Energy-based Seismic Collapse Risk Assessment of Structures

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ABSTRACT: Structural collapse is traditionally associated with the exceedance of a target value of inter-story drift or plastic hinge rotation at structural components. However, such an approach may not accurately estimate the structural collapse potential due to load redistribution and variation of structural damage within the structure. Moreover, collapse prediction may be sensitive to such assumed threshold values. Therefore, in this study, energy balance of a structural system is utilized to represent the severe structural damage history that eventually leads to structural collapse. Performing energy-based collapse analyses, a new dynamic-instability based collapse criterion is developed and key collapse measures are identified. Using the results, a new collapse fragility model is then established for estimating and improving structural reliability against collapse. Moreover, extensive parametric studies are performed to investigate sensitivity of collapse fragilities to variability in structural and earthquake parameters.

1. INTRODUCTION

Accurate estimate of collapse likelihood of buildings under seismic excitations has recently become critical in efforts to promote life safety and hazard-resilience of the society. Despite recent advances in structural collapse assessment, accurate prediction of structural collapse with systematic incorporation of uncertainty still remains a question.

The most commonly used approach to assess the collapse capacity of structures under extreme earthquakes is based on the concept of incremental dynamic analysis (IDA; Vamvatsikos and Cornell, 2002). The IDA approach is based on the behavior of so-called "IDA curves," which track the relationship between an "intensity measure (IM)" and a "damage measure" (DM) evaluated for several ground motions at incrementally increased intensity levels. Uncertainties in structural properties and applied ground motions can be integrated into probabilistic description of structural collapse performance by adopting the probabilistic basis of performance–based earthquake engineering (PBEE) framework together with IDA. The maximum inter-story drift ratio (IDR) is often selected as the measure to represent the global behavior of structural system in the PBEE framework. Likewise, the occurrence of "collapse" is usually indicated by acceleration of IDR towards "infinity" such that the IDA curves become almost flat, but this may not be clear due chaotic structural behavior. Therefore, assumed threshold values based on IDR such as DM-based rule (e.g., exceedance of 10% maximum drift) or IM-based rule on slope of IDA curve between IDR and elastic spectral acceleration (e.g., lower than 20% of the initial IDA slope) are most commonly used limit-states to identify structural collapse capacity. However, collapse assessment approaches based on IDR or other mostly used parameters such as plastic hinge at a component may not accurately represent the overall collapse behavior of structural systems due to load redistribution and variation of structural damage within the structure. Moreover, collapse prediction is found to be sensitive to such subjective collapse limit-states based on some assumed threshold values.

Characterization of overall cumulative and load-path dependent performance of structures considering aforementioned uncertainties is needed for accurate and reliable collapse risk assessment. Since energy parameters at system-level are aggregated quantities considering re-distribution and variation of each individual component-damage within the structural system, they can be excellent indicators to represent total severe structural damage history due to cyclic-loading just before collapse. This paper therefore focuses on energy-based analysis of structures to assess seismic collapse risk.

Among few collapse experiments reported in the literature, three steel experimental case studies are selected to develop computational models validated near collapse. Using the validated computational models, a new dynamic-instability-based collapse limit-state and the most effective collapse descriptor representatives of structural global behavior history are identified based on the energy balance of a structural system. A new collapse fragility model is then introduced using the developed collapse criteria and collapse descriptor for reliable probabilistic evaluation of structural collapse. Finally, the effects of earthquake characteristics and structural parameters on the developed collapse fragilities are investigated for the purpose of estimating and improving structural reliability against collapse.

In the following sections, a brief summary of the study is presented, while giving more details on the development of new probabilistic approaches in structural collapse assessment. To this end, the details of the computational simulation model for only one of the selected case studies (Lignos et al., 2008) is provided in the next section to describe the framework of the study summarized above.

2. COMPUTATIONAL SIMULATON OF A SELECTED COLLAPSE-CASE STUDY

One of the selected case studies considered in this study is the collapse shake-table test by Lignos et al. (2008), which is a 4-story, 2-bay steel frame in 1/8 scale with reduced-beam sections (RBS). Figure 1 shows the setup of the test frame on the NEES mass simulator at the University at Buffalo, which consists of elastic members with plastic hinges at the ends. The mass simulator is connected to the test frame by means of axially rigid horizontal links through which the simulator transfers P-Delta effects acting as a leaning column on the test frame.

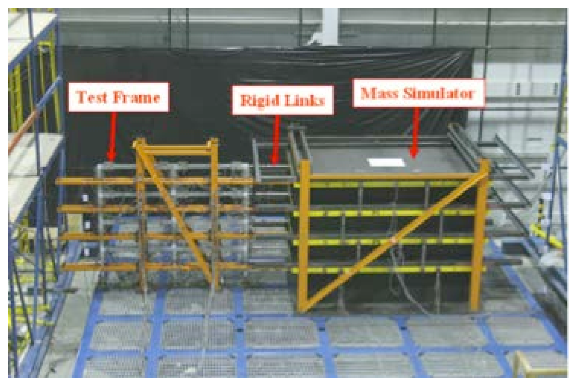

Figure 1: Shake-table-test of a 1/8 scale 4-story, 2-bay steel frame (Lignos et al. 2008).

An analytical clear span model for the test frame was developed in OpenSees (PEER, 2004). A practical model approach was employed to avoid numerical convergence issues, but the model is detailed enough to accurately simulate collapse behavior, especially to perform extensive parametric studies where computational cost is demanding. Following the method used by Lignos et al. (2008), rotational springs were used to analytically model the plastic hinges at ends of elastic elements with a modified Ibarra-Krawinkler deterioration model available in OpenSees, calibrated based on a steel component database of steel beams with RBS under cyclic loading. Moreover, offsets from the element ends were applied to take RBS into account. The nonlinear geometry effects were considered using co-rotational transformations. The nonlinear dynamic analysis results for the developed OpenSees model under the test ground motion show good agreement with the experiment data (Deniz, 2014). Using the validated computational model, we investigated the system's energy balance as summarized in the next section, to introduce a new collapse criteria and a key collapse measure.

3. ENERGY-BASED COLLAPSE CRITERIA

Earthquake loads applied on the structure introduce seismic energy into the system. Some part of this input seismic energy (E_{EQ}) is absorbed as the kinetic energy (E_K) and the strain energy (E_S; i.e., the sum of elastic energy (E_E) and hysteretic energy (E_H)), and the rest is dissipated as damping energy (E_D). As the system experiences loading and unloading repeatedly during an earthquake event, it starts to show highly nonlinear, cyclic and inelastic behavior, which leads to excessive deformations that initiate gravity forces applied on the structure to release gravity energy (E_G). Taking the integral of the dynamic equation of motion with respect to differential displacement, the components of the energy balance can be then described as (Deniz, 2014):

$$E_K + E_D + E_E + E_H = E_{EQ} + E_G \qquad (1)$$

In order to investigate the energy balance with regard to the collapse of a structural system, the energy analyses were performed on the test frame by Lignos et al. (2008) for the test ground motion Canoga Park record at the intensity scale factors of 0.4, 1.0, 1.5, 1.9 and 2.2, which were continuously applied on the frame during the experiment. Figure 2 show the system-energy time histories for the gravity energy (E_G), seismic input energy (E_{EQ}), total input energy (i.e., $E_I = E_{EQ}+E_G$), and strain energy absorbed in springs (E_{SPR}) at the last scale factor of 2.2 where collapse is observed. Note that the beginnings of the energy time histories in Figure 2 indicate the cumulative energy responses obtained from the previous analyses for the scale factor of 1.9.

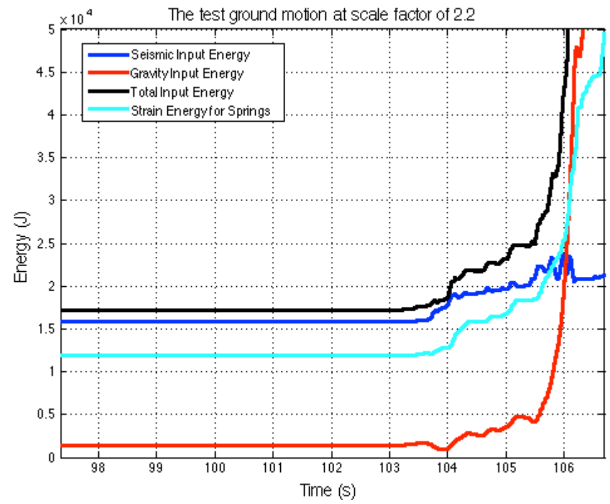

Figure 2: System-energy time histories near collapse for the case study by Lignos et al. (2008).

Figure 2 clearly shows that while gravity energy keeps steady at a comparatively small level comparing to other energy responses at the beginning, it then starts to rapidly increase near collapse, even far exceeds seismic input energy stored in the structure. Considering dynamic instability due to the loss of structural resistance against the gravity loads, a new energy based collapse criteria has been therefore proposed based on the incidence of gravity energy exceeding seismic energy with a sudden increase (i.e., E_G>>E_{EQ} near collapse). Instead of some assumed threshold values, this approach relies on an actual occurrence of collapse due to domination of gravity loads over applied lateral seismic loads. Moreover, using this new approach, one does not need to check structural response at each degree-of-freedom.

Several nonlinear dynamic analyses were performed for the validated test case study of Lignos et al. (2008) using the far-field set of 78 ground motions by Haselton and Deierlein (2007). Figure 3 shows the IDA curves of elastic spectral acceleration to inter-story drift ratio obtained from the OpenSees model. Traditional IM-based rule (green pluses) and DM-based rule (yellow squares) are compared to the new criteria called "energy rule" (red circles) based on the maximum intensity level observed before the dynamic instability occurs, i.e., gravity energy exceeds dynamic energy. Comparing the proposed energy rule with the traditional rules in Figure 3, it is found that the energy rule significantly gives larger collapse drift predictions because it depends on the actual dynamic instability near collapse. On the other hand, it should be noted that the meaning of collapse assumed for traditional rules in the literature does not necessarily represent the real collapse case but mostly "collapse prevention." In Figure 3, much variability is also observed in collapse capacity levels for all rules due to the effect of randomness in the selected ground motions. Large difference is observed especially in drift capacity predictions except for the DM-based rule, which depends on a predetermined threshold drift value. This is due to sensitivity of drift measures near collapse. Therefore, next section investigates key collapse measures with smaller variability based on the energy balance concept to facilitate more effective risk assessment of structural collapse.

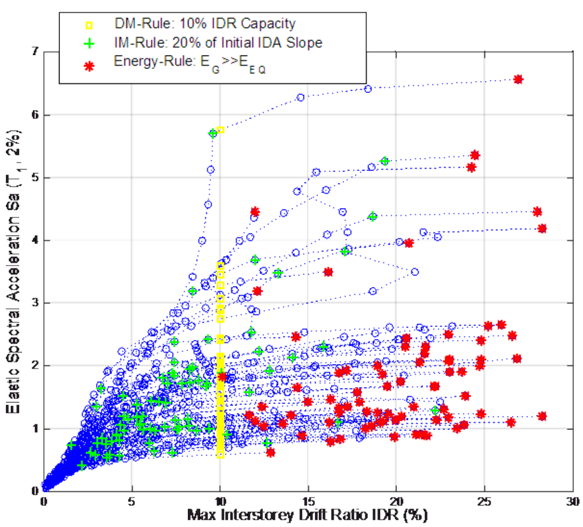

Figure 3: Comparison of collapse data for Sa and IDR obtained by different collapse rules.

4. A NEW COLLAPSE DESCRIPTOR

From the results of validated computational simulations of collapse (e.g. Figure 2), it is noted that inelasticity is concentrated only in rotational springs at the ends of beams and columns. Therefore, E_{SPR} is the total strain energy dissipated from all degrading elements in the frame. In Figure 2, although the strain energy is steady following the seismic input energy, it then shows a rapid increase following the huge release of gravity energy (i.e., $E_I \approx E_G$) near collapse. Considering this energy balance, an "equivalent velocity-ratio (V_R)" of spring energy to seismic energy is investigated, i.e.

$$V_R = \sqrt{\frac{\max(E_{SPR})}{2\ \max(E_{EQ})}} = \frac{\max(|V_{SPR}|)}{\sqrt{2}\ \max(|V_{EQ}|)} \qquad (2)$$

where, V_{SPR} and V_{EQ} are the equivalent velocities for spring and seismic energies respectively based on kinetic energy formulations. When the gravity energy becomes equal to the seismic energy near collapse, the total input energy then becomes almost twice the seismic energy (i.e., $E_I \approx 2E_{EQ}$), which is clear from the energy balance described in Eq. (1). Therefore, the maximum energy that a structure can absorb is less than $2E_{EQ}$, and thus the equivalent velocity-ratio V_R should be less than 1.0.

Figure 4 shows the IDA data in terms of equivalent velocity ratio V_R and spectral acceleration S_a (at first period with 2% damping) for the test case study by Lignos et al. (2008) under the same 78 far-field ground motions. Note that the collapse data is obtained based on the last non-collapse point on the IDA curve according to energy-based collapse criteria.

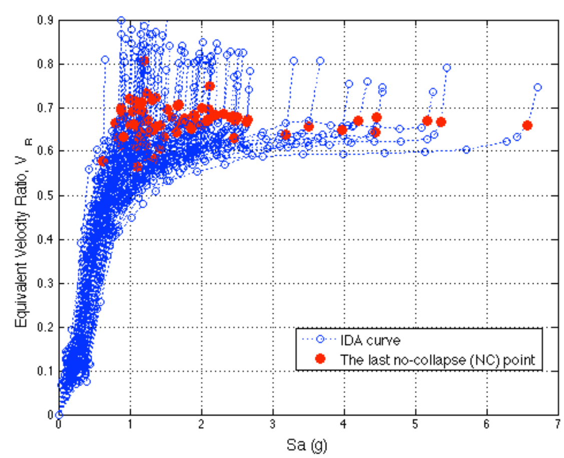

Figure 4: IDA curves and the collapse data for Sa and V_R obtained by energy-collapse criteria.

Using the collapse data (red circles) in Figure 4, statistical analyses were performed on dimensionless V_R as well as on drift ratio D_R and lateral inter-story drift ratio IDR based collapse capacities. The collapse capacity defined by V_R exhibits a significantly reduced variability, which is evidenced by the coefficient of variation (cov) of 0.058 comparing to traditionally used D_R (cov = 0.215) and IDR (cov = 0.226). Therefore, with V_R, one can predict the collapse capacity of a structural system with less uncertainty.

5. A NEW COLLAPSE FRAGILITY MODEL

Assessment of collapse fragility relations are usually obtained in two ways (Zareian et al., 2010): so-called "damage measure (DM) based" approach in which the collapse is described in terms of a structural response parameter such as IDR to evaluate the probability of collapse at a specified intensity level; and so-called "intensity measure (IM) based" approach which directly uses an IM of ground motion (such as S_a) to describe the collapse limit state. Both methods rely on "demand versus capacity" framework, i.e., the probability of collapse is assessed by the likelihood of the event of the seismic demand exceeding the seismic capacity.

Assumptions made for the DM-based approach, e.g., dependency between capacity and demand and approximations in assessment of conditional distributions of DM given IM may make the IM-based approach seem like a more reliable method to get collapse estimates. However, the IM-based approach considers uncertainty only in seismic capacity, which may provide less reliable collapse prediction. On the other hand, using more representative global structural responses with a small sensitivity near collapse such as V_R can improve DM-based approach that traditionally uses IDR measure. Thereby, considering uncertainty in both seismic demand and seismic capacity, this study focuses on DM-based approach using energy based collapse criterion for the development of a new fragility model. Rather than using traditional models such as those based on IDR, new capacity and demand models based on V_R together with S_a (one of the mostly used intensity measures in seismic hazard assessments) are selected instead to assess new more effective collapse fragility relations. The proposed methodology is demonstrated in the next sections using the IDA data in Figure 4.

5.1. Probabilistic Model of Collapse Demand

First, linear regression is employed here to characterize conditional distribution of structural demand V_{R_D} for a given intensity S_a using IDA data in Figure 4. A limited S_a range within one standard deviation from the mean (i.e., 0.22g$\leq S_a \leq$2.23g corresponding to 79 percent of all blue IDA data points in Figure 4) is considered in the linear regression to get a better linear model fitting to IDA points. In addition, logarithms are applied to S_a before the regression in order to achieve an approximate linear relationship.

The conditional probability of structural demand V_{R_D} exceeding a given level of the capacity v at a given spectral elastic acceleration level s of ground motion can be described in terms of cumulative distribution function (CDF) of the conditional demand V_{R_D}:

$$P_{V_{R_D}|\ln S_a=\ln s} = 1 - F_{V_{R_D}|\ln S_a}(v\,|\ln s) \quad (3)$$

In order to find the conditional distribution of demand in Eq. (3), a linear regression model was applied to V_R as in Eq. (4) to find the conditional mean (5) and constant variance (6):

$$V_{R_D}{}' = \mathrm{k}_1 \ln S_a + \mathrm{k}_2 + \sigma\varepsilon \quad (4)$$

$$E[V_{R_D}|\ln S_a] = \mathrm{k}_1 \ln S_a + \mathrm{k}_2 \quad (5)$$

$$Var\left[V_{R_D}|\ln S_a\right] = \sigma^2 \quad (6)$$

where, $V_{R_D}{}'$ is the structural demand in equivalent velocity ratio obtained from approximate linear relationship for a given level of $\ln S_a$, k_1 and k_2 are coefficients found based on linear regression, ε is a normal random variable with zero mean and unit variance, and finally σ represents the magnitude of the linear regression error. Since ε has a normal distribution and other terms in Eq. (4) are deterministic, the structural demand V_R becomes a normal random variable as well. If V_R given an intensity level $\ln S_a$ follows a normal distribution and $\mu_{V_{R_D}|lnS_a}$ and $\sigma_{V_{R_D}|\ln S_a}$ are corresponding conditional mean and deviation obtained from Eq. (5) and Eq. (6) respectively, then the conditional CDF of structural demand based on V_R measure in Eq. (3) becomes:

$$F_{V_{R_D}|\ln S_a}(v \mid \ln s) = \Phi\left[\frac{v-\mu_{V_{R_D}|\ln S_a}}{\sigma_{V_{R_D}|\ln S_a}}\right] \quad (7)$$

Following the methodology described above, the linear demand model between V_R and $ln\, S_a$ is obtained in Figure 5 (green line). Validity of the developed linear approximation made for the conditional distribution in Eq. (3) depends on assumption of a constant conditional variance (the so-called "homoskedasticity" assumption). Although the linear model shows approximate constant variance for a limited intensity range in Figure 5, the distribution of data points indicates some variation in the degree of scatter of data points with increasing intensity level. Therefore, linear regression with "non-stationary" conditional variance was also applied to the IDA data points in Figure 5 to improve the stationary linear demand model (red line). For this purpose, the conditional deviation was assumed as a function of S_a:

$$\sigma_{V_{R_D}|\ln S_a} = \sigma = \sigma_o\, \mathrm{S_a}^{c} \quad (8)$$

where, σ_o is an unknown constant and c is a coefficient larger than zero (note than a zero value of c means stationary variance). It can be reasonably assumed that the IDA data points in the region of smaller variance should have higher "weights" comparing to ones in the region of larger variance. Therefore, the weights w were assigned as inversely proportional to ${S_a}^{2c}$.

After several non-stationary linear regression analyses, the normal demand model with a coefficient c of 0.25 in Figure 5 was found to be the best model for a limited range of S_a based on giving reasonable demand trend as S_a increases while capturing increasing variation in demand. Although both the stationary and non-stationary demand models in Figure 5 seem to be appropriate for a limited intensity range, such models can still be acceptable because they have the benefits of practical applicability and reducing computational expense of estimation. Since the non-stationary model seems to capture the variation in the data points a little better comparing to the stationary model, this regression approach is used in the following sections for developing a new collapse fragility.

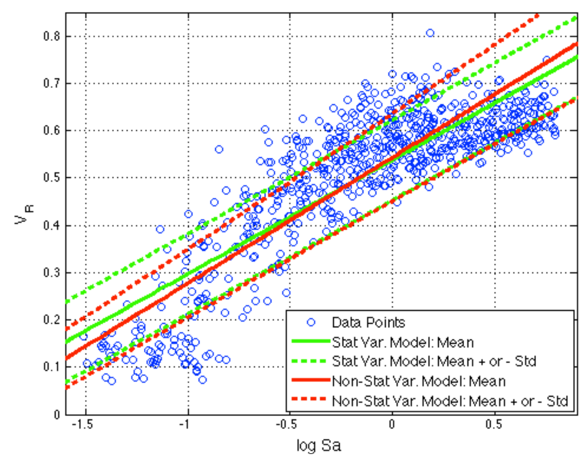

Figure 5: The linear regression models with stationary and non-stationary variance (c=0.25).

5.2. Fragility Function by Probabilistic Model of Collapse Capacity

For a given level of velocity ratio "v" for the demand V_{R_D}, the probability of collapse becomes the CDF of the capacity V_{R_C}:

$$P\left(V_{R_C} < V_{R_D} = v\right) = F_{V_{R_C}}(v) \quad (9)$$

In order to develop the capacity function in Eq. (9), a normal collapse capacity model was assumed for the IDA-collapse data in Figure 4 (red points). The results showed a good agreement between the IDA-collapse data and the fitted normal distribution with a mean $\mu_{V_{R_C}}$ of 0.668 and a standard deviation $\sigma_{V_{R_C}}$ of 0.039.

Note that Eq. (9) provides collapse fragility in terms of only uncertain damage capacity. Next section therefore develops a fragility considering uncertainty in both demand and capacity.

5.3. Fragility Function by Probabilistic Models of Both Collapse Capacity and Demand

Using safety margin approach, structural collapse can be identified considering randomness in a demand/capacity format. If safety margin M_{V_R} of a structural system is a random variable defined as the difference between demand V_{R_D} and capacity V_{R_C}, then probability of collapse becomes the probability of safety margin being less than zero (i.e., demand exceeding capacity) at a spectral acceleration level "s":

$$P_{Col|S_a=s} = P\left(M_{V_R} \leq 0 \middle| ln\, S_a = \ln s\right) \quad (10)$$

It is noted that the linear combination of the developed normal collapse capacity and demand models identified above such as M_{V_R} becomes a normal random variable too. One can reasonably assume that demand and capacity are statistically independent events at a given intensity level. The mean and deviation for M_{V_R} can be then assessed as in Eq. (11) and Eq. (12) respectively:

$$\mu_{M_{V_R}} = \mu_{V_{R_C}} - \mu_{V_{R_D}|lnS_a} \quad (11)$$

$$\sigma_{M_{V_R}} = \sqrt{\sigma_{V_{R_C}}{}^2 + \sigma_{V_{R_D}|lnS_a}{}^2} \quad (12)$$

Then, the probability of collapse is evaluated as the normal CDF of M_{V_R} at the given spectral elastic acceleration level of ln s:

$$P_{Col|S_a=s} = \Phi\left[-\frac{\mu_{M_{V_R}}(S_a)}{\sigma_{M_{V_R}}(S_a)}\right] \quad (13)$$

Following the methodology described above, the new fragility model based on energy-collapse criterion (red curve) is first evaluated in Figure 6. Then it is compared to the common approaches such as "IM-based fragility model using lognormal CDF of S_a (blue curves)" and "DM-based fragility model using IDR (black curves)" in two ways. Figure 6 first compares the new fragility model (red curve) with common approaches obtained using energy-based collapse criteria. The DM-based fragility model using IDR (solid black curve) largely underestimates the probability of collapse due to large conditional cov (0.406) found for the demand model of IDR as well as high sensitivity of IDR to intensity scaling near collapse, thereof, does not work for the energy-collapse rule. On the other hand, probably due to ignoring uncertainty in seismic demand, IM-based fragility model using S_a (solid blue curve) seems to slightly overestimate collapse probabilities in general comparing to the new model (red curve).

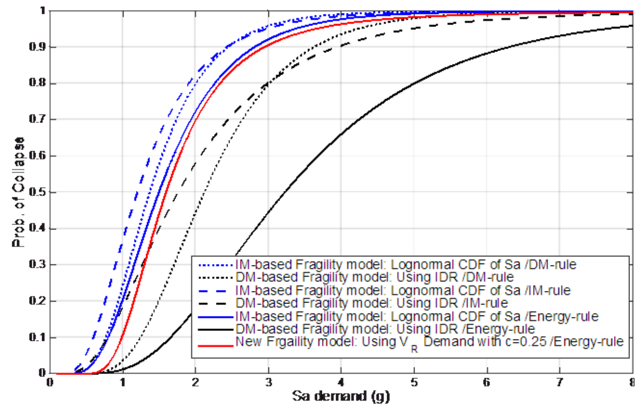

Figure 6: Collapse fragility relations obtained by different rules

In Figure 6, the new fragility model using energy-rule (red curve) is then compared with the common fragility approaches using traditional rules based on IDA data in Figure 3: IM-based fragility model using DM-rule (dotted blue curve) and IM-rule (dashed blue curve); and DM-based fragility model using DM-rule (dotted black curve) and IM-rule (dashed black curve). As seen, there is a large difference between collapse points obtained by the three collapse rules. It is important to indicate that energy-rule assesses the collapse capacity at the maximum intensity before the structure loses its dynamic instability. However, DM-rule is based on a pre-determined threshold value (ignoring variance in capacity), while IM-rule is based on a simple deterministic rule using shape of IDA curves, which can be chaotic due to possible hardening in structural behavior. Therefore, these traditional rules have been found not sufficient to identify when and how a structure collapses under the effect of variable dynamic loads.

5.4. Effect of Uncertainties On Collapse Fragility Models

Extensive parametric studies were performed for the test case study by Lignos et al. (2008) to account for the impacts of a structural model changes on the collapse prediction of structures. Moreover, several subsets of 78 far-field ground motion set by Haselton and Deierlein (2007) were formed to investigate the record-to-record variability of ground motion records on the developed collapse fragilities (Deniz, 2014).

It has been observed that ductile connection model parameters related to strain-hardening ratio and deterioration rate after yielding can result in remarkably different collapse estimates. It is also noteworthy that collapse capacity based on V_R measure is less sensitive to a change in structural properties. Moreover, it is found that a suite of ground motions selected based on the ratio of peak ground displacement to peak ground velocity may reduce the epistemic uncertainty significantly.

6. CONCLUSIONS

A new collapse criterion termed as "energy rule" is based on the actual occurrence of dynamic instability caused by loss of structural resistance against the gravity loads, instead of the behavior of the IDA curves and subjective thresholds. Therefore, it seems to be a more reliable option in collapse assessment of structures. Moreover, a quantitative indication of structural collapse by this approach (in terms of boundless drift) may facilitate developing a mathematical description of dynamic instability, which can be particularly useful for quantitative collapse detection during stochastic collapse analyses.

Integration of identified key collapse measures such as equivalent velocity ratio (V_R) into the collapse fragility models using energy-rule may decrease the dispersion due to record-to-record variability, which in turn corresponds to a reduction in uncertainty level associated to collapse probability computation.

Lastly, parametric studies indicate that uncertainties due to structural and ground motion properties need to be properly incorporated into collapse prediction to promote reliable probabilistic evaluation of structural collapse. If these uncertainties are underestimated, one could obtain unconservative collapse predictions.

7. ACKNOWLEDGEMENTS

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