The Method:

The DLV technique is a procedure for locating damage using changes in flexibility matrices. The procedure is not concerned with how the flexibility matrices are computed but focuses on extracting the damage localization information from the changes. In practice the flexibility matrices are usually synthesized from vibration signals because direct computation from static tests is impracticable. In an experimental setting the flexibility is defined at location where there are sensors.

Strictly speaking the DLV technique does not point to the damage but points to where "the damage is not" once all the regions that can be excluded are eliminated the remaining part of the structure is the "Potentially Damaged Region" (PDR). The question of whether the PDR contains only damaged elements or not depends on the sensor grid and the structure (see ref for details).

The DLV excludes elements from the PDR by computing stress fields using vectors in the null space of the change in flexibility. There are two parts to the computations:

1) Computation of the DLVs
2) Computation of the stress field for the DLVs

Errors and approximations in the experimental extraction of the flexibility matrices affect the accuracy of step#1. Errors in step#2 (if any) are those associated with making a model and computing a static stress field for a known loading condition.

The DLVs are obtained as the null space of the matrix defined by the difference between the flexibility matrices for the two states considered. If there are N sensors then the Null space (if it exists) is a matrix having one or more columns and N rows. Each column is DLV.

The example presented bellow is much simpler than those in the original reference. It is presented to help clarify doubts in the application of the DLV approach.

5-STORY SHEAR FRAME EXAMPLE

Story Stiffness:	: ku={2000, 2000, 2000, 2000, 2000}
Story Masses	: m={2, 1, 1.5, 2, 1}
Damping	: 2% critical in each mode
After damage	: kd={2000, 1200, 2000, 2000, 1000}
Sensors at floors	: {1, 2, 3, 5}

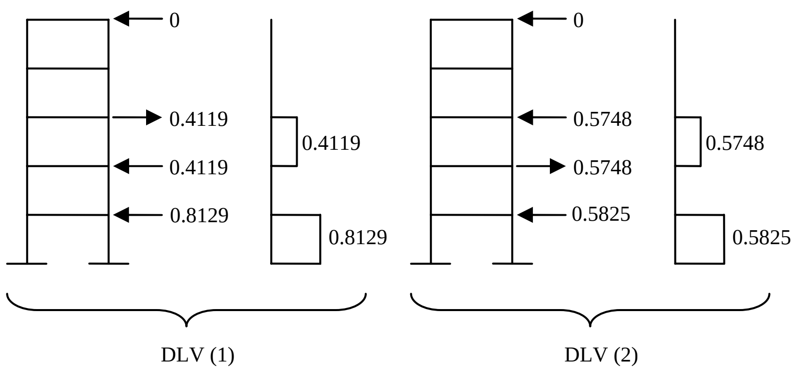
CASE I: Exact matrices

Flexibility matrices for the undamaged (F_u) and damaged (F_d) states are;

$$Fu = \begin{bmatrix} 0.0005 & 0.0005 & 0.0005 & 0.0005 \\ 0.0005 & 0.0010 & 0.0010 & 0.0010 \\ 0.0005 & 0.0010 & 0.0015 & 0.0015 \\ 0.0005 & 0.0010 & 0.0015 & 0.0025 \end{bmatrix} \quad Fd = \begin{bmatrix} 0.0005 & 0.0005 & 0.0005 & 0.0005 \\ 0.0005 & 0.0013 & 0.0013 & 0.0013 \\ 0.0005 & 0.0013 & 0.0018 & 0.0018 \\ 0.0005 & 0.0013 & 0.0018 & 0.0033 \end{bmatrix}$$

Singular value decomposition of the difference, $DF=F_d-F_u=QSV^T$

Q				S				V			
0	0	0.9593	-0.2825	0.0012	0	0	0	0	0	-0.8129	-0.5825
-0.4544	-0.5418	0.1998	0.6783	0	0.0003	0	0	-0.4544	-0.5418	-0.4119	0.5748
-0.4544	-0.5418	-0.1998	-0.6783	0	0	0.0000	0	-0.4544	-0.5418	0.4119	-0.5748
-0.7662	0.6426	0	0	0	0	0	0.0000	-0.7662	0.6426	0	0
							svn=	0.8165	1.0000	0.0000	0.0000



Story	nsi (1)	nsi (2)	wsi
1	1	1	66.6667
2	0	0	0
3	0.5067	0.9868	49.783
4	0	0	0
5	0	0	0

From inspection of the *wsi* one can see that levels 2, 4 and 5 are in the PDR. In reality the damage is on levels 2 and 5. The reason why level 4 is in the PDR is because the sensor arrangement makes it impossible to separate this level from level#5 (if one is using changes in flexibility to locate the damage).

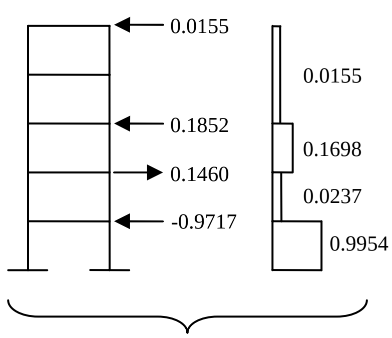
CASE II: System Identification from Noisy Data (first 4 modes are identified)

Flexibility matrices for the undamaged (F_u) and damaged (F_d) states are;

$$
Fu = \begin{bmatrix} 0.0005 & 0.0005 & 0.0005 & 0.0005 \\ 0.0005 & 0.0009 & 0.0010 & 0.0010 \\ 0.0005 & 0.0010 & 0.0014 & 0.0015 \\ 0.0005 & 0.0010 & 0.0015 & 0.0025 \end{bmatrix}
\quad
Fd = \begin{bmatrix} 0.0005 & 0.0005 & 0.0005 & 0.0005 \\ 0.0005 & 0.0012 & 0.0014 & 0.0013 \\ 0.0005 & 0.0014 & 0.0018 & 0.0018 \\ 0.0005 & 0.0013 & 0.0018 & 0.0033 \end{bmatrix}
$$

Singular value decomposition of the difference, $DF = F_d - F_u = QSV^T$

Q				S				V			
-0.0066	0.0187	0.2356	-0.9717	0.0012	0	0	0	-0.0066	0.0187	-0.2356	-0.9717
0.4582	-0.5759	0.6611	0.1460	0	0.0003	0	0	0.4582	-0.5759	-0.6611	0.1460
0.4596	-0.4981	-0.7116	-0.1852	0	0	0.0000	0	0.4596	-0.4981	0.7116	-0.1852
0.7608	0.6480	0.0337	0.0155	0	0	0	0.0000	0.7608	0.6480	-0.0337	0.0155
							svn=	0.8250	1.0000	0.3262	0.0930



DLV (1)

Story	nsi (1)	wsi
1	1	10.7492
2	0.0238	0.2563
3	0.1706	1.8334
4	0.0156	0.1672
5	0.0156	0.1672

The *wsi* points to the same locations as before although the values now are not exactly zero (but are less than 1, which is the threshold specified in the paper).