shifts the effective frequency of the first mode to such a low value that this mode is no longer an important contributor. The second tangent mode, having a higher frequency that the first elastic mode becomes dominant and is the one responsible for the increase in the *IF*. While additional work is needed before the potential of *IF* is fully appreciated, the results of this preliminary investigation do suggest that the technique is unlikely to prove robust in practical cases.

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