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Contents

Freeway Speed-Flow-Concentration Relationships: More Evidence and Interpretations

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In this paper, recent Canadian work challenging long-held theories about speed-flow-concentration relationships in freeway traffic is verified and extended using data from San Diego. Major conclusions of these studies regarding near-constant free-flow speeds and flow-concentration relationships resembling an inverted V are confirmed; other past findings related to the effect of queuing on downstream free-flow speeds and the effect of secondary bottlenecks on flow-concentration relationships are not confirmed. When data are averaged across all lanes (both upstream and downstream of a bottleneck), speed-flow and flow-concentration relationships are found to be consistent with those predicted by queuing and shock wave theory. The functioning of the bottleneck studied is more complicated than had been assumed, however, and this creates further problems in the interpretation of the data. Finally, the inverted-V model of the flow-concentration relationship is shown to imply a simple and plausible model of driver behavior in which speeds and spacings are adjusted to keep the average front-to-back time gaps approximately constant until some desired maximum speed is reached.

Several recent reports dealing with freeway traffic flow have challenged long-held views about relationships among speed, flow, and measures of traffic concentration such as density or occupancy. Although there has never been complete agreement about the nature of these relationships, the representations found in the 1985 *Highway Capacity Manual* (*1*) (for instance, the speed-flow relationships depicted in Figures 3 and 4) may be said to represent the current conventional wisdom. Recent empirical work that challenges these views includes a series of reports by Hurdle and various associates (*2–4*) and a series of papers by Hall et al. (*5–8*). Both of these efforts, which were based on data from Toronto, are indebted to some degree to previous work done by Koshi et al. (*9*) in Japan.

A common point of departure for this work is concern that data may have been misinterpreted in the past due to lack of sensitivity to the effects of location on freeway flow phenomena. Specifically, both groups of Canadian researchers are concerned with the ways in which neglect of queuing phenomena could lead to misinterpretation of speed-flow-concentration data. Particular concerns include

- The effect of queues in limiting flows upstream of bottlenecks, thus creating gaps in the data;
- The implications of data-gathering and data-reduction procedures that may involve averaging data from dissimilar flow conditions; and
- Possible misinterpretation of data taken in the zone of acceleration downstream from a queue.

The primary concern of Hurdle and Datta (*2*), Persaud (*3*), and Persaud and Hurdle (*4*) is speed-flow relationships under free-flow conditions. Their major finding is that there is no precipitous drop in free-flow speeds as flow approaches capacity, although there may be a gradual drop. This is contrary to the popular belief that speeds decline rapidly once the volume/capacity ratio exceeds about 0.80. Other results of this work include the following:

- A suggestion that free-flow speeds of drivers who have been in queues may be noticeably less than those of drivers who have not, even at considerable distances downstream of the bottleneck (*2*);
- A finding that acceleration back to free-flow speeds was still incomplete at a point about 0.8 km (0.5 mi) downstream from the bottleneck (*3*); and
- Speculation that precipitous drops in free-flow speeds reported by previous researchers may have resulted from lumping together data from different locations in the acceleration zone downstream of the bottleneck, where speed would be expected to increase at nearly constant volumes as one moves downstream (*3,4*).

The series of papers by Hall et al. deals with the overall nature of speed-flow-concentration relationships. Major findings are stated in terms of the flow-occupancy relationship. They are that this relationship varies according to both lane and location (*7*) and that the basic relationship is best described (except for shoulder lanes) as a continuous but not continuously differentiable relationship resembling an inverted V, as opposed to the conventional inverted-U shape, the reverse lambda shape proposed by Koshi, and various discontinuous models (*6,7*) (see Figure 1). This finding is consistent with the idea that there is little or no drop in speed with increasing flow under uncongested conditions. The left branch of the flow-occupancy relationship is interpreted as representing free flow. If this branch is linear (or nearly linear), speeds in the uncongested regime are constant (or nearly constant) as flow increases. In addition, Hall and Gunter (*7*) suggest that flow-occupancy relationships may be shifted upward (that is, higher flow at a given occupancy) in secondary bottlenecks.

Hall et al. are also concerned with the conditions under which transitions between congested and uncongested flow take place. They approach this question by studying time

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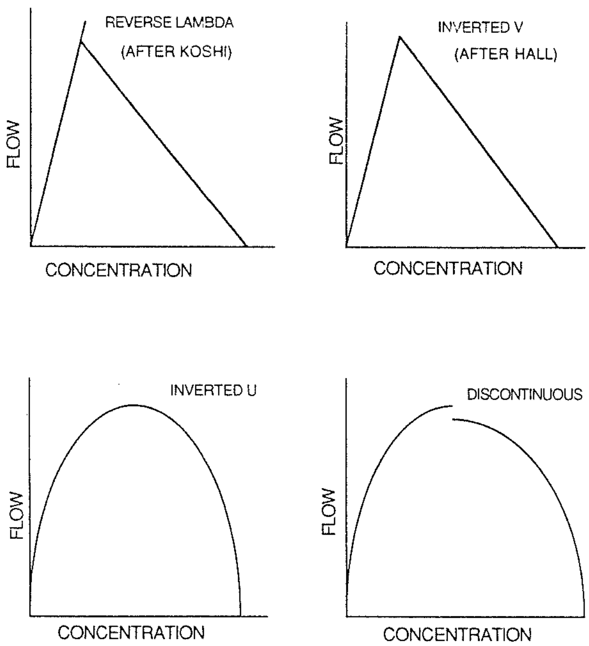

FIGURE 1 Flow-concentration models.

sequences of speed-flow and flow-occupancy states. They report some general patterns (*5,8*) and also use such sequences in the screening and interpretation of data (*5–7*).

RESEARCH OBJECTIVES

The research described in this paper was intended to verify and extend certain aspects of the recent Canadian work. Specific objectives were

- To compare conclusions drawn from the Toronto and Tokyo data with similar data from San Diego,
- To take a more systematic look at the effect of queuing on speed-flow-concentration data, and
- To extend the interpretation of Hall's flow-concentration model by determining its implications as a model of driver behavior.

A major issue considered in the research but not dealt with in this paper is the interpretation of time sequences of speed-flow and flow-concentration states. Interpretation of such data needs to be clearly related to theories of shock wave movement. The discussion of this subject by Hall et al. (*6*) is by no means complete; unfortunately, the matter is too complex to address here.

The first objective of this paper is to verify the recent Canadian findings using data from the San Diego ramp metering system. The San Diego data are quite similar to those used by Hall because they were produced by the metering system's detectors during their routine operation. In this respect they differ from those used by Hurdle, which were produced from time-lapse films. Consequently, the San Diego data set shares the advantages and disadvantages of that used by Hall: the data are abundant, but their exact interpretation is less certain than that of data reduced from a visual record. In general, less screening of data was employed in this study than in the Canadian ones. No effort was made to screen out data resulting from incidents or transitions between congested and uncongested flow. The justifications for this are (a) that incident data should resemble other congested flow data and (b) that allowances can be made for the effects of transitions and the averaging of data from dissimilar flow states without eliminating the data. The major question to be addressed by this part of the work is the extent to which the relationships implied by the Toronto and Tokyo data are confirmed by the San Diego data.

The second objective, to take a slightly different look at the way the data are affected by the relationship between the data-gathering location and the bottleneck, requires a bit of explanation. Although both groups of Canadian researchers are concerned about the effect of queuing on the proper interpretation of speed-flow-concentration data, the only comparison of data gathered both upstream and downstream of the same bottleneck is a discussion in Persaud (*3*) of the process of flow breakdown, which includes data from immediately upstream of the bottleneck as well as from the bottleneck section itself.

A more serious problem arises because Hall's analysis of the effect of queuing on relationships observed upstream of the bottleneck is based on data analyzed on a lane-by-lane basis. At first glance, this appears to be an advantage; it provides more detail and, as it turned out, somewhat different relationships were found in different lanes. However, the whole basis for the concern about queuing is the observation that, once the queue backs up into the upstream section, flow at that point is limited to the capacity of the bottleneck. Thus, maximum flows depend on conditions downstream rather than at the point of observation. This means speed-flow-concentration relationships for near-capacity flows may never be observed; rather, discontinuous data and sudden transitions in speed and concentration can be expected, even if the underlying relationship is continuous (see Hall et al. (*6*), especially Figure 1, for the full argument).

The problem with this reasoning is that it applies only to the facility as a whole, not to individual lanes. In fact, the discontinuous data were found only in the shoulder lane (*6,7*), which suggests that flows up to capacity were occurring in the other lanes (although the average flow across all lanes was less than capacity) during periods of uncongested flow. Consequently, hypotheses about the ways in which queuing and shock wave movement affect speed-flow-concentration data can be verified only if data are averaged across all lanes.

This paper addresses these issues using data collected both upstream and downstream of a bottleneck and data averaged across all lanes. Specifically, the hypothesis to be tested is that, for such data, speed-flow and flow-concentration relationships will appear to be "truncated" in terms of flow (possibly to the point of appearing discontinuous) upstream of the bottleneck but not downstream.

In the process of testing this hypothesis, yet another problem arose: the operation of the bottleneck was more complicated than anticipated. In particular, it does not appear that any one location is always the critical point, so data representing both upstream and downstream conditions may have existed at two of the three locations, and many of the data may actually represent transitional states.

The third objective of this paper is to extend the interpretation of the inverted-V flow-concentration relationship proposed by Hall to determine its implications as a model of driver behavior. Since several models might represent the data equally well, it is interesting to note that this one is consistent with a plausible hypothesis about how drivers determine speeds and spacings.

DATA AND DATA ANALYSIS

The data used in this study were morning-peak (6:00–9:00 a.m.) data produced by loop detectors in the San Diego ramp metering system. Raw data consisted of volumes and occupancies reported at 30-sec intervals for each lane. These had been used in a previous study (*10,11*) and had already been aggregated across all lanes and over 6-min intervals, with the volumes converted to flow rates expressed in vehicles per hour per lane. In addition, average speeds had been calculated from the volume and occupancy data, assuming an average effective vehicle length (including the detector) of 25.75 ft. While there may be some errors in the speed observations due to variations in vehicle length, the speeds appear to be consistent and reasonably accurate.

For purposes of this study, three locations near a major bottleneck on Interstate 8 were selected. Data were collected over a total of 30 days in the months of September and November 1987. Most data represented normal weekday morning commute conditions; however, two holidays were included to provide data related to low-volume free-flow conditions.

Figure 2 is a schematic diagram showing the detector locations and lane configurations. When these locations were selected, it was believed that the section just downstream from the College Avenue onramp, which experiences extremely high flows per lane, is the bottleneck under normal conditions. Further observation and careful examination of the data suggest that no single bottleneck location can be identified, however, and that the detectors upstream of this onramp are often in the zone of acceleration downstream of the queue. At other times, minor queues of short duration form at or downstream from the detectors at Waring Road.

In selecting these locations, the intention was to choose one normally downstream from the bottleneck, one immediately upstream, and one far enough upstream that most queuing would result from traffic backing up from the primary bottleneck. Data were also available from other bottleneck locations in the San Diego area, but this one was used for analysis because of the extremely high volumes just downstream from College Avenue and because there is very little flow on the offramp upstream of the Waring Road detectors. Other bottlenecks for which data were available terminate in freeway branch connectors and involve large drops in flow on the freeway main line upstream of the first set of detectors that are downstream from the bottleneck. In addition, the College Avenue and Waring Road detectors are separated by a comparatively long distance and steep downgrade, which means Waring Road should be beyond the acceleration zone from the College Avenue bottleneck.

As in most recent studies of speed-flow-concentration relationships, the data analysis concentrated on qualitative relationships. A microcomputer with color graphics capability was used to combine and display data. Displays that proved useful in analyzing the data included

- Graphs of speeds and flows versus time of day, for different days at the same location and different locations on the same day;
- Scatter plots of speed versus flow and flow versus occupancy for each location separately and for the three locations combined; and
- Scatter plots of speeds at the same time at two different locations.

In addition, regression analysis was used in the study of speed-flow relationships under free-flow conditions.

Graphs of speeds and flows versus time of day were used to identify linkages between speed and flow conditions at

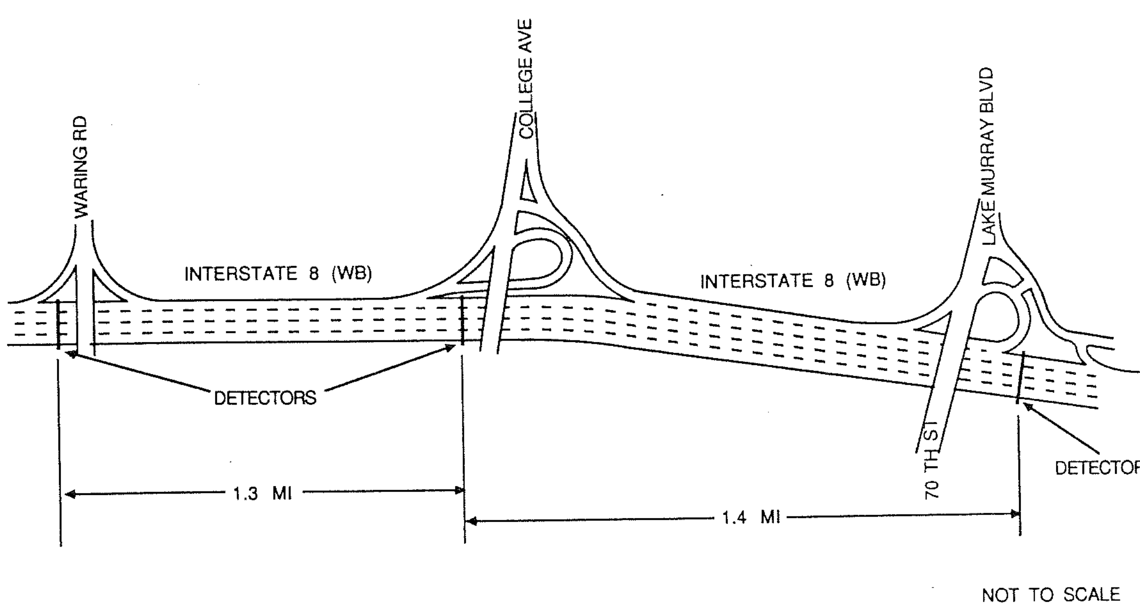

FIGURE 2 Schematic diagram of study site.

different locations and to identify the time of day at which various phenomena occur for purposes of further observation. Scatter plots of speed versus flow and flow versus occupancy were used to investigate these relationships at different locations and under different conditions. Scatter plots of speeds occurring simultaneously at different locations were used to determine whether any correlations between speeds upstream and downstream of the bottleneck might indicate that speed variations downstream of the bottleneck were due to acceleration effects.

VERIFICATION OF PAST WORK

Figure 3 shows scatter plots of speed versus flow at the three locations; similar plots for flow versus occupancy are provided in Figure 4. These plots confirm the major findings of the recent Canadian work:

- There is only a slight drop in speed as flow increases under free-flow conditions, with speeds in excess of 50 mph persisting up to the highest flow levels recorded; and
- The overall relationship between flows and occupancies could well be described as an inverted V, allowing for the effects of queuing and data that might result from averaging over dissimilar flow conditions.

As in Persaud and Hurdle (*4*), there is a slight decline in free-flow speeds as flows increase beyond 1,500 veh/lane/hr. In this study, all data involving flows less than this were gathered on holidays and may not be comparable to the rest of the data. As can be seen in Figure 3, regression analysis confirmed that the slope of the speed-flow relationship varies slightly with location, with the steepest slope occurring at College Avenue. Comparison with the Toronto data indicates that the decline in speed is far more gradual in San Diego, with drops of from 0.3 to 0.5 mph per 100 vph in San Diego as opposed to a rough estimate of 2 to 3 mph per 100 vph for the data in Figures 8 and 9 of Persaud and Hurdle (*4*).

Figure 4 shows the same basic relationships as those discussed by Hall et al. (*6,7*) and Koshi et al. (*9*). The overall flow-occupancy relationship might be described by either the inverted V of Hall or the reverse lambda of Koshi. As noted by Hall, data points are tightly distributed about the left branch of the relationship; somewhat more scatter appears in the right branch. In addition, some points fall "inside" the V or lambda. Time sequences of the data reveal that these often occur during rapid transitions from one branch to the other; hence, they appear to represent averages of data from dissimilar flow conditions. As might be expected, the greatest difficulty in interpreting the data occurs at high volumes in the region where the two branches of the relationship appear to meet.

Other findings by the Canadian researchers concerning the effects of upstream queues and secondary bottlenecks were not confirmed by the San Diego data. Hall and Gunter (*7*) found that flow-occupancy relationships in a "secondary bottleneck" were shifted upward; in other words, higher flows occurred for given occupancies than at other locations. Hall and Gunter's definition of a secondary bottleneck is somewhat unclear. However, the situation appears to have been similar to that at 70th Street and Lake Murray Boulevard in that

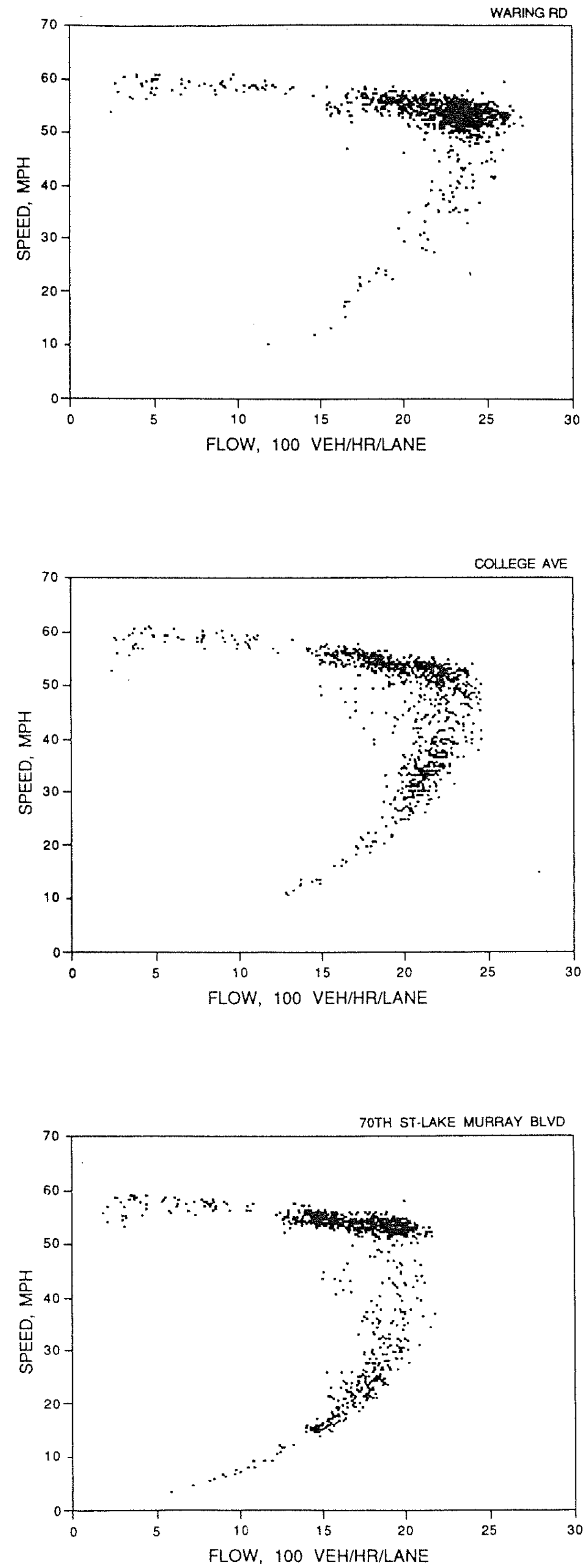

FIGURE 3 Speed-flow scatter plots.

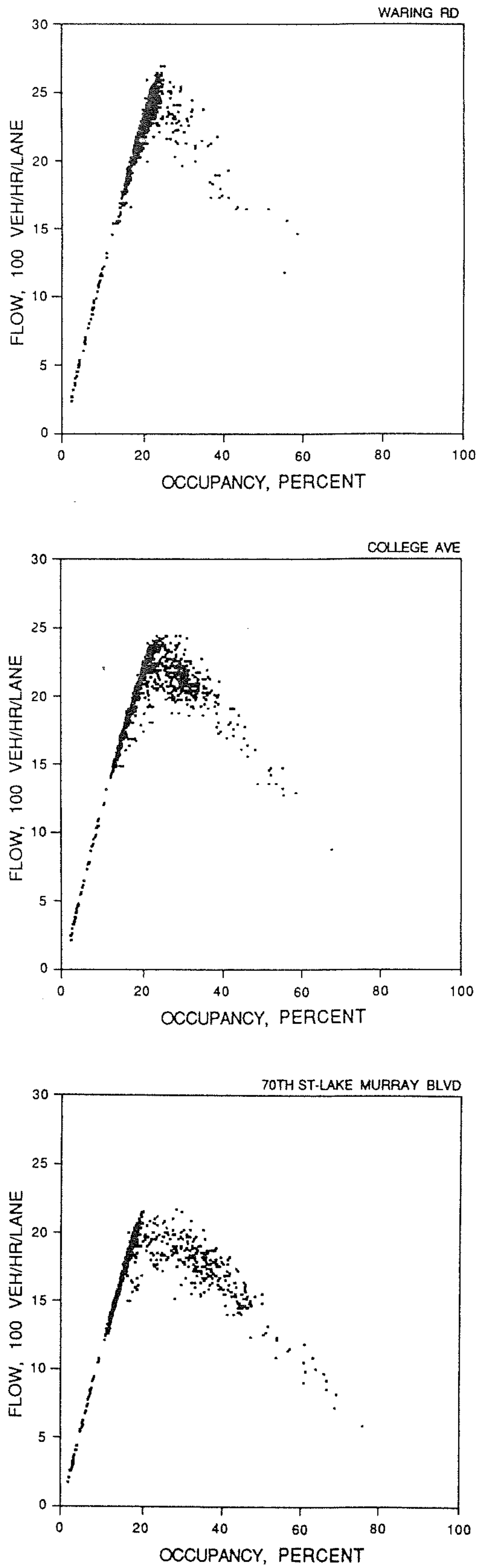

FIGURE 4 Flow-occupancy scatter plots.

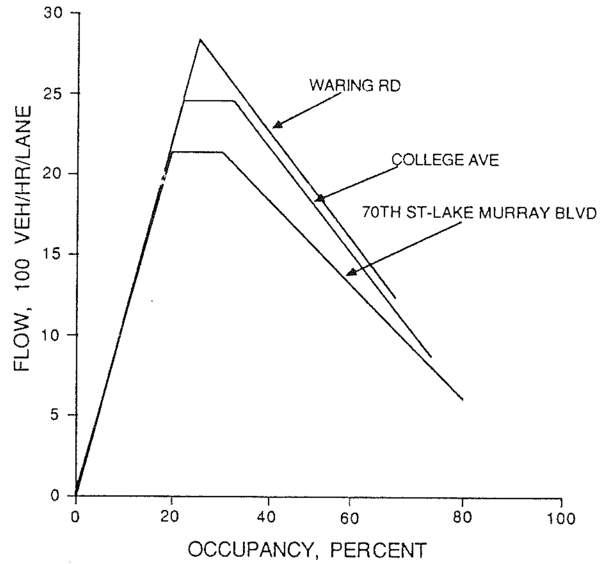

FIGURE 5 Approximate upper bounds of flow-occupancy data.

queues normally backed up into the section from downstream, and heavy onramp flows resulted in smaller mainline flows than occurred in the queue immediately downstream.

If the situations were similar, the San Diego data reveal the opposite result: the flow-occupancy relationship at 70th Street and Lake Murray Boulevard is shifted downward relative to the others. Figure 5 compares the approximate upper bounds of the data at the three San Diego locations. It would appear that, whatever the reasons for the variations in the flow-occupancy relationships at different locations, the presence of "secondary bottlenecks" in the sense of Hall and Gunter is not the cause.

The suggestion by Hurdle and Datta (*2*) that free-flow speeds are reduced when drivers have passed through upstream queues was also not confirmed. Comparisons of speeds taken simultaneously at Waring Road (downstream of the bottleneck) and College Avenue (presumably just upstream) showed that normally there was no relationship between them, and there was no consistent tendency for speeds at Waring Road to drop as soon as the queue formed upstream of College Avenue. When a drop in speed occurred at both locations, it appears most likely that the cause was a minor queue backing upstream from Waring Road or some point downstream. Consequently, these data provide no support for the idea that drivers who have experienced queues upstream alter their free-flow speeds.

EFFECTS OF QUEUING

Figures 3 and 4 show that, when data from all lanes are combined, the speed-flow and flow-occupancy relationships predicted by queuing and shock wave theory do appear: the relationships are progressively more truncated in terms of flow as one moves from locations downstream of the bottleneck to those upstream.

Although the San Diego data appear to confirm expectations about the effects of queuing and shock wave movement on observed speed-flow-concentration relationships, the functioning of the bottleneck was more complex than had been anticipated. As previously mentioned, at the time these locations were selected, the bottleneck was assumed to be the section just downstream from the College Avenue onramp. Flows in vehicles per hour per lane are maximum in this section, and the lower speeds associated with queuing usually begin at College Avenue and move upstream from there. At the beginning, it was assumed that queuing would most often involve slowing in the right-hand lane due to merges into inadequate gaps at the College Avenue onramp; the major question was whether the entire section downstream of the ramp should be viewed as the bottleneck or only the merge point itself.

Subsequent observation showed that this picture was false in two ways. The first, which may not be important, was that there appeared to be quite a bit of sporadic queuing at the Waring Road detectors, which were assumed to be downstream of the bottleneck. In this case, an issue of interpretation exists: do the brief episodes of low speed actually represent queuing, an acceleration effect, or sporadic occurrences of the precipitous drops in free-flow speed that Persaud and Hurdle (*4*) failed to find? Acceleration effects seem to be ruled out by the lack of correlation of speeds at this location with speeds upstream, as discussed above. The most likely explanation is that they represent queuing. In some cases, this can be substantiated by the fact that the speed drop at Waring Road appeared to propagate upstream. To be sure, however, visual observations need to be correlated with the data. This was not undertaken because the phenomenon is rare and a great deal of observation might have been required. The lesson here is probably that, if "capacity" is a random variable, it will sometimes be exceeded at places other than the normal bottleneck. This is especially likely where flows are extremely high throughout the vicinity of the bottleneck. An alternative explanation is that all these queues are the result of potentially identifiable minor incidents and that such incidents are fairly common.

The more important way in which the functioning of the section violated the initial assumptions was that, during queue discharge, the College Avenue detectors (which are upstream of the onramp junction) appeared most often to be in the zone of acceleration downstream of the queue. It was hard to tell whether the location of the downstream end of the queue stabilized or not; if it did, the most frequent location would appear to be at a point of somewhat restricted sight distance where horizontal and vertical curves coincide just downstream of the College Avenue offramp junction. Meanwhile, occasional dense queues, which tended to discharge off the downstream end (so that the downstream end of the queue moved upstream), were observed in the right-hand lane. These were presumably the result of braking at the merge point, although it was difficult to actually see this happening. While it is conceivable that some such incident initiates the queuing at this location, dense queuing in the shoulder lane occurs only a small percentage of the time after the queue is established.

In terms of data interpretation, much of the College Avenue data do not represent what can be considered unambiguously to be congested flow; rather, they represent sporadic dense queuing in one lane combined with longer periods in which the location is in a zone of transition downstream of the queue. Despite this, data for periods in which this behavior typically occurs appear indistinguishable from the rest of the "congested" flow data.

This circumstance illustrates some of the difficulty encountered in interpreting high-volume speed-flow-concentration data. First, the speed-flow and flow-concentration relationships can be expected to show truncation in terms of flow both upstream and immediately downstream of a bottleneck as long as no traffic enters the freeway at the bottleneck. Downstream of the bottleneck, flow is limited to the capacity of the bottleneck, which is presumably less than that of the point of observation, just as it is when the point of observation is upstream of the bottleneck. On the other hand, data taken in the zone of acceleration should be distinguishable from congested flow data. Particular levels of flow should be associated with lower speeds and higher concentrations in congested flow than in the zone of acceleration. The most likely reason that the two conditions are indistinguishable in practice is that the scatter of the congested flow data obscures the slight tendency of the transitional data to be shifted toward higher speeds and lower occupancies.

BEHAVIORAL IMPLICATIONS OF INVERTED-V MODEL

The final objective of this research was to extend the interpretation of Hall's inverted-V flow-concentration model to determine its implications as a model of driver behavior. The inverted-V model implies that drivers maintain a roughly constant average time gap between their front bumper and the back bumper of the vehicle in front of them, provided their speed is less than some critical value. Once their speed reaches this critical value (which is as fast as they want to go), they cease to be sensitive to vehicle spacing, and speeds remain more or less constant with respect to volume or concentration. The average time gaps may vary with location, geometrics, driver population, and the like, and may be subject to some random variation, but they should be reasonably constant at any location under similar conditions.

This can be stated mathematically as follows. For the sake of simplicity, it is assumed that the central tendency of the right-hand branch of the flow-concentration relationship is linear. Concentration is defined in terms of density rather than occupancy to take advantage of the definition of flow as the product of speed and density. The following also apply:

- u = average speed;
- u_m = average speed at capacity, assumed to be roughly equal to free-flow speed;
- q = average flow;
- q_m = maximum flow;
- k = average density;
- k_j = jam density;
- k_m = critical density (i.e., the density at which q_m occurs);
- L = average effective vehicle length, which includes the physical length of the vehicle plus any buffer established by the drivers to provide a margin of safety;
- h = average time headway between common points on two successive vehicles; and

g = average acceptable time gap between the front of a vehicle and the rear of the buffer space behind the preceding vehicle.

The equation of the line representing the congested flow regime is

$$q = \frac{q_m(k_j - k)}{k_j - k_m} \qquad \text{for } k_m < k < k_j \tag{1}$$

The assumption that drivers adjust speeds to maintain average time gap g results in

$$h = g + L/u \qquad \text{for } 0 < u < u_m \tag{2}$$

which is equivalent to

$$q = 1/g - Lk/g \qquad \text{for } 0 < u < u_m \tag{3}$$

If it is further assumed that the average spatial gap between common points on vehicles is equal to L at jam density, and that the average space and time gaps at maximum flow are equal to $gu_m + L$ and $g + L/u_m$ respectively, then

$$q_m = \frac{1}{g + L/u_m} \tag{4}$$

$$k_m = \frac{1}{gu_m + L} \tag{5}$$

and

$$k_j = 1/L \tag{6}$$

By substituting these relationships into Equation 1, it can be shown that Equation 1 is identical to Equation 3. Consequently, a linear flow-density relationship in the congested flow regime is identical to a model of driver behavior that assumes that, on average, drivers maintain constant time gaps between vehicles until they reach the desired maximum speed.

The branches of the flow-concentration relationship do not have to be exactly linear for the behavioral model to be approximately true. For instance, most recent work shows a gradual decline in free-flow speeds with increasing flow, which implies some nonlinearity in the left branch of the flow-density relationship. The explanation of this in the model of driver behavior is that different drivers desire different maximum speeds, and, because maneuverability is not complete in high density flows, not all of them achieve this speed at the same flow.

Similarly, some nonlinearity in the congested-flow branch would indicate a slight change in the acceptable time gap (g) with increasing speed. Both the Koshi and Hall models represented the right branch of the flow-concentration relationship as slightly convex to the origin. Whether this is supported by their data (as opposed to a linear relationship) is questionable. The strongest support for the nonlinear relationship comes from Figures 3 and 4 of Koshi (*9*); however, it should be noted that the "density" data in these figures were actually transformed occupancy data, in which the relationship between density and occupancy was assumed to be nonlinear. This nonlinearity is apparent when their occupancy and density data are compared [(*9*), Figure 1], but no specific information is given as to how the density data were related to the occupancy data and no explanation for the nonlinearity (in terms of detector operation, for example) is provided. The normal assumption would be that the relationship between density and occupancy is linear; if it is, the nonlinearity in Koshi's flow-density relationships might disappear.

It has been shown that the inverted-V model of the flow-concentration relationship yields a simple, plausible model of driver behavior; however, the reverse lambda model is in some ways a better representation of the actual data. The contradiction between the models can be resolved if it is understood that the inverted V is primarily valuable as a behavioral model, whereas the reverse lambda is a good representation of the actual central tendency of the data. The key is that both the maximum flow rate for any given time period and the density at which it occurs are random variables with considerable variance. If the line representing the free-flow branch is extended to the highest flow levels recorded (in situations in which the relationship is not truncated by queuing), it will naturally overlap the line representing the central tendency of the congested-flow branch.

CONCLUSION

The research described in this paper involved the use of data from the San Diego ramp metering system to verify and extend recent Canadian research related to freeway speed-flow-concentration relationships. Specific findings include the following:

- The major results of the recent Canadian work were verified by the San Diego data. In particular, there appear to be only slight declines in speed with increasing flow under free-flow conditions, and the overall speed-concentration relationship is well described by the inverted-V model proposed by Hall et al.
- Certain other results of the Canadian work related to the effects of queuing on downstream free-flow speeds and the effects of secondary bottlenecks in shifting flow-concentration relationships were not confirmed.
- When data are averaged across all lanes and include locations both downstream and upstream of the bottleneck, observable speed-flow and flow-concentration relationships are consistent with those predicted by queuing and shock wave theory. This proved to be true even though the functioning of the bottleneck was more complicated than had been assumed.
- The inverted-V flow-concentration model implies a model of driver behavior in which speeds and spacings are adjusted to keep the average front-to-back time gaps between vehicles approximately constant until some desired maximum speed is reached.

The research described in this paper needs to be extended in a variety of ways. First, data from other bottlenecks in the San Diego area, as well as other geographical areas, need to be examined to further confirm the main conclusions. Second, a complete theory of the effect of shock wave movement on time sequences of speed-flow and flow-concentration states needs to be developed and compared with the data. Finally, extensive film or video records need to be taken at detector locations and compared with automatically collected speed-flow-concentration data to better establish their interpretation.

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DISCUSSION

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Dr. Banks has given us an interesting interpretation of a linear right branch of the flow-density relationship, namely, that in congested flow a driver maintains a constant time gap between the rear of the preceding car and the front of his own car. A parallel interpretation is that the spacing between the front of a car and the rear of the preceding car is proportional to speed. One special case of the model implied by this interpretation is the well-known safety rule that drivers should keep one car length between their car and the car ahead of them for every 10 mph.

If $s = 1/k$ = average spacing between common points on two successive vehicles, then Banks's interpretation leads (by manipulating Equation 2) to

$$s = L + gu \tag{2a}$$

while the proportional distance interpretation, with distance expressed in terms of car lengths, leads to

$$s = L + cLu \tag{4}$$

where c is a proportionality constant. The equivalence between these models is apparent. As long as both models use the same vehicle length L, the models' equivalence implies that

$$c = g/L \tag{5}$$

The old safety rule of one car length per 10 mph specifies $c = 1/(10 \text{ mph})$. With a 20-ft car length, this is equivalent, based on Equation 5, to $g = 1.36$ sec. While Banks does not explicitly estimate g from his data, an examination of Figure 3 suggests maximum flows and corresponding speeds of $q_m = 2{,}000$ to 2,400 veh/hr and $u_m = 52$ mph. The corresponding range for g, using the unnumbered equation following Equation 3 and $L = 20$ ft, is $g = 1.54$ to 1.24 sec, showing very good agreement with the old safety rule.

AUTHOR'S CLOSURE

I want to thank Furth for pointing out that the assumption that the right branch of the flow-density relationship is linear is closely related to the well-known safety rule that drivers should maintain a one-car-length separation for every 10 mph. The close numerical agreement between the safety rule and the data is especially surprising when one considers that the safety rule is stated in very round numbers. It should be remembered, of course, that the safety rule was intended to prescribe minimum separations, whereas the flow-density relationship is based on average separations. In fact, many drivers are following much more closely than the average separation, so it cannot be said that the driver behavior reflected by the data is "safe" in terms of the rule.