COMBINED BENDING AND AXIAL LOAD

The topics covered under this heading vary widely depending on the amount of time permitted. In a first steel course at most universities use of the interaction equations is merely illustrated for no-sway or conditions of individual beam-columns. Second-order behavior of beam-columns in frames is usually discussed in detail in advanced undergraduate or graduate courses. The presentation contained herein will provide a reasonably complete background to the new LRFD interactions since there is currently no published paper on their development. Emphasis is placed on combined bending and axial compression.

ASD and LRFD Interaction Equations. For combined axial compression and uniaxial bending, the following interaction equations are used:

ASD:

\[
\frac{f_a}{F_a} + \frac{C_m f_b}{\left(1 - \frac{f_a}{F'_b}\right) F_{bs}} \leq 1.0 \quad (1.6-1a)
\]

and

\[
\frac{f_a}{0.6 F_b} + \frac{f_b}{F_{bs}} \leq 1.0 \quad (1.6-1b)
\]

LRFD:

For

\[
\frac{P_u}{\phi P_n} \geq 0.2, \quad \frac{P_u}{\phi P_n} + \frac{8}{9} \frac{M_u}{\phi_b M_n} \leq 1.0 \quad (H1-1a)
\]

For

\[
\frac{P_u}{\phi P_n} < 0.2, \quad \frac{P_u}{2 \phi P_n} + \frac{M_u}{\phi_b M_n} \leq 1.0 \quad (H1-1b)
\]

In ASD two equations must be checked; Eq. (1.6-1a) is usually referred to as the stability check and Eq. (1.6-1b) as the yielding or stress check. The bending stress \( f_b \) is based on a first-order structural analysis. Second order effects are considered only in Eq. (1.6-1a) by use of the amplification factor, \( C_m/(1 - f_a/F'_b) \). Effective length factors are used in evaluating \( F_a \) and \( F'_b \).

In LRFD, only one equation is checked, depending on the magnitude of the axial load term, \( P_u/\phi P_n \). A plot of Eqs. (H1-1) is given in Fig. 1. \( P_n \) is based on the effective length of the column. \( M_u \) is based on a second-order elastic structural analysis. The second order analysis can be performed directly and is encouraged. Approximate second order analysis based on amplification factors are given in LRFD when only a first order analysis is performed. The amplification factors are similar to those in ASD but there are some differences which will be explained later.

Development of the LRFD Interaction Equations. In 1973, Task Group #10 was established by AISC to develop recommendations for the design of beam-columns. This effort was originally associated with the ASD Specification but since all comparisons and recommendations were based on ultimate strength solutions, the shift to an LRFD format was rather direct. A large number of individuals contributed to the work of this committee but special acknowledgment must be given to W. LeMessurier for his efforts in the development of the final recommendations contained in LRFD.
The following guidelines were established for the design recommendations.

- The recommendations should be general and apply to a wide range of problems: strong axis bending, weak axis bending, residual stresses, sway, no sway, laterally loaded, inelastic behavior, various L/r, leaned column systems, second order effects. Many suggested solutions which are published are very limited or incomplete.

- The designer should be permitted to perform the second order analysis so the recommendations should keep the second order analysis separate from the strength.

- The method should be based on the use of an elastic second analysis by the designer since no practical inelastic analysis is available for design office use.

- When compared to exact inelastic second order theoretical solutions, the recommended interaction equation with elastic second order analysis should not be more than 5% unconservative.

- Similar problems should give the same results. For example, the four problems shown in Fig. 2 are mathematically identical in both the elastic and inelastic regions. The individual column height varies between L/2 and 2L but the P-V relationship is the same for all cases.

The Task Group concentrated on solutions to unbraced frames since there was considerable literature on no-sway members on which the current ASD interaction equations are based. A large data bank of solutions to frames of the type shown in Fig. 3 was developed on a research project at the U. of Texas using an exact inelastic second order solution method. Some typical results are shown in Fig. 4. In Fig. 4a, the first order moment $H L_e/2$ is plotted against the sway deflection of the frame with an $L/r_w = 40$, a $P/P_v = 0.5$ and two values of $G_T = (I_e/L_e)/(I_0/L_0)$.
Fig. 2 Problems with the Same Solution

Two compressive residual stresses \( \sigma_{rc} = 0 \) and 0.3 \( F_y \) values were also considered. At the peak load, the second order moment was calculated and summarized in Fig. 4b by the solid lines for various values of \( P/P_y \). The girder is assumed to be very long so the axial load in each column is the same and equal to \( P/P_y \). For the weak axis bending shown, the peak load is reached when the second order column moment \( M_c \) is significantly less than the plastic bending strength of a short length of column, \( M_{pc} \). The second order effect \( P \Delta_{inelastic} \) is very significant due to the large plastic bending shape factor ( \( \sim 1.5 \) ) for weak axis bending of \( W \) shapes and its associated nonlinear moment-curvature relationship. The useful first order frame strength interaction is shown dashed. The first order strength is summarized for various \( L/\tau_y \) in Fig. 4c.

Fig. 3 Types of Unbraced Frames That Were Solved

A comparison of the theoretical solutions for weak axis bending and strong axis bending is shown in Fig. 5. For axial load within 60% of the buckling load \( (H L_e/M_p = 0) \), the non-dimensionalized weak axis strength is less than that for the strong axis. For low axial load, the weak axis is slightly stronger. At high axial load, the residual stresses more adversely affect the weak axis buckling load. At low axial load, the large shape factor results in significant inelastic nonlinear behavior which does not permit the section to take advantage of its superior plastic capacity \( M_{pc} \) as previously shown in Fig. 4b. The useful strength of the section is something less than \( M_{pc} \).
A. Typical shear resistance-away relationships of beam-columns in portal frames (weak axis bending)

B. Column end moment at maximum load (2nd order inelastic analysis)

P-delta effect at the maximum strength of beam-columns (weak axis bending) would require 2nd-order inelastic analysis

C. Maximum strength of beam-column in portal frames ($C=3$, weak axis bending)

Fig. 4 Ultimate Strength—Exact Inelastic Solution
Fig. 5 Comparison of Weak Axis and Strong Axis Solutions

Various suggestions for design interaction equations were tested against solutions calculated for the various frames shown in Fig. 3. Two of the popular proposals that did not work are shown in Fig. 6a and b. The proposal shown in Fig. 6a is an attempt to separate member stability from frame stability. The first term of the interaction equation uses $K = 1.0$ and $M_{max}$ is determined from a second order elastic analysis. This proposal is unconservative for high axial load. The $P\Delta$ method as given in the SSRC Guide to Design to Stability Design Criteria for Metal Structures, 3rd Ed., is also shown to be unconservative for weak axis bending in Fig. 6b. In this $P\Delta$ approach it is reasoned that if $M_{max}$ is based on an elastic second order analysis, then the strength can be based on a solution for braced frames ($K = 1.0$). In addition to these two proposals being unconservative, they also give different answers to the various problems shown in Fig. 2 which is not acceptable.

Figure 6c shows that the current ASD interaction equations for unbraced frames ($C_m = 0.85, K = 2.0$) is very accurate for the frame shown. The ASD equations have been merely rearranged into an ultimate strength format rather than allowable stress. A $K$-factor is used in the first term over most of the range. With this comparison and for the cases shown in Fig. 2 in which effective length is constant, the Task Group abandoned its effort to develop interaction equations which did not use $K$ factors.
Fig. 6 Various Interaction Equations

$P = \frac{1.0}{P_{ey}}$ 

$\frac{P}{P_{ey}} = \frac{M_{ey}}{M_{py}}$ 

$P = \frac{1.0}{P_{ey}}$ 

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$\frac{P}{P_{ey}} = \frac{M_{ey}}{M_{py}}$
The LRFD interaction relationship was determined by adding the calculated elastic $P\Delta$ moment to first order moment $HL$ which was developed from the exact solutions. The sum of these two moments is shown by the dashed curve in Fig. 7. The dashed curves were constructed for the wide variety of cases studied (strong axis bending, weak axis bending, leaned columns) and it was found that the LRFD equations shown in Fig. 1 provided a lower bound to all the dashed curves. The equation follows the trend of the simple plastic bending strength for strong axis bending, $P = \frac{P}{P_y} + \frac{1}{18} \frac{M}{M_{pl}} = 1.0$ with $P_y$ substituted for $P$, which is reasonable. For strong axis bending, the moment-curvature relationship is fairly linear up to $M_{pc}$ so an elastic second order analysis will be close to the ultimate strength solution. For weak axis bending, the interaction relationship is similar to that for strong axis bending because the relatively greater $M_{pc}$ strength for weak axis bending is offset by the much larger inelastic $P\Delta$ moments from the highly nonlinear moment-curvature relationship. The LRFD Eqs. H1-1 requires that $P_n$ be based on the effective column length and $M_n$ is the maximum second order elastic moment on the beam-column.

**Fig. 7 Elastic $P\Delta$ Interaction Relationship**

**Elastic Second Order Analysis.** A correct second order elastic analysis must consider both $P\delta$ and $P\Delta$ as illustrated in Fig. 8. The $P\delta$ moments are associated with the no-sway condition for the gravity load on the unbraced frame or when sway is prevented as in braced frames and truss members. If computer programs are used to calculate the second order moments make sure both $P\delta$ and $P\Delta$ are determined. Most commercial programs available do not calculate the $P\delta$ moments.

**$P\delta$ Moments.** The LRFD Specification gives approximations for the $P\delta$ moments for conditions with no joint translation. $M_{n1}$ is the maximum first order moment along the column.
length and the second order moment is \( M_u = B_1 M_n \), where \( B_1 \) cannot be less than 1.0, i.e., the second order moment can not be less than the first order moment.

\[
B_1 = \frac{C_m}{1 - \frac{P_u}{P_e}} \geq 1.0 \quad (H1-3)
\]

The values of \( C_m \) are the same as those used in ASD except that \( C_m = 0.6 - 0.4 \frac{M_1}{M_2} \), Formula H1-4 in LRFD, has no lower limit. In ASC, \( C_m \geq 0.4 \) but this was an error which is now corrected. The solution developed by Ketter in 1961 for the amplified moments is given in Fig. 9 and discussed on p. 6-162 of the LRFD Commentary as follows:

Formulas H1-3 and H1-4 approximate the maximum second order moments in compression members with no relative joint translation and no transverse loads between the ends of the member. This approximation is compared to an exact solution in Fig. C-H1.2. For single curvature, Formula H1-4 is slightly unconservative, for zero end moment it is almost exact, and for double curvature it is conservative. The 1978 AISC Specification limits \( C_m \approx 0.4 \) which corresponds to a \( M_1/M_2 \) ratio of 0.5. However, Fig. C-H1.2 shows that if, for example, \( M_1/M_2 = 0.8 \), the \( C_m = 0.28 \) is already very conservative, so the limit has been removed. The limit was originally adopted from Ref. 29, which was intended to apply to lateral-torsional buckling not second order in plane bending strength. The AISC Specifications, both in the 1978 and LRFD, use a modification factor \( C_m \) as given in Formula F1-3 for lateral-torsional buckling. \( C_m \), which is limited to 2.3, is approximately the inverse of \( C_m \) as presented in Ref. 29 with a 0.4 limit. In Ref. 94 it was pointed out that Formula H1-4 could be used for in plane second order moments if the 0.4 limit was eliminated. Unfortunately, Ref. 29 was misinterpreted and a lateral-torsional buckling solution was used for an in plane second order analysis. This oversight has now been corrected.
Fig. 9 Pe Moments for Straight Line Moment Diagram

For laterally loaded beam-columns, the exact and approximate solutions are given in Fig. 10. Note that the approximate solutions, which are reproduced in the Commentary, p. 163, are very accurate if $K < 1.0$ is used when the ends are restrained. Add the $K$ factors to Table C-H1.1 in the Commentary and replace the $f_a/F'_e$ term with $P_a/P_t$. If the maximum first order moments occur at ends as in the case of a uniformly loaded, fixed end beam, the end moments are amplified.

**PΔ Moments**. Approximate $PΔ$ moments can be calculated using the LRFD $B_2$ factor from Eqs. H1-5 or H1-6, the $PΔ$ and effective length methods, respectively. A general discussion and derivation of these two methods are given on the three sheets entitled, Second Order Moments in Sway Frames.

Example 1 illustrates how $B_1$ and $B_2$ are calculated for an unbraced frame with unsymmetric gravity load, i.e., no lateral load. Two structural analyses are required, (a) with sway prevented and (b) unbraced. The difference between these two analyses gives the $M_{B_2}$ to be used with $B_2$. Of course it will always be conservative to use $B_2$ with the actual first order moment diagram given by analyses (b), the true first order moment diagram.

**Design Examples**. For initial selection of a size, the following formula is useful.

$$P_{equiv} = P + \frac{2M_z}{d} + \frac{7.5M_x}{b} \quad (1)$$

or

$$M_{equiv} = M_z + Pd/2 \quad (2)$$

where $d$ is the nominal depth of the section, generally chosen as 12 in., and $b$ is the nominal width. Column tables can be used with the value of $P_{equiv}$ when axial load dominates the interaction
Fig. 10 Laterally Loaded Beam-Columns

CASE

M_{\text{diag}} \frac{w_0 P}{E I}

\begin{align*}
M_1 &= \frac{W L^2}{12} \\
M_2 &= \frac{W L^2}{24} \\
M_3 &= \frac{W L^2}{12} (\tan u - u) \\
M_4 &= \frac{W L^2}{8} \left( \frac{1}{2} u^2 - \frac{1}{2} \tan u \right)
\end{align*}

Amplified Moments: 

- \text{CASE 1:} M_{\text{END}} = \frac{W L^2}{8} (1 - \frac{0.39 P_{cr}}{P})^k
- \text{CASE 2:} M_{\text{END}} = \frac{W L^2}{8} (1 - \frac{0.39 P_{cr}}{P})^k
- \text{CASE 3:} M_{\text{END}} = \frac{W L^2}{8} (1 - \frac{0.19 P_{cr}}{P})^k
- \text{CASE 4:} M_{\text{END}} = \frac{3 W L^2}{16} (1 - \frac{0.39 P_{cr}}{P})^k

where \( k = 0.5 \) for \text{CASE 1} and \text{CASE 3}, and \( k = 0.7 \) for \text{CASE 2} and \text{CASE 4}.

(1) Timoshenko & Gere, "Theory of Elastic Stability" Chapter 1
Combined Bending and Tension. For simplicity, the same form of the interaction equation is used for axial tension. However, \( M_a \) can be used from a first order analysis and \( O P_a \) is based on the fracture on the effective net area or yielding on the gross area, whichever is less. Example 4 illustrates combined bending and tension.

**EXAMPLE 1 - SECOND ORDER MOMENTS**

\[
G_{1/v} = \frac{I/12}{I/32} = 2.13
\]

\[
G_{1/v} = \infty \quad \Rightarrow \quad k = 2.13
\]

\[
P_c = \frac{n^2 (2900)(890)}{(2.47 \times 15 \times 12)^2} = 1103 k
\]

\[
B_a = \frac{1}{1 - \frac{100}{2 \times 1.03}} = 1.047 \quad \text{(H1-5)}
\]

\[
\text{Braced}
\]

(a)

(b)

(c)

\[
\text{Unbraced}
\]

Fractured Moment Diagrams

or \( B_a \) by \( P \Delta \): \( (H1-5) = \frac{1}{1 - \frac{100}{2 \times 1.03}} = 1.046 \quad \text{(H1-5)}
\]

Note: \( B_a \) by \( H1-5 \) or \( H1-6 \) give the same answer.

**P & S Moments:**

\[
P_a \quad k=1.0 = 7862k \quad C_w = 0.6
\]

Col A: \( B = \frac{0.6}{1 - \frac{25}{7862}} = 0.606 = 6.06k \)

Col B: \( B = \frac{0.6}{1 - \frac{25}{7862}} = 0.602 = 6.02k \)

**Second Order Moments** - use \( H1-6 \) or Illustration

Column A: \( 3442 \times 10 = 907(1.047) = 2662 \text{ in-k = } M_a \)

Column B: \( 1828 \times 10 = 907(1.047) = 2778 \text{ in-k = } M_a \)

65
Second Order Moments in Sway Frames

Exact Solution (A)

From Timoshenko & Gere, Elastic Stability, p. 4-6

\[ M_{\text{max}} = HL \left( \frac{n}{2} \right) \frac{1}{1 - \frac{P_{cr}}{P}} \]

where \( P_{cr} = \frac{H^2}{12E} \)

Approximate Solution (B) - Effective Length Method

Stability Method

If the initial shape is similar to the buckled shape, where \( P_{cr} \) is based on \( KL = 2L \)

\[ \Delta_0 = \frac{H^3}{12EI} \]

\[ \Delta = \frac{\Delta_0}{1 - \frac{P}{P_{cr}}} \]

\[ M_{\text{max}} = HL + PD = HL + \frac{PD_0}{1 - \frac{P}{P_{cr}}} \]

Approximate Solution (C) - PA Method

1st order analysis

1st cycle PA

\[ \Delta_0 = \frac{H^3}{12EI} \]

\[ \Delta = \Delta_0 + \frac{PD_0}{L} \]

Next cycle

\[ \Delta = \Delta_0 + \frac{PD_1}{L} \]

Stop when \( \Delta = \Delta_{k-1} = \Delta \)

Substituting \( \Delta \) for \( \Delta_0 \) and \( \Delta_1 \) in Eq. (3) gives

\[ \Delta = \Delta_0 + \frac{PD}{L} \left( \frac{1}{3EI} \right) \]

\[ M_{\text{max}} = HL + PD = HL + \frac{PD}{1 - \frac{P}{P_{cr}}} \]

Compare Eqs. (1), (2), and (5) for high \( P/P_{cr} \)

<table>
<thead>
<tr>
<th>( P/P_{cr} )</th>
<th>.1</th>
<th>.2</th>
<th>.3</th>
<th>.4</th>
<th>.5</th>
<th>.6</th>
<th>.7</th>
<th>.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{cr}/E )</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.001</td>
<td>1.002</td>
<td>1.003</td>
<td>1.005</td>
<td>1.006</td>
</tr>
</tbody>
</table>

very accurate

<table>
<thead>
<tr>
<th>( P_{cr}/E )</th>
<th>.988</th>
<th>.993</th>
<th>.995</th>
<th>.999</th>
<th>.999</th>
<th>.999</th>
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reasonable except
Second Order Moments in Sway Frames

Annex End Column

Assume that the ends of the beam rotate the same amount i.e. stiffness = \( \frac{6EI_0}{L_b} \)

For Halone, \( \Delta = \frac{HL_c^3}{3EI_c} + \frac{HL_c^2L_b}{EJ_b} \)

Simplifying \( \Delta = \frac{HL_c^2}{6EI_c} (2G) \)

Approximate Soln: Effective Length Method

\[ \Delta = \frac{\Delta_0}{1 - \theta P_{cr}} \]

where \( P_{cr} \) based on \( KL \) from the alignment chart

\[ M_{max} = HL_c + PA = HL_c \left[ 1 + \frac{P_{cr}^2}{3EI_c} (2G) \right] = HL_c \left[ 1 - \left[ 1 - \frac{\pi^2(2G)}{6K^2} \right] \frac{P_{cr}}{P_c} \right] = HL_c \left[ 1 - \frac{P_c}{P_{cr}} \right] \]

Evaluate \( \psi \) for various values of \( G \)

\[
\begin{array}{cccccccc}
G & 0 & 0.5 & 1.0 & 2.0 & 4.0 & 6.0 & 8.0 & 10 \\
K & 2.0 & 2.8 & 2.65 & 2.2 & 3.67 & 4.1 & 4.45 & \\
\psi & 1.17 & 1.05 & 1.06 & 0.64 & 0.36 & 0.25 & 0.20 & 0.035 \\
\end{array}
\]

\( \psi \) varies between 0.18 and 0

Choose \( \psi = 0 \) which is conservative

\( \psi = 0 \) is except at small \( G \) values.

\( \psi \) checks with flagpole solution on previous page

\[ \therefore \text{use } HL_c \left[ \frac{1}{1 - P_{cr}} \right] \]

\[ C_m = 1.0 \text{ for unbraced frames} \]

PA Method - same as flagpole

\[ M_{max} = HL \left[ \frac{1}{1 - P_{cr}} \right] \]

As the girder becomes more flexible, Eqs (8) and (8) approach each other. This is true because the buckled shape becomes more triangular and loses a half sine curve (the buckled shape for an infinitely stiff girders).

For multiple columns in a story, merely replace \( P_{cr} \) and \( P_{cr} \) with \( E \), \( P_{cr} \), \( E \), \( H \) and respectively in Eqs (6), (7) and (8). LRFD uses Eq (7) and (8).

Leaned Columns

This structural arrangement fits the assumptions of the PA method (see Fig. 9) exactly so the PA method will be exact and the effective length method conservative.
Summary:

\[ M_{\text{max}} = E_2 M_{c2} \]

where \( M_{c2} \) is the maximum column end moment from joint translation (CHL on sh/d and #2)

and \( E_2 = \frac{1}{1 - \frac{2P_{c}}{E_k}} \) \[ \text{[H1-5]} \]

PA method

or \( E_2 = \frac{1}{1 - \frac{2P_{c}}{E_k \Delta_y}} \) \[ \text{[H1-6]} \]

Effective Length Method

where \( P_{c} \) is based on \( k = 1.0 \)

For flexible girders, the difference between H1-5 and H1-6 will be small for both fully restrained connections and leaned column systems. The maximum difference occurs for the flagpole which is an extreme case. If the flagpole supports significant axial load, H1-5 will be unconservative and H1-6 conservative. If the flagpole supports mainly lateral force from leaned columns with little axial load of its own, H1-5 will be almost exact and H1-6 conservative.

Most common cases should give similar results. It should be noted that the major differences occur at high axial load. This means that even major differences in H1-5 and H1-6 will not significantly affect designs as shown by the following example:

Say at \( P_{Pr} = 0.70 \) and \( \Delta_y = 0.10 \)

From H1-6 \( E_2 = 3.33 \), so the interaction \( E_2 \) is

\[ 0.70 + 3.33 (0.10) = 1.03 \]

From H1-5 \( E_2 = 2.36 \)

\[ 0.70 + 2.36 (0.10) = 1.04 \]

The difference between the \( E_2 \) values is 4.1%; the difference between the interaction values is only 10%
PORTION OF A MULTISTORY FRAME

CHECK COL A FOR 1.2D + 0.5L + 1.3W

1st order structural analysis performed at factored loads

**DESIGN EX 2**

\[ P_u = 250' \]

\[ 1.2D + 0.5L \]  \[ B_1 \text{ factor} \]

\[ 1.3W \]  \[ B_2 \text{ factor} \]

**Try W 14x74 (F_y = 36 ksi)**

\[ A = 21.8', T_x = 796, T_y = 6.04, r_x = 2.48, r_y = 126, K_x = 1.47, K_y = 1.0 \]

\[ \frac{K_x}{T_y} = \frac{13(12) 1.0}{2.48} = 62.9 \]

\[ \frac{K_y}{T_x} = \frac{147(13) 1.0}{6.04} = 40.0 \]

**COLUMNS FOR P_u**

\[ \lambda_y = \lambda_x = 6.29 \frac{\sqrt{\frac{K_y}{T_y}}}{\sqrt{\frac{K_x}{T_x}}} = 0.705 \]

\[ : \text{ USE E2-2, } F = 0.658 \times 36 = 29.2 \]

\[ \phi P_u = 0.85(29.2 \times 21.8) = 541 \text{ kips} \]

\[ \frac{F_P}{P_u} = \frac{541}{250} = 0.462 \]

\[ : \text{ use H1-1a} \]

**H1-1a**: \[ \frac{P}{\phi P_u} + \frac{E}{q} \frac{M_y}{P_u M_y} = 1.0 \]

\[ B_1: \quad P_{E y} = \pi^2(29000) 796/(126)^2 = 9962, \quad B_1 = \frac{0.6 \times 4.25}{3.35} = 0.323 \]

\[ B_2: \quad P_{E x} = 9363/(1.47)^2 = 4333, \quad B_2 = \frac{1}{1 - 0.250/0.4333} = 1.061 \]

\[ M_{uT} = 35(1.0) + 180(1.061) = 226'k' \]

\[ M_{uB} = 25(1.0) + 180(1.061) = 216'k' \]

**F1-1**: \[ L_{pd} = \frac{[9600 + 2200 \times \frac{248}{226}]}{36 \times 12} = 32.7 > 13' \]

\[ M_p = 126(36)/12 = 378'k' \]

\[ 0.462 + \frac{226}{0.9(378)} = 1.052 > 1.0 \text{ N.G. USE W14x82} \]

\[ \text{Can also use the following approach:} \]

\[ C_b = 2.3, \; \phi H = 218, \; \phi H_p = 340, \; L_r = 40.0' \]

\[ C_b \phi H = 2.3(218) > \phi H_p :: \text{can go at least } L_r \text{ and still use } H_p \]

\[ 13' < 40' \text{ OK use } H_p \]
Frames 20' ea. Out-of-plane braces against sway and exterior columns are supported at mid-height. In-plane sway permitted.

\[ D = 45 \text{ psf}, \quad L = 80 \text{ psf}, \quad W = 40 \text{ psf} \]

1st order analysis:

**Gravitation - \( W \):**
- 1.2D = 1.2 (45 + 20) = 108 psf
- 0.5L = 0.5 (80) = 40 psf

**Wind - \( W \):**
- \( 1.3 \times 108 = 140.4 \text{ psf} \)
- 3400\text{ lb/ft} \times 0.18\text{ ft} = 612 \text{ lb/ft} \times 0.18\text{ ft} = 110 \text{ psf} \)

**Uplift (0.32 k/ft):**
- \( 108 \times 0.32 = 34.56 \text{ psf} \)

**Check:** \( W = 14.74 \text{ k}(E = 36 \text{ ksi}) \) for 1.2D + 0.5L + 1.3 W

- \( P = 50 \text{ ft} \times (1.08 + 0.8) = 20 \text{ ft} \times 1.04 = 73.2 \text{ kips} \)

- \( A = 21.8 \text{ in}^2, \quad I = 796 \text{ in}^4, \quad f_a = 60 \text{ ksi}, \quad 2.48 \text{ ft} = 126 \text{ in} \)

- \( G = \frac{796}{21.8 \times 2 \times 126 \times 12} = 0.487 \text{ ksi}, \quad G_8 = 10 \text{ ksi} \)

- \( k = \frac{1787}{1787} = 1.787 \)

Since exterior columns lean on interior, \( k \) must be adjusted for \( f_a \). See p. 9

- \( k_a = \frac{1}{k} \frac{f_a}{f_y} = \frac{1787}{1787} \times \frac{60}{248} = 2.729 \)

- \( k = 9.68 \times 2.729 = 26.1 \text{ kips} \)

- \( \lambda = \sqrt{9.68/26.1} = 1.08 \text{ ksi} \)

- \( f_e = 0.65 \times 21.98 \text{ ksi} \)

- \( P_a = \frac{73.2 \text{ kips}}{0.85 \times 21.98} = 0.18 \text{ ksi} \)

Use H1-11b

\[ H1-11b \quad \frac{P_a}{2} \leq 1.0 \]

- \( M_w = 200 \text{ kips} \times 200 \text{ ft} = 40,000 \text{ kip-ft} \)

- \( F_2 = \frac{1}{1 - 1.38} \frac{3}{2} = \frac{1}{1 - 1.38} \frac{3}{2} = \frac{1}{1 - 1.38} \frac{3}{2} = 1.198 \)

- \( M_w = 208 \text{ kip-ft} \)

- \( P_m = [320 \times 2200 \times 0.566] \times 2 = 20.67 \text{ ksi} \)

- \( M_p = 126 \text{ kip-ft} \)

- \( M_p = 378 \text{ kip-ft} \)

**H1-2:** \( \frac{180}{2} + \frac{234.6}{0.9(378)} = 0.090 + 0.645 = 0.735 \leq 1.0 \)

Choose lighter section:

W14 x 61
DESIGN EV. #4

WELDED TRUSS
STRUCTURAL TEE FOR BOTTOM CHORD
LOADS SHOWN ARE NOT FACTORED

\[ P_{u, max} = 1.2(50) + 1.6(85) = 196 \, k \]

\[ P_{u, min} = 1.2(50) = 60 \, k \]

\[ M = 1.6 \left( \frac{2 \times 9}{4} \right) = 72 \, k \text{ft} \]

Try WT 9 x 30

\[ A = 8.82, \quad S_x = 9.29, \quad F_y = 36 \, ksi \]

\[ \frac{P_u}{P_{n}} = \frac{196}{0.9(8.82 \times 36)} = 0.686 > 2 \quad \therefore \text{use HI-1a} \]

HI-1

\[ 0.686 + 0.9 \left( \frac{7.2(12)}{0.9(36 \times 9.29)} \right) = 0.686 + 2.55 = 0.941 \quad \text{OK} \]

CHECK COMP IN STEM, use \[ P_{min} = \frac{50}{0.9(8.82)} = 6.30 \, ksi \text{ Tension} \]

BENDING STRESS = \[ \frac{7.2(12)}{0.9(9.29)} \times 10.33 \, ksi \text{ comp} = 4.03 \, ksi \text{ -low} \]

Slenderness ratio and buckling will be OK

\[ \text{ck formula F1-15 - homework} \]

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