ABSTRACT: Many structural systems are subjected to the risk of cascading system-level failures initiated by local failures. For efficient reliability analysis of such complex system problems, many research efforts have been made to identify critical failure sequences with significant likelihoods by an event-tree search coupled with system reliability analyses; however, this approach is time-consuming or intractable due to repeated calculations of the probabilities of innumerable failure modes, which often necessitates using heuristic assumptions or simplifications. Recently, a decoupled approach was proposed (Kim 2009; Kurtz et al. 2010): critical failure modes are first identified in the space of random variables without system reliability analyses or an event-tree search, then an efficient system reliability analysis is performed to compute the system failure probability based on the identified modes. In order to identify critical failure modes in the decreasing order of their relative contributions to the system failure probability, a simulation-based selective searching technique was developed by use of a genetic algorithm. The system failure probability was then computed by a multi-scale system reliability method that can account for statistical dependence among the component events as well as among the identified failure modes (Song & Kang 2009; Song & Ok 2010). This paper presents this decoupled approach in detail and demonstrates its applicability to complex bridge structural systems that are subjected to the risk of cascading failures induced by fatigue. Using a recursive formulation for describing limit-states of local fatigue cracking, the system failure event is described as a disjoint cut-set event (Lee & Song 2010). Critical cut-sets, i.e. failure sequences with significant likelihood are identified by the selective searching technique using a genetic algorithm. Then, the probabilities of the cut-sets are estimated by use of a sampling method. Owing to the mutual exclusiveness of the cut-sets, the lower-bound on the system cascading failure probability is obtained by a simple addition of the estimated probabilities of the identified cut-sets. A numerical example of a bridge structure demonstrates that the proposed search method skillfully identifies dominant failure modes contributing most to the system failure probability, and the system failure probability is accurately estimated with statistical dependence fully considered. An example bridge with 97 truss elements is considered to investigate the applicability of the method to realistic large-size structures. The efficiency and accuracy of the method are demonstrated through comparison with brute-force Monte Carlo simulations.

1 INTRODUCTION

Most research efforts to estimate the failure probabilities of structural systems (Freudenthal et al. 1966; Thoft-Christensen & Baker 1982; Ditlevsen & Madsen 1996; Melchers 1999; Der Kiureghian 2005) have been aimed at component reliability analysis, which characterizes the failure event by a single limit state; however, it is widely accepted that complexity of system-level failure of a structure requires system reliability analysis (Lee 1989; Moses 1990; Park 2001; Song & Der Kiureghian 2003; Liu & Tang 2004), in which the failure event is described by a Boolean function of multiple limit state functions. For example, a cut-set system event is described as

\[ E_{sys} = \bigcup_{k=1}^{N_{cut}} C_k = \bigcup_{k=1}^{N_{cut}} \bigcap_{i \in I_k} E_i \]  

where \( E_i \) is the \( i \)-th component event representing the failure at a location or member, \( i = 1, \ldots, N_{comp} \); \( C_k \) is the \( k \)-th cut-set event, i.e. a failure mode, \( k = 1, \ldots, N_{cut} \), where the cut-sets are a joint realization of component events that constitutes a realization of the system event \( E_{sys} \); and \( I_k \) denotes the index set of components that appear in the \( k \)-th cut-set.

Component failure events, \( E_i \), are often statistically dependent on each other due to correlated or common random variables in the limit state definitions.
by the selective search and the corresponding system failure probability are computed by system reliability analyses. While brute-force Monte-Carlo simulation of failure sequences could provide the system reliability accurately given sufficient time to converge, the selective searching method provides not only the system failure probability but also critical failure modes without prior knowledge of the system response.

In this paper, the selective searching method is applied to a bridge structural system subjected to the risk of fatigue-induced cascading failures. Using an efficient characterization of fatigue-induced failure modes developed by Lee & Song (2010), cascading failure events are described as mutually exclusive (or disjoint) cut-set events, making the system failure probability simply the sum of the probabilities of all identified critical failure modes. This paper first introduces the simulation based selective searching technique, followed by a summary of the efficient formulation of fatigue-induced failure modes and methods used for calculating the probabilities of the identified cut-sets. The proposed risk assessment framework is then demonstrated by a large-size planar-truss bridge structure.

2 SELECTIVE SEARCHING TECHNIQUE FOR DOMINANT FAILURE MODES

Most of the methods developed to identify failure modes of structural systems can be placed into the following two types of approaches (Shao & Murotsu 1999): the so-called probabilistic approach, which includes the branch and bound method (Murotsu et al. 1984; Thoft-Christensen & Murotsu 1986; Karamchandani 1987); and simulation based techniques (Grimmett & Schueller 1982; Rasheed 1983; Moses & Fu 1988; Ditklevsen & Bjerager 1989; Melchers 1994); and the so-called deterministic approach, which includes the incremental loading method (Moses & Stahl 1978; Moses 1982; Lee 1989), the β-unzipping approach (Thoft-Christensen & Murotsu 1986), the methods based on mathematical programming (Corotis & Nafday 1989), or methods employing heuristic techniques (Xiao & Mahadevan 1994; Shetty 1994).

In general, the probabilistic approach is considered theoretically rigorous but computationally costly, whereas the deterministic approach is computationally efficient but has the risk of overlooking important failure modes (Shao & Murotsu 1999). To remedy these issues, Shao & Murotsu (1999) proposed an improved simulation-based selective searching technique in which a genetic algorithm (GA) (Holland 1975; Goldberg 1989) is used to find the few most dominant failure modes that contribute the most to the system failure probability. Noting that GA works with a population of multiple searching points, Kim
By contrast, the searching method by Kim (2009) intends to reverse the searching direction. This “outward” search identifies multiple dominant failure modes in the decreasing order of their likelihoods until their contributions become negligible. The system failure probability can then be accurately evaluated from the identified critical failure modes. First, generate random points in the space of uncorrelated standard normal variables for the initial population of the GA search. To search outward, the points are generated on the surface of a hypersphere with a smaller radius. If one has an idea of the expected system reliability index, the range of hypersphere radii must encapsulate the expected value and address the uncertainty of the expected value. In accordance with the joint PDF of the standard normal space, points on a hypersphere with radius $R$ are generated by

$$
\mathbf{u}^i(R) = R \cdot \mathbf{d}^i = R \cdot \frac{\mathbf{u}^i}{||\mathbf{u}^i||}, \quad i = 1, ..., N_{pop}
$$

where $\mathbf{d}^i = [d^i_1, d^i_2, ..., d^i_n]^T$ is a “direction” vector, i.e. a point randomly generated on the surface of a unit-radius hypersphere, which can be obtained by normalizing randomly generated standard normal vectors $\mathbf{u}^i = [u^i_1, u^i_2, ..., u^i_n]^T$. The direction vectors constitute the initial population of chromosomes for the GA search. The $\mathbf{u}^i$’s can be generated by any sampling method. In this study, Latin Hypercube sampling (McKay et al. 1979) is used for efficient sampling.

Second, transform the sampling points $\mathbf{u}^i(R)$ to the corresponding values in the original random variable space, i.e. $\mathbf{x}(R) = T^{-1}[\mathbf{u}^i(R)]$. For a structural system, $\mathbf{x}$ may denote the uncertainties in loadings, material properties and resistances of the structural members. For each $\mathbf{x}(R)$, the structural analysis is performed to check if local failures occur. If any members have failed, the structural analysis is performed again with the failed member removed. Progressive failures can be found using this framework. These procedures are repeated for each $\mathbf{x}(R)$. If system failures occur according to a set of system failure criteria, the corresponding failure modes and sampling points are recorded. All chromosomes corresponding to these detected system failure modes are imported into a mating pool, i.e. a group of individuals that will later produce offspring as the population of the next generation.

Third, perform a selective search in the vicinity of the $\mathbf{x}(R)$ that caused system failures. Additional failure modes are often identified since one structural element is often involved in multiple system failures, making failure modes relatively close together in the random variable space. This selective search is executed by creating offspring from the parent population of the mating pool through the evolution operators of crossover and mutation. Although several options are available for these evolution operators.
(Goldberg 1989; Deb & Agarwal 1995; Vahdati et al. 2009), the methodologies most suitable for this study are those illustrated in Figure 2. For the crossover operation, a real value between 0 and 1 is randomly generated for each gene, i.e. the rectangular sections in Figure 2. If this value is larger than 0.5, parent 1’s gene is selected as the offspring’s; otherwise, parent 2’s gene is selected. This multi-point crossover operation generates the next-generation searching points, i.e. offspring in the vicinity of the parent populations. This keeps the cases of analyses diverse. Additionally, the mutation operation is used to search for failure modes far from the identified ones, by inverting the signs of the genes, as seen in Figure 2b. This turns the search direction for that gene in the opposite direction.

$$\Delta K = S \cdot Y(a) \cdot \sqrt{\pi a}$$

where $S$ represents the far-field stress range, and $Y(a)$ is the “geometry” function. By integrating the differential equation that arises from Equations 3 and 4, one can describe the time until a truss member under cyclic loading fails as

$$T_i^0 = \frac{1}{Cv_0(S_i^0)^{\nu_i}} \frac{1}{[Y(a)\sqrt{\pi a}]^m} da$$

where $T_i^0$ represents the time until the failure of the $i$-th member given that no other members have failed; $v_0$ is the frequency of the applied loading; $a_i^c$ is the critical crack length of the $i$-th member that leads to the crack failure; $a_i^0$ is the initial crack length for the $i$-th member given that no members have failed; and $S_i^0$ denotes the far-field stress range of the $i$-th component in the original (i.e. no damage) stress distribution.

As for sequential system failures induced by local failures, it becomes necessary to model load redistributions and find how long it takes for other members to fail after the previous members have failed. Using further inspiration from Lee & Song (2010), one can formulate these times efficiently in terms of the redistributed stresses. For example, the time until the $i$-th component fails after the occurrence of the local failure sequence $\{ 1 \rightarrow 2 \rightarrow \ldots \rightarrow (i-1) \}$ can be evaluated by the following recursive formula (Lee & Song 2010):
For a two dimensional structure, the system failure event caused by the failure sequence \( \{1 \rightarrow 2 \rightarrow \ldots \rightarrow (i-1)\} \) is described as follows (Lee & Song 2010):

\[
\left[ \bigcap_{j \neq i} (T_i^0 < T_j^0) \right] \bigcap \left[ \bigcap_{l \neq i, 2} (T_i^0 < T_l^0) \right] \bigcap \ldots \\
\bigcap \left[ \bigcap_{l \neq i \ldots j} (T_i^{l-1} < T_l^{l-1}) \right] \bigcap \left( T_i^0 + T_1^1 + \cdots + T_i^{l-1} < T_{ins} \right) 
\]

The events in the brackets describe the occurrence of the particular failure sequence (“1 fails first” and “2 fails next” and so on) while the last event indicates that the system failure occurs within the inspection cycle. The event in Equation 8 constitutes one of the cut-sets for the system failure event shown in Equation 1. Owing to the mutually exclusiveness of the cut-sets formulated as above, the lower-bound on the system cascading failure probability is obtained by a simple addition of the probabilities of the cut-sets. Therefore, the sum of the probabilities of the cut-sets identified by the selective searching technique provides a lower bound, i.e.

\[
P(E_{sys}) = \sum_{k=1}^{N_{id}} P(C_k) \geq \sum_{k=1}^{N_{id}} P(C_k)
\]

where \( N_{id} \) denotes the number of the critical failure sequences identified by the selective searching technique. It is noted that there is no need to characterize the statistical dependence between failure modes or to perform additional system reliability analyses to get \( P(E_{sys}) \), as the cut-sets formulated in Equation 8 are all mutually exclusive. Lee & Song (2010) computed the probability of each failure mode, \( P(C_k) \), \( k = 1, \ldots, N_{id} \), by performing component reliability analyses for each of the events in Equation 8 using the first- or second-order reliability method (FORM or SORM; Der Kiureghian, 2005), followed by a system reliability analysis using an efficient sampling method (Genz 1992). For the numerical example of this paper, high nonlinearity of limit state functions in Equation 5 and Equation 6 prevented FORM and SORM from obtaining accurate estimates on the probabilities of the events in Equation 8. A sampling method was thus used to estimate the probabilities of the identified cut-set events. The probability of the cut-set event is directly estimated by a Monte Carlo sampling method instead of performing component reliability analyses and a system reliability analysis. The estimated probabilities are added up as in Equation 9 to obtain a lower bound of the system failure probability.
The proposed methodology is demonstrated through a numerical example of a truss bridge system shown in Figure 4. The structure consists of 97 elements (E1,…,E97) and 50 nodes (N1,…,N50). There are pin supports at the nodes N2 and N50, and roller supports at the nodes N1 and N49. This planar structure is both internally and externally statically indeterminate to the third degree. This model was inspired by the Grand Sung-Soo bridge in Seoul, South Korea (before the re-construction), and has the same members lengths and areas as described in KSCE (1995). In this example, three truss members were added at the internal hinges to add complexity.

Since proper field strain data was unavailable for the particular example, any sort of direct strain history based method, such as the one shown in Zhou (2006) was deemed inappropriate, in favor of using the fatigue analysis recommended by the LRFD Bridge Specifications (AASHTO 2004). This entails executing a full influence load analysis using a truck weighing 75% of the AASHTO design truck. From this analysis, one can obtain stress ranges from each member in a given damage state during the selective search. If none of the stresses for a given member are large enough to initiate crack growth, that member’s limit state can be neglected for that time of analysis.

Table 1. Distribution types and statistical parameters of random variables.

<table>
<thead>
<tr>
<th>Random variables</th>
<th>Distribution</th>
<th>Mean</th>
<th>c.o.v.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$</td>
<td>Lognormal</td>
<td>$1.202 \times 10^{13}$</td>
<td>0.533</td>
</tr>
<tr>
<td>$m_i$</td>
<td>Lognormal</td>
<td>3</td>
<td>0.02</td>
</tr>
<tr>
<td>$a_i^0$</td>
<td>Exponential</td>
<td>0.11 mm</td>
<td>1</td>
</tr>
<tr>
<td>$I$</td>
<td>Normal</td>
<td>1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

See Table 1 for the distribution types and statistical parameters of the random variables used in this study: material parameters of the Paris-Erdogan crack growth model, i.e. $C_i$ (mm/cycle/(MPa-mm)$^m$) and $m_i$, the initial crack lengths $a_i^0$ of the truss members, $i=1,…,97$, and the stress range multiplier $I$, to model the randomness in the traffic loading. Each of these random variables is modeled based on examples in the literature (Lee & Song 2010). Each member is assumed to have an elastic modulus of 200 GPa. The average daily truck traffic (ADTT) for the Grand Sung-Soo bridge was 4,483 (Cho et al. 2000). The ADTT was multiplied by 365 days to determine the annual loading frequency $v_0$.

A total of 63 significant failure modes were identified by the selective searching method, whose reliability indices range from 2.7752 to 4.7534. It is noted that 30 modes with higher likelihood have similar reliability indices between 2.7752 and 3. See Table 2 for a list of the most critical failure modes and their associated reliability indices. The existence of many critical failure modes with similar likelihood reflects the high degree of symmetry and redundancy of the bridge. It is also noted that all of these 30 critical modes originate at members 61, 62, 63, and 64, which are the diagonals in the central part of suspended truss. The nature of these “competing” failure modes made it necessary to identify 63 modes.

In using the selective searching method, an $N_{same}$ value of 10 was used. The lower bound on the system failure probability by Equation 9 using 63 modes is $7.03 \times 10^{-2}$ (generalized reliability index 1.4509). This result is verified by brute-force Monte Carlo Simulation (MCS) which produces the reliability index of 1.4735 with a c.o.v. of 3.4%. The relative error is only 4.22%. See Table 3 for a list of CPU time costs for the proposed method and MCS. The brute-force MCS simply generates $x$ repeatedly and check if the system fails within the inspection cycle or not in order to tally the number of times a system failure occurs, disregarding the way the system failure happens. By contrast, the proposed method identifies the most significant failure modes in the decreasing order of likelihood.

Table 3. Computational cost for the proposed method and MCS.

<table>
<thead>
<tr>
<th></th>
<th>Proposed Method</th>
<th>Brute-force MCS</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPU time (sec)</td>
<td>441.04 sec</td>
<td>132,620 sec (≈36.84 hours)</td>
</tr>
</tbody>
</table>

5 CONCLUSION

This paper develops an efficient and accurate method to identify dominant failure modes of a structural system subjected to the risk of fatigue-induced cascading failures and compute the probabilities of the overall system and failure mode events. Using the proposed approach, identification of dominant failure
modes and evaluation of the system failure probabilities are decoupled. Dominant failure modes are first identified using the selective searching technique employing a genetic algorithm. The failure modes are formulated as mutually exclusive events, and their probabilities are calculated by sampling. The system failure probability can then be found by simply summing up the failure mode probabilities while fully considering dependence between dominant failure modes. This approach has several advantages:

- Decoupling failure mode identification and system reliability analysis helps to prevent the computational cost from rapidly increasing with the complexity of the structure.
- This simulation-based technique identifies cascading fatigue failure modes.
- The mutually exclusive formulation for sequential failure modes accurately accounts for statistical dependence between failure mode events.

In order to demonstrate this method in system reliability analysis of complex bridge systems, a 97 member planar-truss numerical example was analyzed. The proposed method identified 63 failure modes. A brute-force Monte Carlo simulation confirmed that the proposed method can compute the system failure probability accurately and efficiently.

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