8.1 AISI 4340 steel (Fig. 8.31), $\sigma_o = 800$ and 1600 MPa.

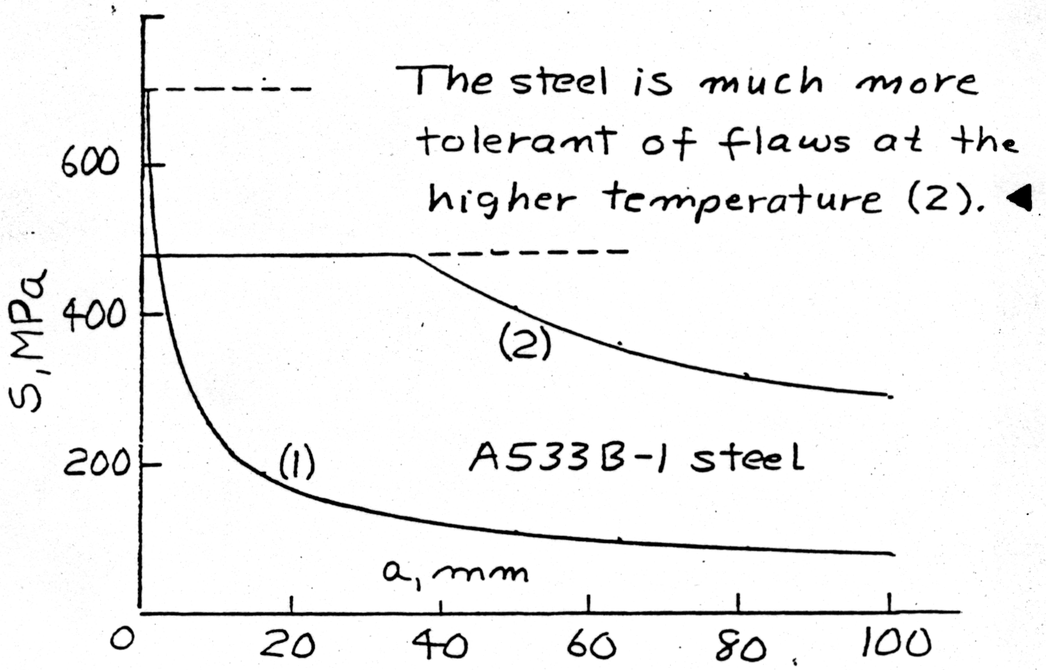
(a) $K_{Ic} = ?$ (b) $a_t = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma_o}\right)^2$

σ_o, MPa	K_{Ic}, MPa$\sqrt{m}$	a_t, mm
800	185	17.0
1600	40	0.20

The much smaller a_t for the higher σ_o indicates a greater sensitivity to flaws, so that brittle fracture would be an important design consideration.

8.2 $a_t = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma_o}\right)^2$, $K_{Ic} = S\sqrt{\pi a}$

	Temp. °C	K_{Ic} $MPa\sqrt{m}$	σ_o MPa	a_t m	a_t mm
(1)	−150	42	700	0.00115	1.15
(2)	+10	160	480	0.0354	35.4



(1) $S = \frac{K_{Ic}}{\sqrt{\pi a}} = \frac{42\ MPa\sqrt{m}}{\sqrt{\frac{\pi a\ mm}{1000\ mm/m}}} = \frac{749}{\sqrt{a}}$ MPa

(2) $S = \frac{K_{Ic}}{\sqrt{\pi a}} = \frac{160}{\sqrt{\frac{\pi a}{1000}}} = \frac{2855}{\sqrt{a}}$ MPa

8.5 Center-cracked plate, AISI 1144 steel, $b = 40$, $t = 15$ mm. $K_{Ic} = 66\ MPa\sqrt{m}$ (Tbl. 8.1)

$P = ?$ for $X = P_c/P = 3$ if $a = 10, 24$ mm

(a) $a = 10$ mm, $\alpha = a/b = 10/40 = 0.25$

$K = FS_g\sqrt{\pi a}$, $F \approx 1$ (Fig. 8.12)

$$S_g = \frac{P_c}{2bt} = \frac{XP}{2bt}, \quad P = \text{allowable design load}$$

$$K_{Ic} = F\,\frac{XP}{2bt}\sqrt{\pi a}$$

$$66\ MPa\sqrt{m} = \frac{3(P, N)}{2(40)(15)\ mm^2}\sqrt{\pi(0.010\ m}$$

$P = 148{,}900\ N = 148.9\ kN$ ◀

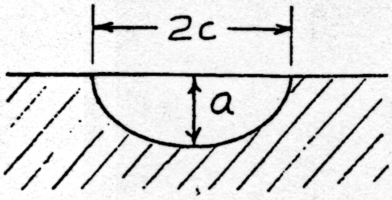
(b) $a = 24$ mm, $\alpha = a/b = 0.6$

$$F = \frac{1 - 0.5\alpha + 0.326\alpha^2}{\sqrt{1-\alpha}} = 1.292 \quad \text{(Fig. 8.12)}$$

$$66 = 1.292\,\frac{3P}{2(40)(15)}\sqrt{\pi(0.024)}$$

$P = 74{,}400\ N = 74.4\ kN$ ◀

8.9 ASTM A470-8 steel, $S = 250$ MPa



$2c = 50$ mm

$a = 15$ mm

$K_{Ic} = 60\ \mathrm{MPa}\sqrt{m}$

(Table 8.1)

Safe?

Fig. 8.19 applies

$$K = FS\sqrt{\frac{\pi a}{Q}}, \quad F = 1.12 \quad \left(\frac{a}{t}, \frac{c}{b} \text{ small}\right)$$

$$Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65} = 1 + 1.464\left(\frac{15\ \mathrm{mm}}{25\ \mathrm{mm}}\right)^{1.65}$$

$$Q = 1.630$$

$$K = 1.12\,(250\ \mathrm{MPa})\sqrt{\frac{\pi\,(0.015\ \mathrm{m})}{1.630}}$$

$$K = 47.6\ \mathrm{MPa}\sqrt{m}$$

$$X = \frac{K_{Ic}}{K} = \frac{60\ \mathrm{MPa}\sqrt{m}}{47.6\ \mathrm{MPa}\sqrt{m}} = 1.26 \quad \text{No!}$$

X is inadequate to safely operate.

8.12 Shaft, $d = 50$ mm, with circumferential crack, $a = 5$ mm. 18 Ni maraging steel
$K_{Ic} = 123$ MPa$\sqrt{m}$ (air melted, Table 8.1)

(a) $X = ?$ if $M = 1.5$ kN·m
Fig. 8.14 (b) applies: $S_g = 4M/(\pi b^3)$
$K = F S_g \sqrt{\pi a}$, $\alpha = a/b$, $\beta = 1 - \alpha$
$b = d/2 = 25$ mm, $\alpha = 5/25 = 0.2$, $\beta = 0.8$

$$F = \frac{3}{8\beta^{2.5}}\left[1 + \frac{\beta}{2} + \frac{3}{8}\beta^2 + \frac{5}{16}\beta^3 + \frac{35}{128}\beta^4 + 0.537\beta^5\right]$$

$F = 1.368$

$$K = F S_g \sqrt{\pi a} = 1.368\,\frac{4(1.5\times10^6\ \text{N·mm})}{\pi\,(25\ \text{mm})^3}\sqrt{\pi(0.005)}\ \text{m}$$

$K = 20.95$ MPa$\sqrt{m}$, $X = K_{Ic}/K = 5.87$ ◀

(b) Add 120 kN tension (Fig. 8.14(a))

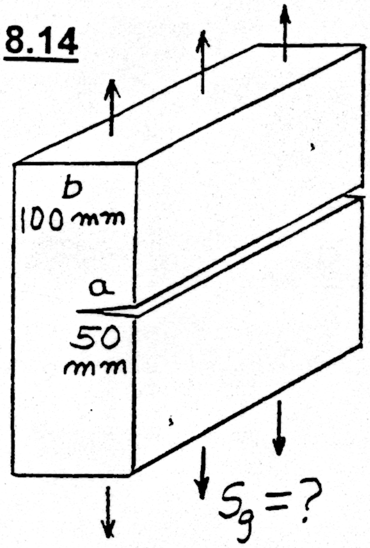
$$F_2 = \frac{1}{2\beta^{1.5}}\left[1 + \frac{\beta}{2} + \frac{3}{8}\beta^2 - 0.363\beta^3 + 0.731\beta^4\right] = 1.225$$

$K_2 = F_2 S_{g2}\sqrt{\pi a}$, $S_{g2} = P/(\pi b^2)$

$$K_2 = 1.225\,\frac{120{,}000\ \text{N}}{\pi\,(25\ \text{mm})^2}\sqrt{\pi\,(0.005\ \text{m})} = 9.39\ \text{MPa}\sqrt{m}$$

$K = K_1 + K_2 = 20.95 + 9.39 = 30.33$ MPa$\sqrt{m}$
where K_1 is K due to bending from (a).
$X = K_{Ic}/K = 123/30.33 = 4.05$ ◀

8.14



A533B-1 steel, $-75\,°C$

$K_{Ic} = 52\ MPa\sqrt{m}$

$\sigma_o = 550\ MPa$

$K = F S_g \sqrt{\pi a} = K_{Ic}$

$F = F(a/b = 0.5)$

$F = 2.82$ from equation for Fig. 8.12(c)

$$S_g = \frac{K_{Ic}}{F\sqrt{\pi a}} = \frac{52\ MPa\sqrt{m}}{2.82\sqrt{\pi \times 0.05\ m}} = 46.5\ MPa \blacktriangleleft$$

Discussion: The usual basis of ductile weld design is to keep the net stress below yield by some safety factor X_o.

$$S_n = \frac{P}{(b-a)t} = \frac{P}{bt(1-a/b)} = \frac{S_g}{1-a/b} = \frac{46.5}{1-0.5} = 93\ MPa$$

$$X_o = \frac{\sigma_o}{S_n} = \frac{550}{93} = 5.9$$

This X_o suggests that there is considerable safety, but if the flaw acts as a crack, the safety factor that controls the behavior is $X_K = 1$ at $S_g = 46.5\ MPa$.