7.1 $\sigma_x = 60,\ \sigma_y = 0,\ \tau_{xy} = 30$

(a) $X_o = ?$ if AISI 1020 steel, $\sigma_o = 260$ MPa

(b) $X_u = ?$ if SiC, $\sigma_{ut} = 307$ MPa

(Tables 3.10 and 4.2)

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$\sigma_1, \sigma_2 = 30 \pm 42.43 = 72.43,\ -12.43$ MPa

$\sigma_3 = \sigma_z = 0$

(a) $\bar{\sigma}_S = MAX(|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|)$

$\bar{\sigma}_S = 84.85$ MPa

$X_o = \sigma_o / \bar{\sigma}_S = 260/84.85 = 3.06$ ◀

(b) $\bar{\sigma}_N = MAX(|\sigma_1|, |\sigma_2|, |\sigma_3|) = 72.43$ MPa

$X_u = \sigma_{ut} / \bar{\sigma}_N = 307/72.43 = 4.24$ ◀

7.2 Cylindrical pressure vessel, closed ends. $t = 5\ mm$, $d_1 = 3\ m$, $p = 2\ MPa$
(Same situation as Prob. 6.5)

$$\sigma_x = \frac{p r_1}{2t} = \frac{(2\ MPa)(1500\ mm)}{2\,(5\ mm)} = 300\ MPa$$

$$\sigma_y = \frac{p r_1}{t} = 600\ MPa$$

$$\sigma_1, \sigma_2, \sigma_3 = \sigma_x, \sigma_y, \sigma_z = 300,\ 600,\ 0\ MPa$$

18 Ni maraging steel (250 gr.)

$\sigma_o = 1791\ MPa$ (Table 4.2)

$$\bar{\sigma}_S = MAX(|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|) = 600\ MPa$$

$$X_o = \frac{\sigma_o}{\bar{\sigma}_S} = 2.99$$

Second solution:

$$\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$

$$\bar{\sigma}_H = 520\ MPa, \quad X_o = \frac{\sigma_o}{\bar{\sigma}_H} = 3.45$$

7.4 $\sigma_x = 150,\ \sigma_y = 30,\ \tau_{xy} = -45\ MPa$

$\sigma_o = ?$ for $X_o = 3.0$

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1, \sigma_2 = 90 \pm 75 = 165, 15\ MPa$$

$$\sigma_3 = 0$$

$$\bar{\sigma}_S = MAX(|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|)$$

$$\bar{\sigma}_S = 165\ MPa$$

$$X_o = \sigma_o / \bar{\sigma}_S, \quad \sigma_o = X_o \bar{\sigma}_S = 3(165) = 495\ MPa$$

Second solution:

$$\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)}$$

$$\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{(150-30)^2 + 30^2 + 150^2 + 6(45)^2}$$

$$\bar{\sigma}_H = 158.0\ MPa$$

$$\sigma_o = X_o \bar{\sigma}_H = 3(158.0) = 474\ MPa$$

7.11 Block confined in y dir., free in x.

$\varepsilon_y = 0$, $\sigma_x = 0$, $\sigma_x, \sigma_y, \sigma_z = \sigma_1, \sigma_2, \sigma_3$

$\sigma_z = ?$, at yieding. Poisson's ratio effect?

$$\varepsilon_y = \frac{\sigma_y}{E} - \frac{\nu}{E}(\sigma_x + \sigma_z), \quad 0 = \sigma_y - \nu\sigma_z, \quad \sigma_y = \nu\sigma_z$$

$$\bar{\sigma}_S = \mathrm{MAX}(|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|) = \sigma_0$$

$$\mathrm{MAX}(|0 - \nu\sigma_z|, |\nu\sigma_z - \sigma_z|, |\sigma_z - 0|) = \sigma_0$$

$\sigma_z = \sigma_0$, No effect of confinement or ν.

Second Solution:

$$\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} = \sigma_0$$

$$\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{(0 - \nu\sigma_z)^2 + (\nu\sigma_z - \sigma_z)^2 + (\sigma_z - 0)^2} = \sigma_0$$

$$\sigma_z = \frac{\sigma_0}{\sqrt{1 - \nu + \nu^2}}$$

ν	0	0.3	0.5
σ_z / σ_0	1	1.125	1.155

In this case a, modest effect is predicted that depends on ν.

Note: σ_z and σ_0 above both have negative values,

7.15 Solid circular shaft. $T = 1.5$ kN·m
$M = 1.0$ kN·m, $X_o = 2$. Tabulate σ_o for each material using Tables 4.2 and P4.29. Assume ρ and C_m are the same as the most similar materials in Table 3.13.

Material	σ_o MPa	ρ g/cm^3	Cost C_m	d mm	Mass f_2 $\rho/\sigma_o^{2/3}$	Mass Rank	Cost f_2 $C_m\rho/\sigma_o^{2/3}$	Cost Rank
1020 steel	260	7.9	1	50.5	0.194	8	0.194	4
2024 Al	303	2.7	6	47.9	0.060	4	0.359	8
7075 Al	469	2.7	6	41.4	0.045	1	0.268	7
4140 (205)	1583	7.9	3	27.6	0.058	2	0.174	1
4140 (315)	1560	7.9	3	27.8	0.059	3	0.176	2
4140 (425)	1399	7.9	3	28.8	0.063	5	0.189	3
4140 (540)	1158	7.9	3	30.7	0.072	6	0.215	5
4140 (650)	872	7.9	3	33.7	0.087	7	0.260	6

(a) From the Prob. 7.9 solution using $\bar{\sigma}_H$, diameters are given by

$$d = \left(\frac{32 X_o}{\pi \sigma_o} \sqrt{M^2 + \frac{3}{4} T^2} \right)^{1/3}$$

$$d = \left[\frac{32 \times 2}{\pi (260 \text{ MPa})} \sqrt{\underset{\text{N·mm}}{(1 \times 10^6)^2} + 0.75 \underset{\text{N·mm}}{(1.5 \times 10^6)^2}} \right]^{1/3}$$

$d = 50.5$ mm (1020 steel; others similarly)

(b) Minimize mass and cost as in Sec. 3.8.
Requirements; T, M, X, L Geometry: d
Material: $\sigma_o, \rho, (C_m)$

(7.15, p.2)

$$m = \frac{\pi d^2}{4} L\rho = \left[\frac{\pi L}{4}\left(\frac{32 X_o}{\pi}\sqrt{M^2 + \frac{3}{4}T^2}\right)^{\frac{2}{3}}\right]\left[\frac{\rho}{\sigma_o^{2/3}}\right]$$

$m = f_1 f_2, \quad f_2 = \rho/\sigma_o^{2/3}$ and $C_m \rho/\sigma_o^{2/3}$

7075 Al has the lightest weight, but a considerably higher cost than the higher strength 4140 steels, which rank 2 and 3 as to light weight. However, the 4140 (205°C temper) has low ductility, 7% RA from Table P4.29. Use either 4140 (315) or 4140 (425).