Solution to
HW #3
MIM 1440

Problem 4.1

Stiffness : Resistance to Deformation

Strength : Resistance to Failure

Ductility : Ability to Deform (Indication of How Deformable)

Yielding : Strength at the boundary of elastic Plastic Regime.

Toughness : Energy/Volume to break a Material

Strain Hardening : Indication of Material hardening with Deformation.

Problem 4.2

Brittle : No Plastic Deformation Prior to Failure

Ductile : Deform Plastically Prior to Failure

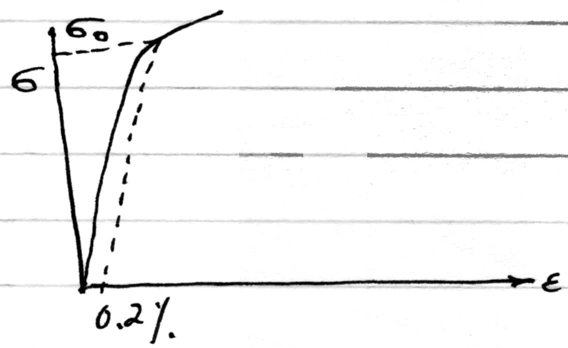
Resilient : Energy stored in the elastic Regime.

Tough: enery Required to break

Stiff : Resistance to Deformation: Such spring constant

Strong: high stress for failure.

4.3

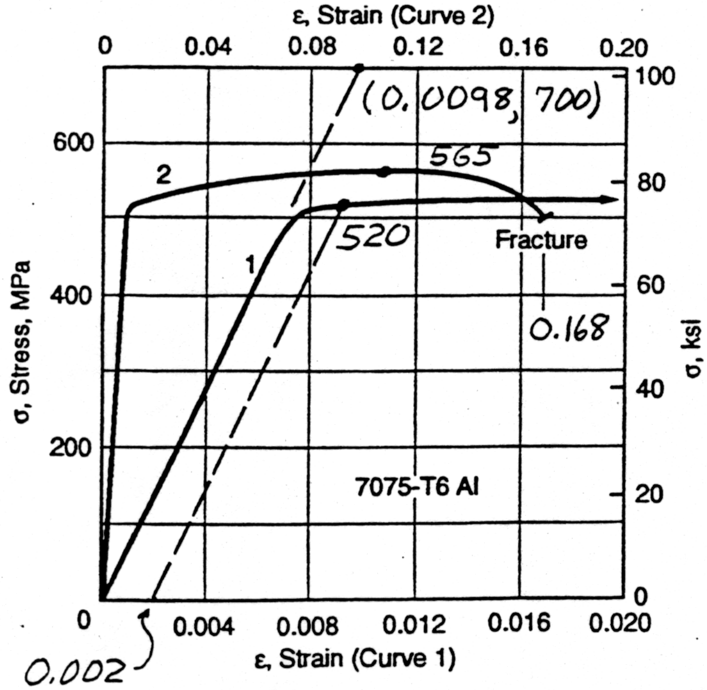


The offset is better, because it depends less on the sensitivity of the Instrument used.

offset may not be useful if you would like to avoid permanent defoTmation.

4.5

4.5 $7075\text{-}T6\ Al,\ d_i = 9.07,\ d_f = 7.29\ mm$



0.0098 from Curve 1 scale

0.168 from Curve 2 scale

$\sigma_f = 500$ MPa

$\varepsilon_f = 0.168$

$$E = \frac{\Delta\sigma}{\Delta\varepsilon} = \frac{700\ MPa}{0.0098} = 71,400\ MPa = 71.4\ GPa$$ ◀

At $\varepsilon_{po} = 0.002,\ \sigma_o = 520\ MPa$ ◀

$\sigma_u = 565\ MPa$ ◀

$100\varepsilon_f = 16.8\ \%,\ 100\ \varepsilon_{pf} \approx 16.1\ \%$ ◀

$$\varepsilon_{pf} \approx \varepsilon_f - \frac{\sigma_f}{E} = 0.168 - \frac{500}{71,400} = 0.161$$

$$\%RA = 100\frac{d_i^2 - d_f^2}{d_i^2} = 100\ \frac{9.07^2 - 7.29^2}{9.07^2} = 35.4\ \%$$ ◀

Problem: 4.12

Engineering stress is force / original cross section = F/A_o

true stress = $\frac{F}{A}$ ↳ cross section at any time.

Engineering strain is $\frac{L-L_o}{L_o}$ percentage based on the original length.

True strain is the summation of all incremental strain

$$\tilde{\varepsilon} = \frac{\Delta L_1}{L_1} + \frac{\Delta L_2}{L_2} + \frac{\Delta L_3}{L_3} + \cdots = \sum \frac{\Delta L_i}{L_i} = \int \frac{dL}{L} = l$$

problem 4.15

4.15 Find σ_u in terms of H and n.

$$\tilde{\sigma}_u = H\tilde{\varepsilon}_u^{\,n}, \quad n \approx \tilde{\varepsilon}_u, \quad \tilde{\sigma}_u = Hn^n$$

$$\sigma_u = \frac{\tilde{\sigma}_u}{1+\varepsilon_u} = \frac{\tilde{\sigma}_u}{e^{\tilde{\varepsilon}_u}} = \frac{\tilde{\sigma}}{e^n} = H\left(\frac{n}{e}\right)^n$$

Note: Since $\tilde{\varepsilon}_u = \ln(1+\varepsilon_u)$, we have

$$e^{\tilde{\varepsilon}_u} = 1+\varepsilon_u$$

Problem 4.21

$E = 73.1$ GPa $\quad \sigma_o = 303$ MPa $\quad \sigma_u = 476$ MPa $\quad$ Elongation $= 20\%$ $\quad$ Reduction of Area 35%

$\tilde{\sigma}_{fB} = 631$ MPa $\quad \tilde{\varepsilon}_f = 0.43 \quad H = 806 \quad n = 0.2$

$$\tilde{\sigma} = H(\tilde{\varepsilon})^n \implies \tilde{\sigma} = 806(\tilde{\varepsilon})^{0.2}$$ good up to necking $\tilde{\varepsilon} = 0.2$

having true stress vs. true strain, engineering stress vs. engineering strain will be

$$\tilde{\sigma} = \sigma(1+\varepsilon)$$

$$\tilde{\varepsilon} = \ln(1+\varepsilon) \qquad \tilde{\varepsilon} = 0.2 = \ln(1+\varepsilon)$$

$$\varepsilon = 0.221$$

$$\sigma(1+\varepsilon) = 806(\ln(1+\varepsilon))^{0.2}$$

$$\sigma = \frac{806(\ln(1+\varepsilon))^{0.2}}{1+\varepsilon}$$

[illegible]

$$\frac{\sigma_o}{E} \le \varepsilon \le 0.221$$

You can plot this curve

$$4.14 \times 10^{-3} \le \varepsilon \le 0.221$$

Results

In the elastic Regime $\sigma = E\varepsilon$ $\quad \varepsilon \leq 4.14\times10^{-3}$

In the plastic Regime upto necking

$$\sigma = \frac{806\,(\ln(1+\varepsilon))^{0.2}}{1+\varepsilon}$$

$$4.14\times10^{-3} \leq \varepsilon \leq 0.221$$

$$\tilde{\sigma} = 806\,(\tilde{\varepsilon})^{0.2} \qquad 0 \leq \tilde{\varepsilon} < 0.2$$

Assuming Bridgman constant holds for Aluminum.

$$\tilde{\sigma}_B = B\,\tilde{\sigma}$$

$$B = 0.83 - 0.186\log\tilde{\varepsilon}$$

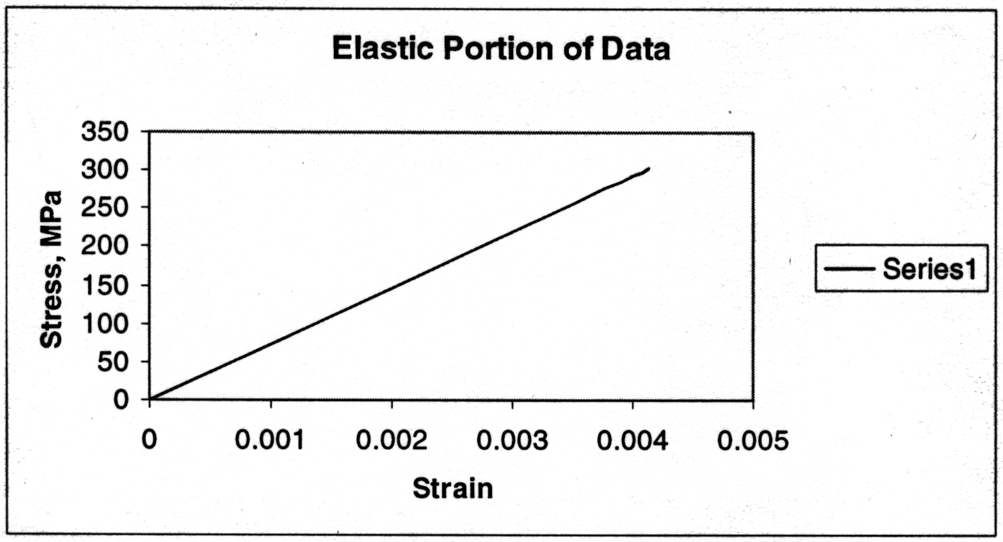
$$\tilde{\sigma}_B = H\,(\tilde{\varepsilon})^{0.2}$$

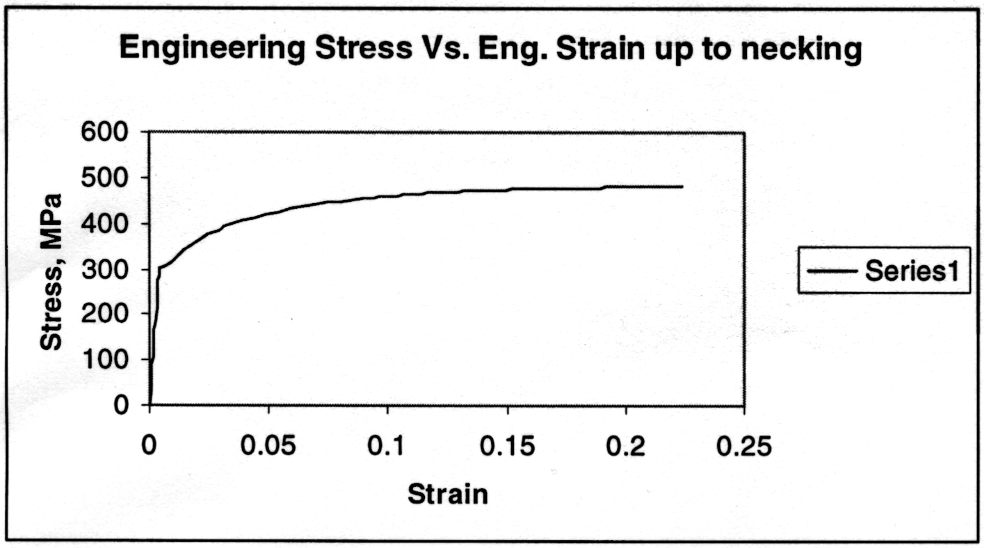
$$\tilde{\sigma} = \frac{H(\tilde{\varepsilon})^{0.2}}{(0.83 - 0.186\log\tilde{\varepsilon})} \qquad 0.2 \leq \tilde{\varepsilon} < 0.43$$

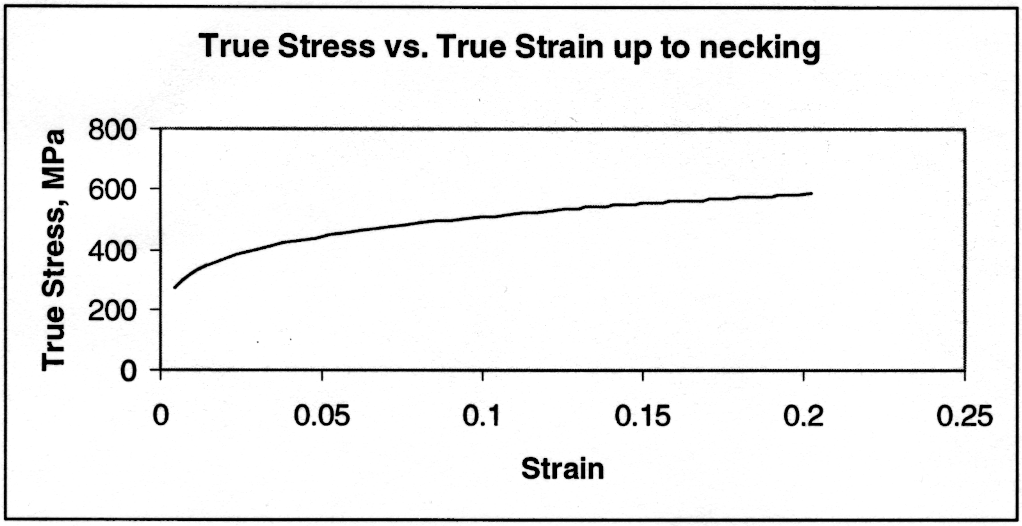
$$\tilde{\sigma} = \sigma e^{\tilde{\varepsilon}} \Rightarrow \sigma = \frac{\tilde{\sigma}}{e^{\tilde{\varepsilon}}}$$

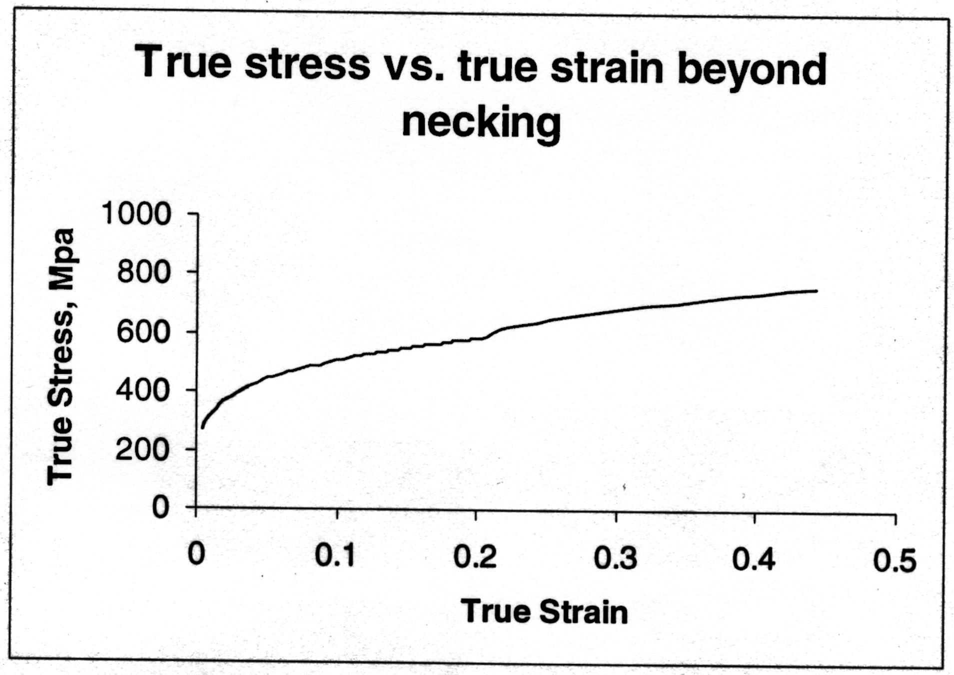
$$\sigma = \left(\frac{806(\tilde{\varepsilon})^{0.2}}{(0.83 - 0.186\log\tilde{\varepsilon})}\right) / e^{\tilde{\varepsilon}}$$

$$0.2 \leq \tilde{\varepsilon} \leq 0.43$$


Elastic Portion of Data
Stress, MPa
350
300
250
200
150
100
50
0
0
0.001
0.002
0.003
0.004
0.005
Strain
Series1


Engineering Stress Vs. Eng. Strain up to necking
Stress, MPa
600
500
400
300
200
100
0
0
0.05
0.1
0.15
0.2
0.25
Strain
Series1


True Stress vs. True Strain up to necking
True Stress, MPa
800
600
400
200
0
0
0.05
0.1
0.15
0.2
0.25
Strain


True stress vs. true strain beyond necking
True Stress, Mpa
1000
800
600
400
200
0
0
0.1
0.2
0.3
0.4
0.5
True Strain