Example

Beam span: 48'
Beam spacing: 10' = 120"
\( t_s = 6" \)
\( t_c = 4" \)
\( h_r = 2" \)  \( \perp \) deck  \( \therefore L_b = 12" \)
\( w_r = 6" \)  \( H_s = 5" \)  \( C_b = 1 \)
\( F_y = 50 \text{ ksi} \)  \( F_u \) (headed stud) = 65 ksi
\( f'_c = 3.5 \text{ ksi} \)  normal weight
\( \Delta_{\text{max}}^{\text{dead}} = L/240 = 2.4" \)
\( \Delta_{\text{live}} = L/360 = 1.6" \)

Unshored construction

Costs
\[ \$2.00/\text{stud} \]
\[ \$2.00/\text{lb} \]

\[ \text{DL} \quad \text{Concrete:} \quad \frac{5''}{12\%} \left(145 \text{pcf}\right) (10') = 604 \text{ lb/ft} = 0.604 \text{ k/ft} \]

\( \begin{align*}
W_D & \left\{ \\
\text{Steel:} & \quad \text{Assume} \quad 100\text{ lb/ft} = 0.1 \text{ k/ft} \\
\text{Superimposed dead:} & \quad 15 \text{ psf} \quad (10') = 150 \text{ lb/ft} = 0.15 \text{ k/ft} \\
\end{align*} \]

\[ \begin{align*}
A_I & = 2 \\
A_{\text{trib}} & = 2 (48') (5' + 5') = 960 \text{ ft}^2
\end{align*} \]

\[ \begin{align*}
\text{LLR:} & \quad L = (100 \text{ psf}) \left(0.25 + \frac{15}{\sqrt{960}}\right) = 73.4 \text{ psf} \\
W_L & = 73.4 (10') = 734 \text{ lb/ft} = 0.734 \text{ k/ft} \\
\text{Construction} & \quad W_C = 20 \text{ psf} (10') = 200 \text{ lb/ft} = 0.2 \text{ k/ft}
\end{align*} \]
Composite Beam

Beam, typical

Plan View

(deck orientation)

Girder

Girder

10'

typical

48'
Composite Beam

Section View

Steel beam

\( t_c = 4'' \)

\( h = 2'' \)

\( H_s = 5'' \)

\( W_r = 6'' \)

\( t_s = 6 \)
Can either select bare steel beam based on factored DL+construction, or composite beam based on factored DL+LL.

- Bare steel is often easier to do first and may control, especially if deck is parallel and $L_b = \text{length of beam}$.

Let's do composite first though:

$$W_u = 1.2 W_d + 1.6 W_L = 1.2 (0.604 + 0.1 + 0.15) + 1.6 (0.734) = 2.2 \text{ k}\ell$$

$$m_u = \frac{W_u L^2}{8} = 633 \text{ k}\ell$$

Two methods for first guess

I. Bump down from a bare steel beam \(\approx 2 \text{ sizes}\)

$$Z_x^{req} = \frac{633(12)}{0.9(50)} = 169 \text{ in}^3$$

Use \(W_{24\times68}^{50}\)

Bump down two sizes: say \(W_{24\times55}\), etc.

II. Use composite beam tables (AISC Table 3.19)

Assume PNA is in slab (i.e., fully composite)

$$Y_2 = t_s - \frac{a}{2}$$

Take \(a/2\) as halfway down from top of slab to top of flange as an initial guess

$$Y_2 = 6'' - \frac{4''}{2} = 4''$$

Use TFL = top of flange \(\Rightarrow\) that's the row corresponding to PNA in slab.
For $Y_2 = 4''$, $M_u^{req} = 633 \text{ k}\cdot\text{lbf}$, $F_y = 50 \text{ ksi}$.

For TFL row, best section is p. 3-172 W21×44

Breaches $\frac{L}{d} < 24$

Note: W21×44 is lighter than assumed 100 plf service load, W21×44 is lighter than assumed 100 plf service load.

Check deflection:

$I_t^{req} = \frac{5W_L L^4}{384EI_{max}} = \frac{5(0.734+0.15)12 (48.12)^4}{384 (29000) (1.6)}$

= $2276 \text{in}^4$

Could compute $I_t$ (accounting for metal deck!), or could use $I_{LB}$ (Table 3-20).

For $Y_2 = 4''$, TFL, W21×44:

$I_{LB} = 2180 \text{in}^4 \approx I_t^{req} = 2276 \text{in}^4$

Let's see why real $Y_2$

Now compute $M_n$:

First: $\frac{b}{tw} = 53.6 < \lambda_p = 3.76 \sqrt{\frac{F_y}{E}} = 91$ OK for $W_{LB}$

Section I 3.2

Then: 

$C = A_s F_y = 13.0 (50) = 650 \text{k}$

$C = 0.85 f'c A_c$

$A_c = t_c b e$

$t_c = 4''$ Sec. I 3.2 c

$be = \min \left\{ \frac{1}{8} + \frac{1}{8} = 6 + 6 = 12' = 144'' \right\}$

$\frac{1}{2} + \frac{1}{2} = 5 + 5 = 10' = 120''$ Controls

Edge of slab: not relevant here.
\[ 2C = 0.85 (3.5)(4)(120) = 1428 \text{ kN} > 'C \]
\[ 3C = \sum F_n = 650 \text{ kN} \rightarrow \text{we'll discuss later} \]
\[ C = \min \left\{ \begin{array}{c} 2C \\ 3C \end{array} \right\} = \frac{2C}{3} = 650 \text{ kN} \]
\[ a = \frac{\frac{f'_c}{0.85f'c_b}}{b_e} = 1.82'' \]

From AISC LRFD commentary:
\[ M_n = C(d_1 + d_2) + P_y(d_3 - d_2) \]
\[ C = 0.85C = 650 \text{ kN} \]
\[ P_y = 'C = 650 \text{ kN} \]
\[ d_1 = t_s - \frac{a}{2} = 6 - \frac{1.82}{2} = 5.09'' \]
\[ d_2 = 0 \text{ (no compression in steel)} \]
\[ d_3 = \frac{d_1}{2} = 20.7''/2 = 10.35'' \]
\[ M_n = 640 (5.09 + 0) + 650 (10.35'' - 0) = 10,031 \text{ kN}'' \]
\[ = 836 \text{ kN}'' \]
\[ \phi_{bc} M_n = 0.9 (836 \text{ kN}'' ) = 752 \text{ kN}'' > M_u = 633 \text{ kN}'' \text{ ok} \]
\[ Y_2 = \frac{d_1}{5} = 5.09'' \]
AISC p.3-174 w/ \[ Y_2 = 5.09'' \& TFL. \text{ Interpolate} \]
\[ \text{between } Y_2 = 5 \& 5.5: } \]
\[ \phi_{bc} M_n = 751 \text{ kN}'' \text{ confirmed} \]
Checks

1) \( \Delta_{SL+L} \Rightarrow \text{For } y_2 = 5.09', I_{LB} \geq \) at least \( 2370 \text{ in}^4 > I_c \text{req} = 2275 \text{ in}^4 \)

\[ \Delta_L \text{ OK (AISC Table 3.20)} \]

2) Construction (unshored)

\( W_{u,\text{const}} = 1.2 \left( 0.604 + 0.044 \right) + 1.6 \left( 0.2 \right) = 1.10 \text{ k/ft} \)

\[ W_{u,\text{const}} L^2 \] \[ \text{FLB: } \lambda = \frac{b f}{2 c_f} = 7.22 < \lambda_p = 0.38 \sqrt{\frac{E}{f_y}} = 9.2 \text{ OK} \]

\[ \text{LTB: } L_B = 12'' = 0 \]

\[ \phi_b m_n = \phi_b m_p = \phi_b Z_x f_y \]

\[ = 0.9 \left( 95.4 \right) \left( 50 \right) = 358 \text{ k/ft} \geq \frac{12}{M_u = 317 \text{ k/ft}} \]

3) Camber:

\( W_d = 1.0 \left( 0.604 + 0.044 \right) = 0.648 \text{ k/ft} \)

no superimposed included here

\[ \Delta_D = 5 \frac{W_d L^4}{384 E I_x} = 5 \frac{0.648/12 \left( 48.12 \right)^4}{384 \left( 29000 \right) \left( 8/3 \right)} \]

of W21x44 = 3.17''

Camber \text{ it out:}

See limits in handout. Round down to 3'' cap (normally go to nearest 1/2'' anyway)
Then check: \( A_D - \text{camber} \leq \frac{1}{240} \) (i.e., limit)

4) Studs

\[
Q_n = 0.5 A_{sc} \sqrt{f'_{c}} E_c \leq R_g R_p A_{sc} F_u \quad \text{(EQ. I8-1)}
\]

\[
E_c = \frac{W}{1.5 \sqrt{f'_{c}}} = (145) \frac{1.5 \sqrt{3.5}}{3.267} = 3.267 \text{ ksi}
\]

\( A_{sc} \Rightarrow \frac{3}{4}'' \text{ Stud} \Rightarrow A_{sc} = 0.442 \text{ in}^2 \)

\( R_g = 0.85 \)

\( R_p = 0.60 \)

Assume "weak" stud position – see Comm. Fig. C-I8.1

\[
Q_n = 0.5 (0.442) \sqrt{3.5 (3.267)} \leq 0.85 (0.6) (0.442) (65)
\]

\[
= 23.6 \text{ k} \leq 14.7 \text{ k} \quad \text{controls}
\]

\[
N_{req} = \frac{C}{Q_n} = \frac{650}{14.7} = 44.1 \text{ call it 45 studs over distance from } M=0 \text{ to } M_{max} \text{ (sect. I8.2)}
\]

For our symmetric beam, that means 90 studs total

Stud spacing: Theoretically could be

\[
s = \frac{48(12)}{90} = 6.4''
\]

But with deck \( \perp \), ribs are @ 12''

:: Put 1 stud every 12'' \( \Rightarrow \) that's 24 over \( M=0 \) to \( M_{max} \). Then put extra stud/rib for 21 ribs starting from end where \( V_{max} \) is
Now you could fine-tune this (because $R_g = 1.0$ for ribs with one stud) and use fewer than 90 studs, but we're about to do partial composite.

Also check:

- Min. spacing: $4 \times (3\frac{3}{4}) = 3'' < 12''$ \textbf{OK}
- Max. spacing: $8 \times t_s = 8(6) = \frac{48}{3''} > 12''$ \textbf{OK}
- $t_s = 0.45'' > 2.5(0.45) = 1.1'' > 3\frac{3}{4}''$ \textbf{OK}

**Partial Composite Action**

Lower # of studs such that

$$C = \sum Q_n < C_{\text{Full comp}} = F_y A_s$$

in our case

$$\geq 0.25 C_{\text{Full comp}}$$

- Rule of thumb

Note: For % composite check, always use $A_s F_y$ for fully composite.

But of course: $\phi_b C_n > M_u$

and $I_{LB} \approx I_{eff} > I_{req}$

$$\% \text{ Composite} = \frac{\sum Q_n}{C_{\text{Full comp}}} \geq 0.25$$
So trial and error is used to determine # of studs
For example, try 1 stud/rib ⇒ 24 studs from M=0 to M

\[ R_g = 1.0 \text{ now} \]

\[ Q_n = 1.0 \left( 0.6 \right) \left( 0.442 \right) \left( 65 \right) = 17.2 \text{ k} \]

\[ \therefore \sum Q_n = 24 \left( 17.2 \text{ k} \right) = 414 \text{ k} \]

\[ a = \frac{\sum Q_n}{0.85 f' c b_e} = \frac{414}{0.85 (3.5)(120)} = 1.16" \]

\[ y_2 = 6 - \frac{a}{2} = 5.42" \]

Where is PNA?

If PNA is in slab, then full steel section is in tension:

\[ T = P_y = A_s F_y = 650 \text{ k} \]

This is greater than \( C = 414 \text{ k} \), so PNA must be in steel.

Is it in flange? To see, compute:

\[ P_y - 2 P_y f = A_s F_y - 2 b_f t_f F_y \]

\[ b_f = 6.5" \]

\[ t_f = 0.45" \]

\[ P_y - 2 P_y f = 358 \text{ k} < \sum Q_n = 414 \text{ k} \]

So yes, it's in the flange.

\[ 414 = A_s F_y - 2 b_f y_f \]

\[ y_f = 0.36" \text{ from top of flange} \]

So \( M = C(d_1 + d_2) + P_y (d_3 - d_2) \)

\[ d_1 = y_2 = 5.42" \]

\[ d_2 = \frac{y_f}{2} = 0.18" \]

\[ d_3 = 10.35" \]
\[ \phi_{bc} M_n = 0.9 \left[ 414 \left( 5.42 + 0.18 \right) + 650 \left( 10.35 - 0.18 \right) \right] = 8,036 \text{ k}^{-1} \]

\[ = 670 \text{ k}^{-1} > M_u = 633 \text{ k}^{-1} \quad \text{OK!} \]

To use tables: (See Fig. 3-3 on p. 34)

\[ Y_2 = 5.42'' \]

\[ Y_1 = 0.36'' \Rightarrow Y_1/t_f = 0.36/0.45 = 0.8 \quad \text{close to position 4} \]

\[ \Sigma Q_n = 414 \text{ K} \]

Use double interpolation on W21x44 w/ \( \Sigma Q_n \) & \( Y_2 \)

<table>
<thead>
<tr>
<th>PNA</th>
<th>( Y_1 )</th>
<th>( \Sigma Q_n )</th>
<th>( Y_2 )</th>
<th>5.0''</th>
<th>5.5''</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0.338''</td>
<td>434</td>
<td>663</td>
<td>674</td>
<td>84%</td>
</tr>
<tr>
<td>BFL</td>
<td>0.45''</td>
<td>358</td>
<td>632</td>
<td>647</td>
<td>84%</td>
</tr>
</tbody>
</table>

Gets 668 k^{-1} \quad \text{OK}

Similarly \( I_{LB} \approx 2110 \text{ in}^4 < I_{t \, \text{req}} = 2275 \text{ in}^4 \)

\[ I_t = 2525 \text{ in}^4 \text{ from elastic transformed section...} \]

So compute \( I_{\text{eff}} = I_s + \sqrt{\frac{\Sigma Q_n}{C_{\text{Full}}}} (I_t - I_s) \)

(AISC C-I3 - g)

\[ I_{\text{eff}} = 843 + \sqrt{\frac{414}{650}} (2525 - 843) = 2,185 \text{ in}^4 \]

\[ < I_{t \, \text{req}} = 2275 \text{ in}^4 \]

but close enough (\( \sim 4\% \) low) \quad \text{OK}
Finally check shear on bare steel: \( V_u < \phi_v V_n \) max. shear (AISC Sect. G2)

\[
\frac{h}{t_w} = \frac{53.6}{2.24} < 2.24 \sqrt{\frac{E}{F_y}} = 53.9
\]

So \( \phi_v V_n = 1.0 \left( 0.6 F_y \right) A_w C_v \)

\[
A_w = d t_w = 20.7 \left( 0.35 \right) = 7.25 \text{ in}^2
\]

\( C_v = 1.0 \)

\[
\phi V_n = 1.0 \left( 0.6 \right) \left( 50 \right) \left( 7.25 \right) \left( 1.0 \right) = 218 \text{ K}
\]

\[
V_u = \frac{W_u L}{2} = \frac{2.2 \left( 48 \right)}{2} = 52.8 \text{ K}
\]

\( \phi V_n > V_u \quad \text{OK} \)

And: Because \( \phi_{bc} M_n \) exceeds \( M_u \) by 25 k-in, you could reduce studs further, but 1 stud @ 24" doesn’t work here, & doing something in between is a little picky for construction workers.

Use: \( W21 \times 44 \left( 50 \right) \)

\( < 3^\prime > \left( 48 \right) \)

Camber \# studs

(Workers know how to place if Standard)