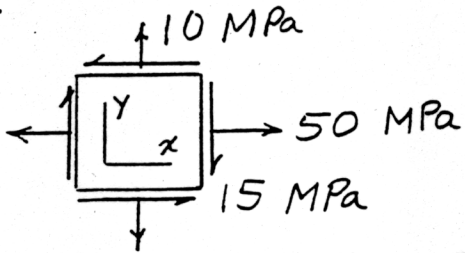


6.1

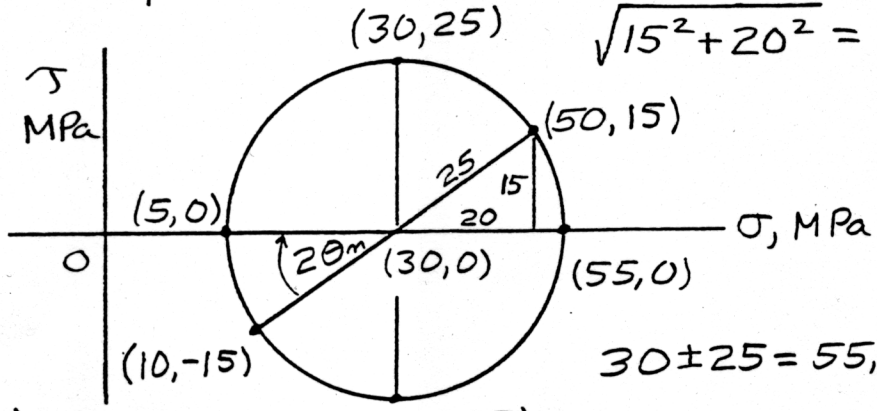


$$\tan 2\theta_m = \frac{15}{20}$$

$$\theta_m = 18.4^\circ$$

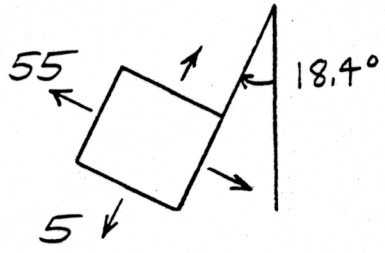
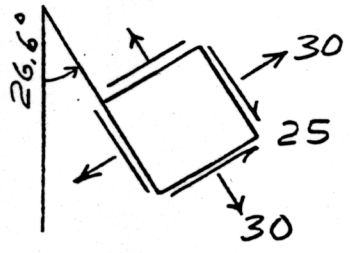
$$\theta_s = 45 - \theta_m = 26.6^\circ$$

$$\sqrt{15^2 + 20^2} = 25$$



$$30 \pm 25 = 55, 5$$

(a)



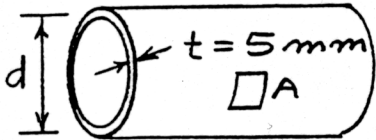
$$\sigma_1, \sigma_2 = 55, 5 \text{ MPa}, \tau_3 = 25 \text{ MPa}$$

(b)  $\sigma_3 = 0$

$$\tau_{max} = \text{MAX} \left( \frac{|\sigma_1 - \sigma_2|}{2}, \frac{|\sigma_2 - \sigma_3|}{2}, \frac{|\sigma_3 - \sigma_1|}{2} \right)$$

$$\tau_{max} = \text{MAX} (25, 2.5, 27.5) = 27.5 \text{ MPa}$$

6.5

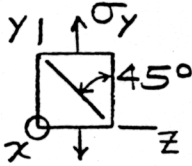
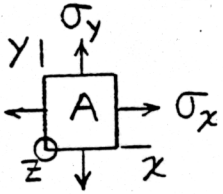


$d = 3 \text{ m}$  (inner)

$p = 2 \text{ MPa}$

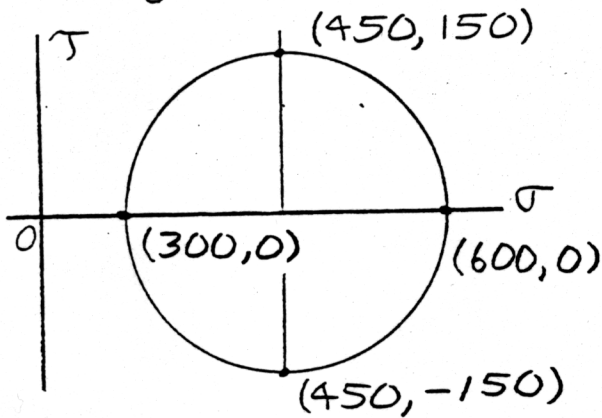
$\sigma_{\max}, \tau_{\max} = ?$

What planes?



$$\sigma_x = \frac{pr}{2t} = \frac{(2 \text{ MPa})(1500 \text{ mm})}{2(5 \text{ mm})} = 300 \text{ MPa}$$

$$\sigma_y = \frac{pr}{t} = 600 \text{ MPa}$$



$(x, y)$  are  $(1, 2)$

$\sigma_1, \sigma_2, \sigma_3 =$

$600, 300, 0 \text{ MPa}$

$\sigma_{\max} = 600 \text{ MPa}$

$$\tau_{\max} = \text{MAX} \left( \frac{|\sigma_1 - \sigma_2|}{2}, \frac{|\sigma_2 - \sigma_3|}{2}, \frac{|\sigma_3 - \sigma_1|}{2} \right)$$

$$\tau_{\max} = \text{MAX} (150, 150, 300) = 300 \text{ MPa}$$

$\tau_3 = 150 \text{ MPa}$  in  $x$ - $y$  plane is not  $\tau_{\max}$   
 $\tau_{\max}$  acts on a plane  $45^\circ$  to surface  
 ( $x$ - $y$  plane), as shown above.

**6.10** Find  $\sigma_1, \sigma_2, \sigma_3$  and  $(l, m, n)$  for each.

$$\sigma_x = 50, \sigma_y = 10, \sigma_z = -20, \tau_{xy} = -15 \text{ MPa}$$

$$\tau_{yz} = \tau_{zx} = 0$$

(a)  $\sigma_3 = \sigma_z = -20 \text{ MPa}$  ◀

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1, \sigma_2 = 30 \pm 25 = 55, 5 \text{ MPa} \quad \blacktriangleleft$$

$$\tau_1, \tau_2, \tau_3 = \frac{|\sigma_2 - \sigma_3|}{2}, \frac{|\sigma_3 - \sigma_1|}{2}, \frac{|\sigma_1 - \sigma_2|}{2}$$

$$\tau_1, \tau_2, \tau_3 = 12.5, 37.5, 25 \text{ MPa} \quad \blacktriangleleft$$

(b)  $\sigma_1 = 55, \quad l_i^2 + m_i^2 + n_i^2 = 1$

$$\left. \begin{aligned} (\sigma_x - \sigma_i) l_i + \tau_{xy} m_i + \tau_{zx} n_i &= 0 \\ \tau_{xy} l_i + (\sigma_y - \sigma_i) m_i + \tau_{yz} n_i &= 0 \\ \tau_{zx} l_i + \tau_{yz} m_i + (\sigma_z - \sigma_i) n_i &= 0 \end{aligned} \right\} \begin{array}{l} \text{one} \\ \text{red.} \end{array}$$

$$\left. \begin{aligned} -5l_1 - 15m_1 &= 0, \quad l_1 = -3m_1 \\ -15l_1 - 45m_1 &= 0 \quad (\text{redundant}) \\ -75m_1 &= 0, \quad m_1 = 0 \end{aligned} \right\}$$

$$l_1^2 + m_1^2 + 0 = 1$$

$$(-3m_1)^2 + m_1^2 = 1, \quad m_1 = 1/\sqrt{10} = 0.316$$

$$l_1, m_1, n_1 = -0.949, 0.316, 0 \quad \blacktriangleleft$$

(6.10, p.2)

$$\sigma_2 = 5, i = 2$$

$$45l_2 - 15m_2 = 0, m_2 = 3l_2$$

$$-15l_2 + 5m_2 = 0 \quad (\text{redundant})$$

$$-25n_2 = 0, n_2 = 0$$

$$l_2^2 + m_2^2 + n_2^2 = 1$$

$$l_2^2 + (3l_2^2) + 0 = 1, l_2 = 1/\sqrt{10} = 0.316$$

$$l_2, m_2, n_2 = 0.316, 0.949, 0$$

$$\sigma_3 = -20, i = 3$$

$$\begin{array}{l} 70l_3 - 15m_3 = 0 \\ -15l_3 + 30m_3 = 0 \end{array} \quad \left. \begin{array}{l} \text{Satisfied by only} \\ l_3 = m_3 = 0 \end{array} \right\}$$

$$0 = 0$$

$$l_3^2 + m_3^2 + n_3^2 = 1, n_3 = 1$$

$$l_3, m_3, n_3 = 0, 0, 1$$



(6.16, p.2)

(b)  $r_1 = 40$ ,  $r_2 = 50$  mm,  $p = 100$  MPa

$R = 40$  mm (inner wall)

$$C = \frac{p r_1^2}{r_2^2 - r_1^2} = \frac{(100 \text{ MPa})(40)^2 \frac{\text{mm}^2}{\text{mm}^2}}{50^2 - 40^2} = 177.78 \text{ MPa}$$

$$\sigma_1 = \sigma_t = C \left( \frac{r_2^2}{R^2} + 1 \right) = C \left( \frac{50^2}{40^2} + 1 \right) = 455.6 \text{ MPa} \blacktriangleleft$$

$$\sigma_2 = \sigma_r = -C \left( \frac{r_2^2}{R^2} - 1 \right) = -C \left( \frac{50^2}{40^2} - 1 \right) = -100 \text{ MPa} \blacktriangleleft$$

$$\sigma_3 = \sigma_x = C = 177.8 \text{ MPa} \blacktriangleleft$$

$$\tau_1 = C \frac{r_2^2}{R^2} = C \frac{50^2}{40^2} = 277.8 \text{ MPa} \blacktriangleleft$$

$$\tau_2 = \tau_3 = \frac{C}{2} \frac{r_2^2}{R^2} = 138.9 \text{ MPa} \blacktriangleleft$$