Northeastern University
College *of* Engineering

DEPARTMENT OF MECHANICAL AND INDUSTRIAL ENGINEERING

ME 7205 Advanced Mathematical Methods for Mechanical Engineers

CATALOG DESCRIPTION

ME 7205 Advanced Mathematical Methods for Mechanical Engineers (4SH)
Topics include complex variables; generalized functions; integral transforms (including Fourier transforms); variational calculus and applications; approximate methods of engineering analysis including asymptotic expansions and regular and singular perturbation methods; and numerical solution of partial differential equations with emphasize on parabolic and elliptic problems occurring in mechanical engineering. Examples are drawn from solid mechanics, vibration, and fluid mechanics. *Prerequisite: ME 6200*.

COURSE DISCUSSION

As the catalog description implies this is a multifaceted course, the most advanced in the sequence of mathematics methods courses for MEs. Although the first (or equivalent) course is a prerequisite, it is really a graduate level of mathematical maturity that is required, since the course will be largely self-contained. I will attempt to treat the topics listed in the catalog description in a connected way (except for variational and numerical methods) tying them all to complex analysis.

The course will begin with a development of the main ideas behind functions of a complex variable, including complex algebra, simple functions, elementary transcendental functions and series, the integral theorems, contour integration in the complex plane, and integral transforms as applied to ordinary and partial differential equations. Generalized functions will also be covered, including their Fourier transforms. This portion of the course will cover 9-10 weeks. This is followed by asymptotic expansions, including the asymptotic expansions of functions defined by integrals, regular and singular perturbation theory.

Students are expected to be familiar with the exercises from the textbooks in the sections listed on the syllabus. Problem Sets will be collected, graded and solutions provided. The Problem Sets will serve as more challenging assignments and will apply toward the grade. In this sense they can be considered to be a series of take home quizzes.

REQUIRED/REFERENCE COURSE TEXTBOOKS

Advanced Engineering Mathematics 2nd Ed.; M.D. Greenberg (Prentice Hall, 1998) (Used in the first course in this series, ME 6200). This has an excellent elementary introduction to complex variables in Chapters 21-24. We will cover these chapters in some depth. In addition preparatory materials for these chapters can be found in Sec 4.2.1. *Review of Power Series*, as well as sections 4.5.1 *Singular Integrals* and 4.5.3 *Order of Magnitude* which you should read.

Mathematical Methods for the Mechanical Sciences (abbreviated MMMS), Howe. Available as a pdf from http://www.bu.edu/me/people/faculty/gn/howe/ (with permission of the author). This is an excellent compact treatment of complex variables, contour integration, generalized functions and Fourier Transforms as applied to PDEs. See especially sections: 1.11, 4.9, 4.10.

Advanced Calculus for Applications: 2nd Ed. Hildebrand. (Prentice-Hall, 1976). See especially Chapters 10 and 11 for Complex Variables and Applications as well as Sec. 4.1 for power series.

Complex Variables and Applications: 8th Ed. Churchill and Brown (McGraw-Hill, 2009). This is a very complete treatment at about the same level as Greenberg.

Complex Variables: Introduction and Applications 2nd Ed. Ablowitz and Fokas. (Cambridge University Press, 2003) This is a somewhat deeper and more complete treatment with excellent sections on Laplace and Fourier Transforms as well as asymptotic expansions of integrals.

Functions of a Complex Variable: Theory and Technique: Carrier, Krook and Pearson (McGraw Hill 1966). This is an excellent text for applications, but the treatment of fundamentals is terse, for those who are already comfortable with complex analysis. Reading Chapters 1 and 2 after we have covered the fundamentals to see how they can be presented completely in a short span is recommended. Transform methods are done very well in Chapter 7 and Asymptotic Methods in Chapter 6.

Differential Equations A. C. King, J. Billingham and S. R. Otto. (Cambridge U.P. 2003). See Chapter 4 on *Fourier Transforms and Generalized functions*, Chapter 6 *Laplace Transform* and Chapters 11 and 12 on *Asymptotics and Perturbation Methods.*

A First Course in Partial Differential Equations with Complex Variables and Transform Methods H. F. Weinberger. (Dover, 1995). Here the treatment of complex variables is done with a view to problems in PDEs.

Introduction to Mathematical Physics M. T. Vaughn. (Wiley, 2006). See especially Chapter 1: Infinite Sequences and Series and Chapter 4: Functions of a Complex Variable.

Mathematical Methods for Physicists 6th Ed. Arfken and Weber. (Elsevier, 2005) See especially Chapter 5: Infinite Series, Chapters 6 and 7: Functions of a Complex Variable and Chapter 15: Integral Transforms.

Fourier Transforms I. N. Sneddon (Dover, 1995). This is the classic reference with lots of great problems solved in mechanical engineering areas.

Introduction to Fourier Analysis and Generalized Functions: Lighthill (Cambridge, 1964). This is the first really good treatment of generalized functions and Fourier Transforms at an elementary level, but written for those with a good foundation in analysis. Requires very careful reading but is the complete word on generalized functions, Fourier transforms and series.

Asymptotics:

A First Look at Perturbation Theory: Simmonds and Mann (Dover, 1998) This is an excellent and well-written introduction to perturbation methods, including both singular and regular perturbations. Written for an undergraduate course, this text is easy reading, but with the main theoretical points developed during examples and dispersed throughout the examples.

Perturbation Methods: Hinch (Cambridge, 1991) Very readable book, starting with simple problems and leading to some truly hard problems.

Asymptotic Expansions of Integrals: Bleistein and Handelsmann (Dover, 1986). A very complete reference at the research applied mathematician level.

Perturbation Methods in Applied Mathematics: Kevorkian and Cole (Springer, 1996). Ditto

TOPICAL SYLLABUS AND READING

Week	Topics	Reading
#1	Complex Algebra, Simple functions, powers and roots; Polar form of complex numbers; Sequences, series and convergence in the complex plane. Comparison Test, Ratio test.	G: 21.1-21.3 G:24.2.1
#2	Integral test, Order of magnitude symbol (Big O), relative growth and decay. Complex Exponential, logarithm, general powers. Definition of e^z. Complex trigonometric and hyperbolic functions and their inverses. Multi-valued functions, branches, branch cuts.	G:4.5.1, 4.5.3 G:21.4, 5
#3	Derivatives, analyticity, Cauchy Riemann Eqns. Complex Integration Cauchy Integral Theorem,	G: 21.5, 23.2, 23.3
#4	Antiderivatives, examples. Cauchy Integral Formula, Morera's Theorem, Liouville's Theorem.	G: 23.4, 23.5
#5	Taylor and Laurent Series	G: 24.2, 24.3
#6	Classification of Singularities, Residue Theorem	G: 24.4, 24.5
#7	Contour Integration.	G: 24.5
#8	Fourier Transforms: Generalized functions	G:17.9-11: KBO 5.2-3
#9	Laplace Transforms	G: 5(all): KBO 6 (all)
#10	Applications of Integral Transforms, ODEs , PDEs.	
#11	Asymptotic Expansions; Asymptotic Expansions of Integrals; Integration by Parts,	KBO: 11
#12	Laplace's Method, Stationary Phase, Steepest Descents	KBO: 11
#13	Perturbation methods: Algebraic equations, regular and singular perturbations. Applied to ODEs.	KBO: 12
#14	Applied to ODEs. Multiple scales, Poincare's Method.	KBO:12

Textbook abbreviations
KBO: King, Billingham, Otto, *Differential Equations*
G: Greenberg, *Advanced Engineering Mathematics*