Solution to HW #4
MIM 1440

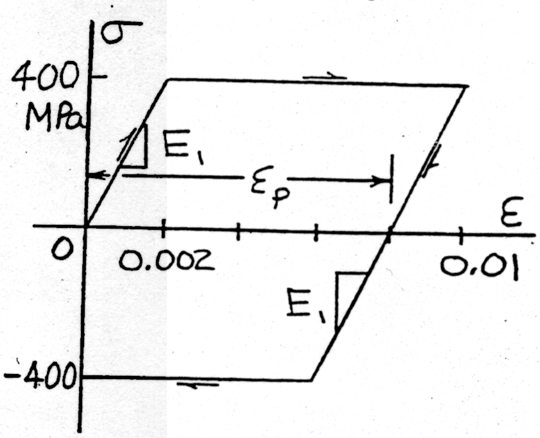
Problem 1:

a) Elastic strain, elastic part of the deformation which is recovered upon unloading
b) Plastic strain, plastic part of the deformation, which is not recovered upon unloading and is independent of time.
c) Creep, continuous plastic deformation with time.
d) Tensile viscosity, for the steady state plastic deformation under constant stress, the rate of plastic deformation is related to material viscosity analogous to a dashpot in a mechanics course.
e) Recovery is return of the material to the original geometry upon unloading with time.
f) Relaxation is the stress relief in a material under a constant stretch.

5.2 Elastic, perfectly plastic model:

$E_1 = 200$ GPa, $\sigma_o = 400$ MPa

(a) ε to 0.01, then (b) back to zero.



$$\varepsilon = \frac{\sigma}{E_1} + \varepsilon_p$$

$$\varepsilon_p = 0.01 - \frac{400 \text{ MPa}}{200{,}000 \text{ MPa}}$$

$$\varepsilon_p = 0.008$$

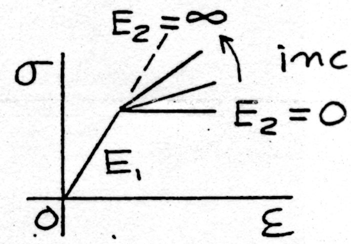
$$\varepsilon_e = 0.002$$

5.3 Elastic, linear-hardening model.

$$\varepsilon = \frac{\sigma}{E_1} + \frac{\sigma - \sigma_o}{E_2} \qquad (\sigma > \sigma_o)$$

$$\frac{d\varepsilon}{d\sigma} = \frac{1}{E_1} + \frac{1}{E_2}, \quad \frac{d\sigma}{d\varepsilon} = \frac{1}{\frac{1}{E_1} + \frac{1}{E_2}}$$

This is the stiffness of E_1 and E_2 in series.



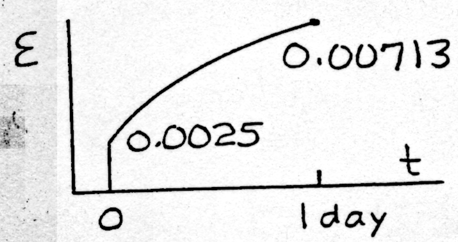
5.7 Elastic, transient creep model.

$E_1 = 6$, $E_2 = 3$ GPa, $\eta_2 = 10^5$ GPa·s

$\sigma = 15$ MPa applied for 1 day = 86,400 s.

$$\varepsilon = \frac{\sigma}{E_1} + \frac{\sigma}{E_2}\left(1 - e^{-\frac{E_2 t}{\eta_2}}\right)$$

Substitute various values of t in seconds and calculate correspon ing ε



Then plot these ε versus t.

5.21 $\sigma_y = \lambda \sigma_x, \ \varepsilon_z = 0$

(a) Yes

$$\varepsilon_z = \frac{\sigma_z}{E} - \frac{\nu}{E}(\sigma_x + \sigma_y) = \frac{\sigma_z}{E} - \frac{\nu \sigma_x}{E}(1+\lambda)$$

$$\sigma_z = \nu \sigma_x (1+\lambda)$$

(b) $\varepsilon_x E = \sigma_x - \nu(\sigma_y + \sigma_z)$

$$\varepsilon_x E = \sigma_x - \nu \lambda \sigma_x - \nu^2 \sigma_x (1+\lambda)$$

$$\frac{\sigma_x}{\varepsilon_x} = E' = \frac{E}{1 - \nu\lambda - \nu^2(1+\lambda)}$$

λ	-1	0	$+1$
E'	$0.77E$	$1.10E$	$1.92E$

(c) The effect of λ on E' for $\nu = 0.3$ is substantial.

5.14

$\varepsilon_x = -0.002, \ \varepsilon_y = 0.003, \ \sigma_z = 0$
$E = 212{,}000$ MPa, $\nu = 0.293$

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_y + \sigma_z)$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \frac{\nu}{E}(\sigma_x + \sigma_z)$$

$$\left.\begin{aligned} -0.002(212{,}000) &= \sigma_x - 0.293\,\sigma_y \\ 0.003(212{,}000) &= \sigma_y - 0.293\,\sigma_x \end{aligned}\right\} \text{Solve}$$

$\sigma_x = -260, \ \sigma_y = 560$ MPa

If (x, y) are principal (1,2) axes, then $\tau_{xy} = 0$, and the state of stress is completely determined. Otherwise, another measurement is needed, such as the strain $45°$ to the x-y axes.

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$$\sigma_z = \nu \sigma_x (1+\lambda)$$

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