

# A HEADWAY CONTROL STRATEGY FOR RECOVERING FROM TRANSIT VEHICLE DELAYS

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## **Abstract**

*Suppose a subway train is delayed due to, say, a medical emergency. What adjustments to the following trains' itineraries should be made in order for the schedule to recover from this initial delay? An optimization framework for finding the optimal schedule adjustments is devised. It accounts for the impact of those adjustments on ride time and waiting time, and has as its objective the minimization of total passenger time. Optimality conditions and a solution algorithm are developed. Realistic constraints such as a safety headway, vehicle capacity, and maximum delay at the start of the line are explicitly incorporated.*

*Examples illustrate the main features of the optimal adjustment pattern. After adjustment, a train will follow its leader by the scheduled headway minus an amount called that train's schedule recovery. Because the optimal solution involves a tradeoff between minimizing the ride time impact, which is accomplished by immediately recovering from the initial delay, and the wait time impact, which is minimized by spreading the recovery over a large number of following trains, the optimal recovery pattern lies between these two extremes. In general, there is an S-shaped pattern to the recovery distribution: large recovery for the first one or two trains following the initially delayed train, then rapidly diminishing recoveries per train, and finally small recoveries for the last few trains. The location of the initial delay influences the recovery pattern. Delays that occur on a boarding section, where many waiting passengers will be affected, tend to benefit most from an optimal recovery as opposed to a policy of immediate recovery. On alighting sections, where there are few waiting passengers, immediate recovery is often the optimal policy. Simulated application to ideal routes and to Boston's Orange Line suggest that the savings in passenger time by using an optimal recovery policy instead of immediate recovery vary from 0 to about 100 passenger-hours, depending on the location and duration of the initial delay and on characteristics of the route. The benefits are not large, but appear to be great enough to merit incorporation in automated train control.*

When a train is delayed due, say, to a temporary malfunction of the doors or a medical emergency, how should that delay be propagated to the following trains? One strategy would be to recover the schedule as quickly as possible, by holding the following trains only enough to make them follow the previous train by the minimum safe headway (the safety headway). Alternatively, the recovery could be spread over more trains, holding trains longer than the minimum required for safety. As with any holding strategy, spreading the headways offers some benefit to waiting passengers because it reduces headway variance in comparison with a strategy of having a few very small headways and then returning to full headways. Stated otherwise, spreading the recovery will allow more passengers to enjoy boarding during a time of reduced headway. However, as with any holding strategy, passengers already on board will suffer more delay. The problem at hand, then is to find the headway control / recovery strategy that minimizes overall delay, subject to realistic constraints.

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This problem was originally posed by Turnquist [1989], who built upon original work showing the effect of headway variance on wait time by Welding [1957] and a considerable amount of work done in the late 70's and early 80's on service reliability that is reviewed in Turnquist [1982]. However, the derivation in Turnquist's 1989 paper missed some important aspects; for example, it did not account for the effect of headway variation on the number of waiting passengers, nor did it account for the spatial nature of the problem.

This problem is of increasing relevance as more real-time information on train operations becomes available, and as trains move toward automated control. With real-time information on train location, passenger loads, and passenger boarding and alighting patterns, a more finely tuned solution can be devised. An optimal solution for schedule adjustments can also be more easily implemented if trains are subject to real-time automated control. However, the general pattern of optimal adjustments that will be developed can also illuminate manual decisions regarding schedule recovery for systems that do not have automatic control.

## PROBLEM FORMULATION

Suppose trains are evenly spaced, one headway (H) apart. Then suppose a train, train 0, is delayed an amount  $d_0$ . The trains behind it are numbered train 1, 2, ....

The initial delay will increase train 0's headway to  $H + d_0$ . After a short time required for detecting the delay, the trains behind train 0 can also be delayed. If there are several stations between a train and the train ahead of it when the delay is first detected, one issue is where that train should be delayed. Whenever possible, trains between stations should at least be advanced to the next station before being delayed, since this hurts no one and eliminates delay for passengers on board who are destined for that next station. For this analysis, we have limited the decision scope by assuming that trains that are in a station when the delay is detected will be delayed at that station (if they are to be delayed at all), and trains between stations will be delayed at the next station they reach. The station at which a train will be delayed will be called its decision station.

The control issue is how much, if any, to delay the trains following train 0. Our notation will be that train  $i$  will be delayed so that it follows its predecessor by  $H$  minus an amount called  $r_i$ , the schedule recovery of train  $i$ . In this way, the initial delay  $d_0$  is recovered gradually over several trains. If the recovery is spread over  $N$  trains,

$$\sum_1^N r_i = d_0 \tag{1}$$

The total delay imposed on each train, and its headway with respect to the preceding train, is as follows, as illustrated in Figure 1:

<u>train</u>	<u>delayed by</u>	<u>headway (time since previous train) after point of initial delay</u>
0	$d_0$	$h_0 = H + d_0$
1	$d_1 = d_0 - r_1 = \sum_2^N r_i$	$h_1 = H - r_1$
k	$d_k = d_0 - \sum_1^k r_i = \sum_{k+1}^N r_i$	$h_k = H - r_k$

$$N \quad d_N = d_0 - \sum_1^N r_i = 0 \quad h_N = H - r_N$$

Because headway must always be at least as great as the given "safety headway"  $h_S$ ,  
 $H - r_k \geq h_S$

or

$$r_k \leq H - h_S, \quad k = 1, \dots, N \quad (2)$$

We assume that vehicle delays continue unchanged for the remainder of a vehicle's trip (no "catching up"). We also assume there is sufficient recovery time in the schedule that delays do not propagate to the return trip. Therefore there is no operating cost impact to this control issue. For now, we will also assume that each train's capacity is effectively unlimited so that no passengers are forced to wait for the next train due to crowding. The only impacts are on passenger ride time and wait time.

Let  $V_i$  = volume on train  $i$  at the decision station.  $V_i$  should not include passengers boarding or alighting at the decision station. The total ride time delay is:

$$R = \sum V_i d_i = \sum_{i=0}^{N-1} V_i \left( d_0 - \sum_{k=1}^i r_k \right) \quad (3)$$

To evaluate the wait time impact, it will be helpful to aggregate stations into groups of stations for whom the first delayed train is train  $i$ . For our purposes, then, station group 0 is the station at which train 0 is (or the station it will first reach) when initially delayed, plus all downstream stations. Station group  $i$  ( $i > 0$ ) will consist of train  $i$ 's decision station, plus all stations downstream up to, but not including, those in station group  $i-1$ . If trains  $i$  and  $i-1$  have the same decision station, station group  $i$  will be empty. Let:

$\lambda_j$  = aggregate passenger arrival rate for station group  $j$

(headway) $_{ij}$  = time between train  $i$ 's departure from a station in station group  $j$  and the previous train's departure from that station.

For any train  $i$  and station group  $j$ , the wait time is the product of the number of people affected,  $\lambda_j$ (headway) $_{ij}$ , and their average wait per person,  $0.5$ (headway) $_{ij}$ . Wait time delay is the difference between wait time and the wait time that occurs under a normal schedule, when (headway) $_{ij} = H$ . The headway of train  $i$  at station group  $j$  depends on station group  $j$ 's location, as follows:

$$\text{(headway)}_{ij} = \begin{cases} H & \text{if } j < i & \text{(upstream of station group } i) \\ H + d_i & \text{if } j = i & \text{(at station group } i) \\ H - r_i & \text{if } j > i & \text{(downstream of station group } i) \end{cases}$$

The total wait time delay is then

$$\begin{aligned} W &= \sum_{j=0}^{N-1} 0.5 \lambda_j [(H - d_j)^2 - H^2] + \sum_{i=1}^N \sum_{j=0}^{i-1} 0.5 \lambda_j [(H - r_i)^2 - H^2] \\ &= 0.5 \sum_{j=0}^{N-1} \lambda_j \left( H + \sum_{i=j+1}^N r_i \right)^2 + 0.5 \sum_{i=1}^N (H - r_i)^2 \left( \lambda_{\text{Tot}} - \sum_{j=1}^N \lambda_j \right) - 0.5 H^2 \sum_{j=0}^{N-1} (N+1-j) \lambda_j \end{aligned} \quad (4)$$

where  $\lambda_{\text{Tot}} = \sum_0^N \lambda_j$

## OPTIMALITY CONDITIONS

Minimizing the sum (W + R), and inserting constraint 1 into the objective function with Lagrange multiplier  $\mu$ , the first derivative of the resulting Lagrangian with respect to  $r_k$  reduces to

$$\frac{\partial L}{\partial r_k} = r_k \left( \lambda_{\text{Tot}} - \sum_k^N \lambda_j \right) - \sum_{j=k}^{N-1} \lambda_j \sum_{i=j+1}^N r_i - \sum_k^{N-1} V_i - \mu$$

Equating to zero and solving for  $r_k$  yields

$$r_k = \frac{\sum_{j=k}^{N-1} \lambda_j \sum_{i=j+1}^N r_i + \sum_k^{N-1} V_i + \mu}{\lambda_{\text{Tot}} - \sum_k^N \lambda_j} \quad (5)$$

which can be solved recursively beginning with  $k = N$ , followed by  $k = N-1$ , etc. When  $k = N$ ,

$$r_N = \frac{\mu}{\lambda_{\text{Tot}} - \lambda_N} \quad (6)$$

and so

$$\mu = r_N (\lambda_{\text{Tot}} - \lambda_N) \quad (7)$$

Using this last relationship to substitute for  $\mu$  (making  $\mu$  implicit),

$$r'_k = \frac{r_N (\lambda_{\text{Tot}} - \lambda_N) + \sum_k^{N-1} V_i + \sum_{j=k}^{N-1} \lambda_j \sum_{i=j+1}^N r_i}{\lambda_{\text{Tot}} - \sum_k^N \lambda_j} \quad \text{for } k = N-1, N-2, \dots, 1 \quad (8a)$$

where  $r'_k$  is the value of  $r_k$ , ignoring the safety headway constraint (equation 2). To account for the ceiling imposed by that constraint,

$$r_k = \min (r'_k, H-h_S) \quad (8b)$$

An important and interesting corollary of this result is that

$$r_k \leq r_{k-1} \tag{9}$$

(This relationship is evident, since as  $k$  increases, the numerator shrinks and the denominator grows.) In other words, the recovery is not spread equally over  $N$  trains, which would result in equal headways among those  $N$  trains. Rather, the recovery decreases with each subsequent train, which results in a gradual increase in the overall headway from the first train, whose headway is quite short, until the last train, whose headway returns to the base headway  $H$ .

## A BASIC SOLUTION ALGORITHM

The solution process can be divided into three steps:

Step 1: Determine  $N$ .

Step 2: Given  $N$ , determine  $r_N$

Step 3: Given  $N$  and  $r_N$ , determine  $r_k$  for  $0 < k < N$ .

Working backwards, Step 3 is accomplished directly by recursive application of equation 8. That is, once  $r_N$  is determined, the remaining recoveries follow directly from equation 8. Step 2 can be accomplished by a one-dimensional search for  $r_N$ , searching for the value that, upon application of Step 3, satisfies equation 1. If we call  $\sum r_i$  the total recovery, it is clear that as  $r_N$  increases, so does the total recovery. Therefore, Step 2 is a search for the  $r_N$  for which the resulting total recovery equals the initial delay. A standard interval reduction method such as bisection search can be employed. For now, boundaries for  $r_N$  can be set at  $[0, d_0/N]$ , although for some situations tighter boundaries can be established.

Step 3, solving for  $N$ , is simplified by recognizing that, because  $r_N = 0$  is a permitted solution, a feasible solution with larger  $N$  will always be preferred to one with smaller  $N$ . Of course,  $N \geq d_0 / (H - h_s)$ , for otherwise it would be impossible to satisfy equation 2 even if all the  $r_k$ 's were equal.

Beginning with this lower bound (rounded up), successively increasing values of  $N$  can be tested. As long as  $r_N = 0$  applied in Step 3 yields a total recovery  $\sum_{k=1}^N r_k \leq d_0$ , there will be a feasible solution by simply raising  $r_N$ . As soon as the value of  $N$  is found for which, when  $r_N = 0$ ,  $\sum_{k=1}^N r_k > d_0$ , set  $N=N-1$  and use that value for  $N$ . Furthermore, a tighter upper bound on  $r_N$  will then the value of  $r_{N-1}$  obtained for the last trial value of  $N$  (i.e., the value of  $N$  for which total recovery exceeded  $d_0$ ).

## START OF LINE EFFECT

If the initial delay occurs near the beginning of the line, or if  $N$  is large, the recovery will extend backwards in time to trains that have not yet begun their trip; these trains are called "on-deck trains." Assuming that the delay imposed on an on-deck train  $k$  will be small enough that it doesn't imply delaying train  $k$ 's preceding trip, the decision station of train  $k$  is station 1. Its station group is an empty set, implying that  $\lambda_k = 0$ .  $V_k = 0$  as well. For every on-deck train, then, equation 8 makes it clear that the  $r_k$ 's are all equal to  $r_N$ , that is, all on-deck trains contribute equally to the schedule recovery. Furthermore, if in the optimal solution the recovery extends to even one on-deck train, it is easy to show that it must extend to an infinite number of trains, with each recovering an equal, infinitesimal amount, essentially spreading the delay evenly over remaining trips in the day. This is obviously the best way to minimize wait time since there are no through passengers and wait time is minimized by equalizing headways.

Practically, there will usually be two limitations against spreading the recovery over an infinite number of trains. First, there will be a maximum delay that can be imposed on on-deck trains beyond which their previous trips (in the opposite direction) will be affected. Because the first on-deck train has the greatest delay, it is necessary to impose this limit on the first on-deck train only. This limit depends on the headway, the scheduled recovery time, and the turnaround facilities at the end of the line. For example, on rapid transit lines in Boston with a yard at the end of the line, operations staff estimate that a 5 min delay in the start of a trip can easily be absorbed, but at terminals with only a crossover, the maximum is about 3 min. If this value is called  $d_{\max}$ , the total recovery by on-deck trains is limited to  $d_{\max}$ , since the delay of the first on-deck train equals the total recovery of all on-deck trains. If Step 3 calls for a larger total recovery by on-deck trains, this limit must be imposed, fixing the recovery of the on-deck trains. Then, letting  $f$  denote the latest train that is not on-deck (i.e., the first train whose decision station is station 1), Step 2 becomes a search for  $r_f$  such that

$$\sum_{k=1}^f r_k = d_0 - d_{\max} .$$

If  $d_0 - d_{\max} > f(H - h_S)$ , there is no feasible solution, i.e., there is no way to avoid congestion at the turnaround with its effects on the opposite direction of travel.

Second, it is impractical and of insignificant marginal value to make schedule adjustments of only a few seconds. In programming this algorithm, a minimum schedule recovery of 30 sec was imposed on all but the latest on-deck train. For example, if the optimal solution called for on-deck trains to have a total recovery of 160 sec, the delay would be spread over six trains, the first five each with  $r_k = 30$  sec and the last with  $r_N = 10$  sec.

## CAPACITY CONSTRAINTS

Whenever there is a delay, there is a risk of overcapacity, causing overflow queues on the platforms (passengers unable to board a crowded train). Accurately modeling passenger waiting time in the presence of overflow queues requires some adjustment to the waiting time formula (equation 4). In addition to their large impact on waiting time, overflow queues are highly undesirable because of the aggravation felt by passengers and because congestion near the doors can hamper operations. It has been assumed, therefore, that the subway operator's first objective will be to clear overflow queues, running trains at the safety headway until all queues are cleared. In the process of clearing the platforms, a portion of the initial schedule delay will be recovered. The second objective is then to recover the remainder of the schedule delay in a way that minimizes wait plus ride delay. Essentially, then, if it takes  $m$  trains running behind the originally delayed train at the safety headway to clear the platforms, their recoveries are each fixed at  $(H - h_S)$ . That  $m$ 'th train can then be considered train 0 in our analysis framework, and its delay (relative to its schedule) the original delay.

The capacity of a train for this kind of problem should reflect the number of passengers that can be expected to fit on a train in the presence of overflow queues. This value, sometimes called crush capacity, is greater than the design capacity normally used in scheduling, so that, even in peak periods, it will not usually be necessary to recover the entire schedule as quickly as possible to avoid overflow queues.

## EXAMPLES

To illustrate the types of solutions given by this optimality framework, we begin with an ideal route. It has 26 stations, spaced 2 min apart, divided into five sections. Within each section the stations are identical. First there is an "inbound section" with eight stations at which passengers board only. Next, an "inbound shoulder" section with four stations having equal

boarding and alighting rates. Next, a "downtown" section with three stations which have heavy alighting and some boardings. Next, an "outbound shoulder" section with three stations with equal boarding and alighting rates half those of the inbound shoulder stations. Finally, there is an outbound "alighting section" of eight stations with alightings only. With a 300 sec headway, the peaks load is 1200 passengers, occurring between Stations 8 and 12. Further details are given in Table 1.

The first example illustrates how the schedule is optimally recovered for an initial delay at Station 8, the last station of the boarding section. Capacity is assumed to be unlimited. Results for initial delays of 180 sec to 900 sec (3 min to 15 min) are displayed in Figure 2. As expected, recovery is greatest for the first train, and decreases with each later train. Also, recovery from longer initial delays is spread over more trains. The recovery distribution has an S-shape, with recoveries decreasing at first slowly, then rapidly, and then slowly again as the recoveries approach zero. Initial delays of 660 sec and greater propagate back to trains that have not yet left Station 1 (on-deck trains), whose recoveries are equal (except perhaps for the last one). With the longer delays, the effects of two of the constraints become evident. First, the early trains have their recoveries capped at 210 sec so that they do not violate the 90 sec safety headway. Second, the total recovery by on-deck trains is capped at  $d_{\max} = 300$  sec, limiting to 15 the number of on-deck trains, each recovering 30 sec, the user-set minimum recovery for on-deck trains. For comparison, the "immediate recovery" policy would have trains (except the final one) recovering 210 sec until the initial delay was recovered.

For the same example, the impact on wait, ride, and total time impacts are shown in Figure 3 in comparison with the immediate recovery policy. Because the immediate recovery policy minimizes ride time delay, the ride time savings are negative, but are outweighed by the wait time savings. Not surprisingly, benefits generally increase with initial delay, although not linearly. For a 900 sec (15 min) delay, the total savings compared to policy of immediate recovery is about 68 passenger-hours, not a very large amount. Averaging over the number of passengers on the five trains that would be involved under the immediate recovery policy, this amounts to 13 sec per person. (Averaging over passengers on all trains involved in the optimal policy is deceptive, since the optimal policy involves tiny impacts on a huge number of trains.)

To show the effect of shorter and longer headways, the time savings from using an optimal policy vs. an immediate recovery policy is shown in Table 2 for varying headways and varying initial delays. As headway was varied,  $d_{\max}$  was kept equal to the headway or 300 sec, whichever was greater. In general, savings increases with both headway and initial delay. However, for this example the demand, in passengers/sec, was kept constant. Travel time savings is proportional to the overall level of demand. Therefore the results in Table 2 can be scaled up or down to match an overall demand level. One reasonable scaling would be inversely proportional to the headway, such that total load on the train remains constant. For example, if demand were half as big as the base case and the headway were therefore changed from 300 sec to 600 sec, the travel time savings of the optimal policy vs. the immediate recovery policy for an initial delay of 900 sec would be  $173 / 2 \approx 87$  passenger-hours. Even with this scaling, savings still generally increase with headway.

The location of the initial delay is important. If the initial delay is near the beginning of the line, there are few passengers on board to worry about, and so the optimal policy approaches the minimize wait time policy, i.e., small recoveries for many trains. If the initial delay is near the end of the line, there are few passengers boarding downstream to worry about, and so the optimal policy approaches the minimize ride time policy, i.e., immediate recovery. Between these extremes, the S-shaped recovery distribution prevails. In Figure 4, two summary measures are shown vs. the location of the initial delay: the number of trains over which the recovery is spread, and the travel time savings in comparison with an immediate recovery policy. Number of trains and savings are both large when the delay is near the start of the line. Depending on the headway and the length of the initial delay, the immediate recovery policy becomes optimal or near optimal between station 9 (beginning of the inbound shoulder, after peak volume has been reached) and station 15 (end of the downtown section, after which there are few boardings). In general, delays on the outbound section of a route should be recovered immediately, or nearly immediately, while delays on the inbound section should be recovered more gradually.

The demand distribution on many subway lines differs from the ideal route in that the first station often has a heavier demand than normal. When the demand at the first station is high, the ride time impact of spreading out the recovery increases, and there is a tendency to move toward more immediate recovery. To illustrate this effect, the ideal route was altered by attributing to Station 1 three-eighths of the demand of the boarding section (versus one eighth in the base case). Results are shown in Figure 5 (distribution of recovery for an initial delay at Station 8) and Table 3 (travel time savings in comparison with immediate recovery for an initial delay at Station 8). Comparing these results with the base case results found in Figure 2 and Table 2, the tendency to move toward immediate recovery is clear. However, there still are substantial benefits from spreading the delay in certain cases, particularly when the recovery extends to on-deck trains.

An analysis of the effect of capacity constraints found them to have very little impact. Typically, if the initial delay is large, the optimal solution calls for running the first few trains after the initially delayed train at the safety headway. The capacity constraint calls for doing just the same thing, although if the volume / capacity ratio is close to one, nearly all recovering trains will have to follow at the safety headway. However, for typical cases that were analyzed, the differences were very small.

Finally, examples were run using demand and running time data from Boston's Orange Line. The southbound direction was used. In the a.m. peak, it operates at a 270 sec headway, with 11,000 boardings per hour (3.1 / sec) and a peak volume of 8000 / hr (2.2 / sec). There are 19 stations, roughly grouped into a 4-station boarding section, a 3-station inbound shoulder, a 5-station downtown section, and a 7-station alighting section (there is no outbound shoulder section). Boardings are heavy at stations 2, 3, and 4. Running time is about 33 min. Crush capacity was set at 900 (150 passengers per car, consistent with the maximum observed in a recent count), which is about 1.5 times greater than the average peak load.

The distribution of the optimal recovery from an initial delay at Station 4 (Sullivan Square) is illustrated in Figure 6. Because of the short length of the boarding section, it consists essentially of a mixture of one or two trains with maximum recovery (in this case, 180 sec) followed by trains with small recoveries. Minimizing ride time and alleviating overflows dominates for the first one or two trains, while minimizing wait time by equalizing headways dominates once the recovery reaches trains whose decision station is near or at the start of the line.

## **USING REAL-TIME INFORMATION**

The optimization framework presented in this paper depends to some extent upon availability of real-time information, and as the amount of real-time information increases, still better solutions can be found. Of course, it is necessary to know where the initially delayed train is, and to estimate the duration of the delay. While the formulation presented assumed that the following trains were on schedule, that assumption is not necessary. By incorporating the actual location of each following train, a more precise recovery pattern can be derived. Perhaps the most important contribution real-time information can make to improving the solution is in the area of passenger counts. One important variable is the load on each train. In the absence of real-time information, loads must be estimated from historical on / off patterns. A subway system with automated fare collection could design a system that estimates load on every train from actual turnstile counts and actual train departure times at each station. Likewise, in estimating the impact on waiting passengers, instead of using arrival rates that may be years old, an automated fare collection system should be capable of producing predicted arrival rates over the next 5, 10, 15, and 20 minutes based on a combination of historical counts, recent counts, and counts on that day. Integrating the fare collection system with the train control system will then result in train control that is responsive to the actual loads on each train and the best estimates of arrival rate of waiting passengers so that the chance of overflow queues is reduced and the tradeoff between reducing ride time and reducing wait time is more finely tuned.



## CONCLUSIONS

An optimization framework for finding the optimal schedule adjustments to recover from an initial delay has been devised. It accounts for the impact of the schedule adjustments on ride time and waiting time, and has as its objective the minimization of total passenger time. Optimality conditions and a solution algorithm have been developed. Realistic constraints such as vehicle capacity and maximum delay at the start of the line have been incorporated.

Examples illustrate the main features of the optimal recovery pattern. Because the optimal solution involves a tradeoff between minimizing the ride time impact, which is accomplished by immediately recovering from the initial delay, and the wait time impact, which is minimized by spreading the recovery over a large number of following trains, the optimal recovery pattern lies between these two extremes. In general, there is an S-shaped pattern to the recovery distribution: large recovery for the first one or two trains, then rapidly diminishing recoveries per train, and finally small recoveries for the last few trains. The location of the initial delay influences the recovery pattern. Delays that occur on a boarding section, where many waiting passengers will be affected, tend to benefit most from an optimal recovery as opposed to a policy of immediate recovery. On alighting sections, where there are few waiting passengers, immediate recovery is often the optimal policy. Simulated application to ideal routes and to Boston's Orange Line suggest that the savings in passenger time by using an optimal recovery policy instead of immediate recovery vary from 0 to about 100 passenger-hours, depending on the location and duration of the initial delay and on characteristics of the route. The benefits are not large, but appear to be great enough to merit incorporation in automated train control.

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# Table 1

## Summary of Ideal Route

<b>Section</b>	<i>Stations</i>	<i>Ons/station (pax/sec)</i>	<i>Offs/station (pax/sec)</i>	<i>End Volume (pax/sec)</i>
Boarding	8	0.500	0.000	4.000
Inbound Shoulder	4	1.000	1.000	4.000
Downtown	3	1.000	2.000	1.000
Outbound Shoulder	3	0.500	0.500	1.000
Alighting	8	0.000	0.125	0.000
<b>Total</b>	26	12.500	12.500	

Headway (sec)	300
Safety headway (sec)	90
Dmax (sec)	300
Detection time (sec)	60

**Table 2**  
**Travel Time Savings (pax-hr)**  
**With Initial Delay at Station 8**

Initial Delay (sec)	Headway (sec)			
	180	300	480	600
180	0.53	3.79	1.23	1.16
300	0.27	0.82	16.19	20.98
420	2.19	14.10	34.39	66.08
540	8.25	12.32	28.89	105.30
660	14.03	32.39	53.42	90.39
780	22.94	44.58	107.95	109.49
900	34.76	68.01	84.48	173.44

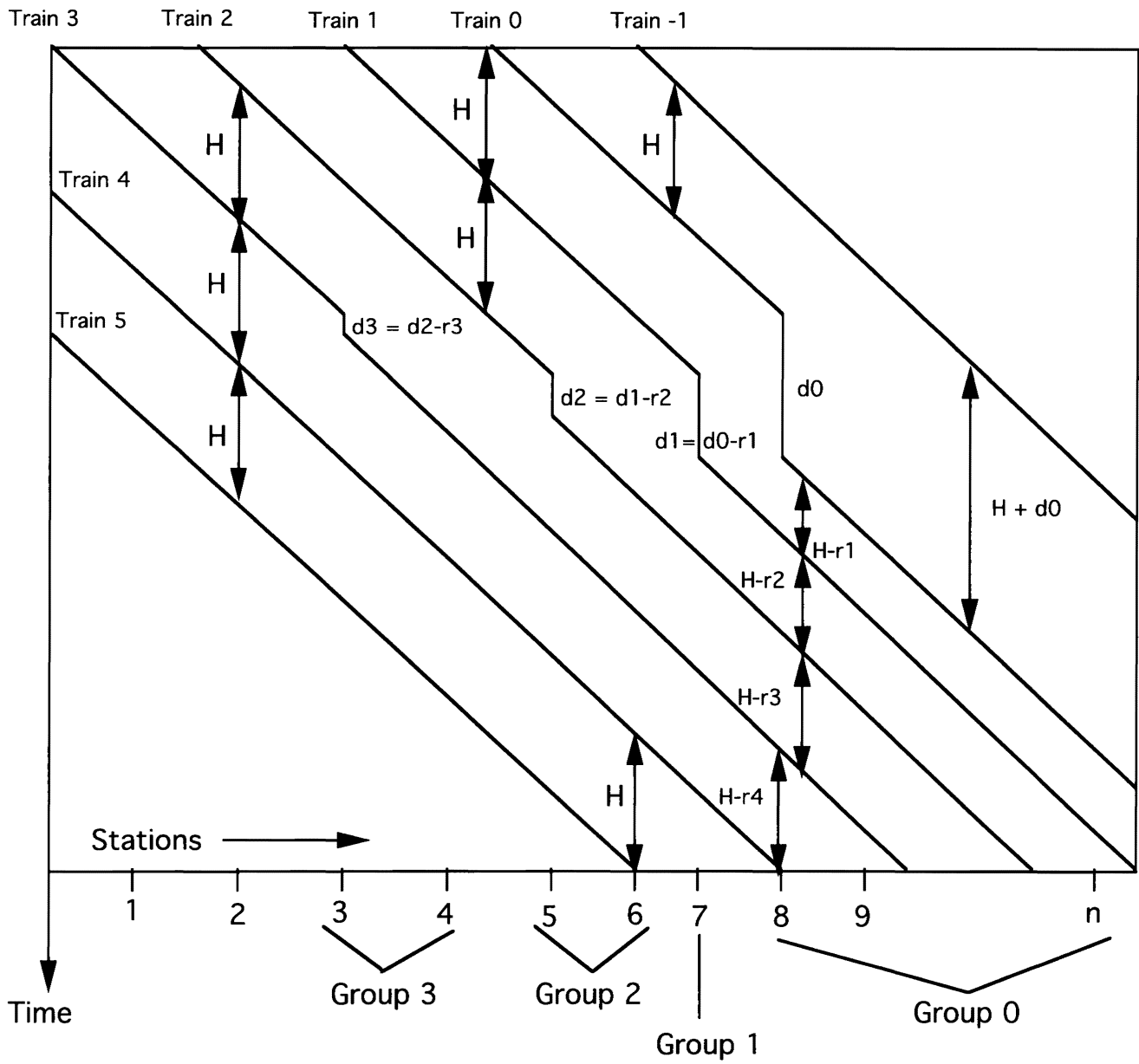


**Table 3**  
**Travel Time Savings (pax-hr),**  
**Heavy Demand at Station 1**

Initial Delay (sec)	Headway (sec)			
	180	300	480	600
180	0.32	2.79	0.26	0.01
300	0.02	0.04	11.36	10.84
420	0.52	9.47	22.56	47.93
540	3.84	4.02	9.23	81.52
660	7.85	19.54	29.52	67.78
780	16.34	33.11	76.59	83.46
900	28.24	46.65	59.79	128.58

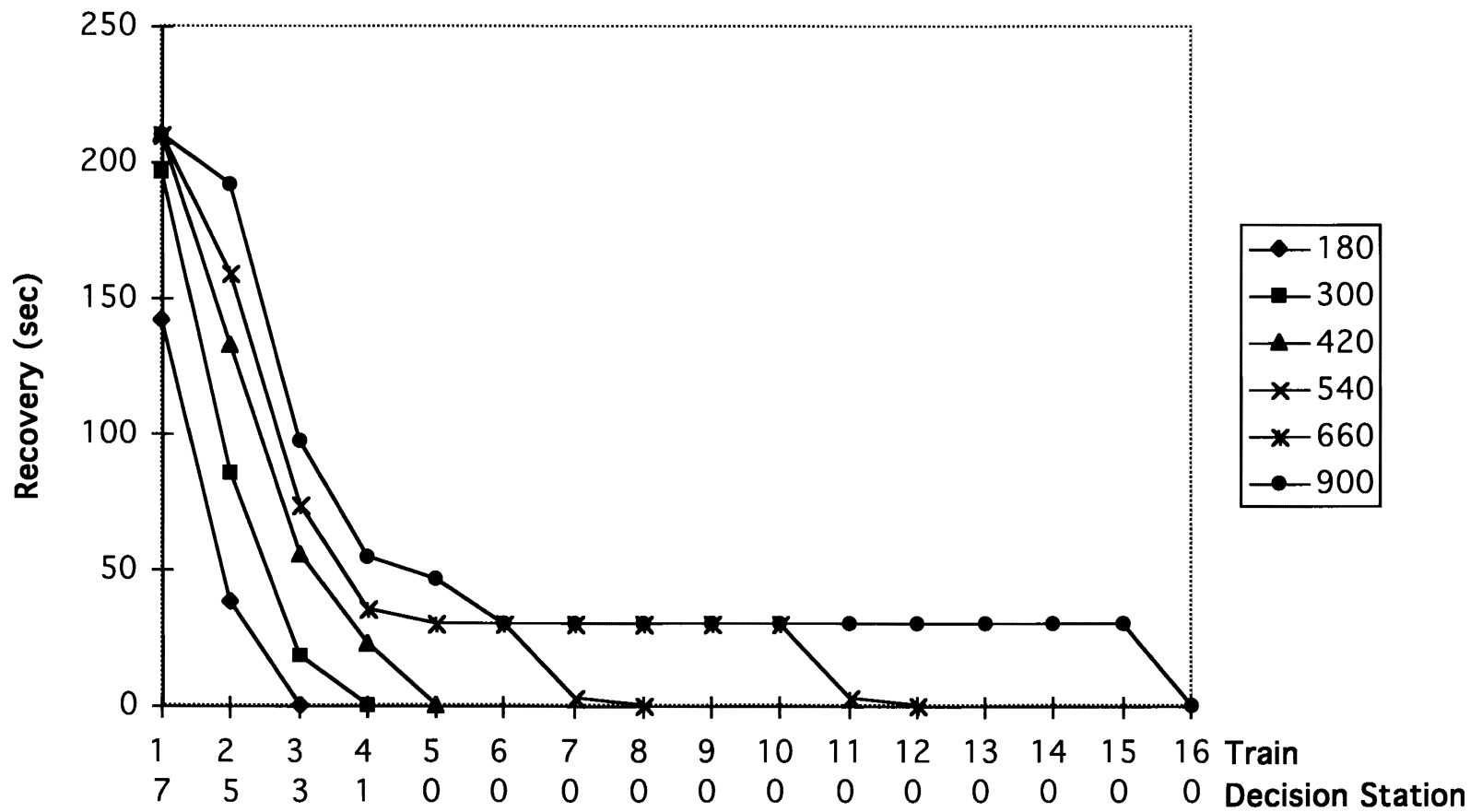


Figure 1. Schedule Recovery Variables for  $N=4$

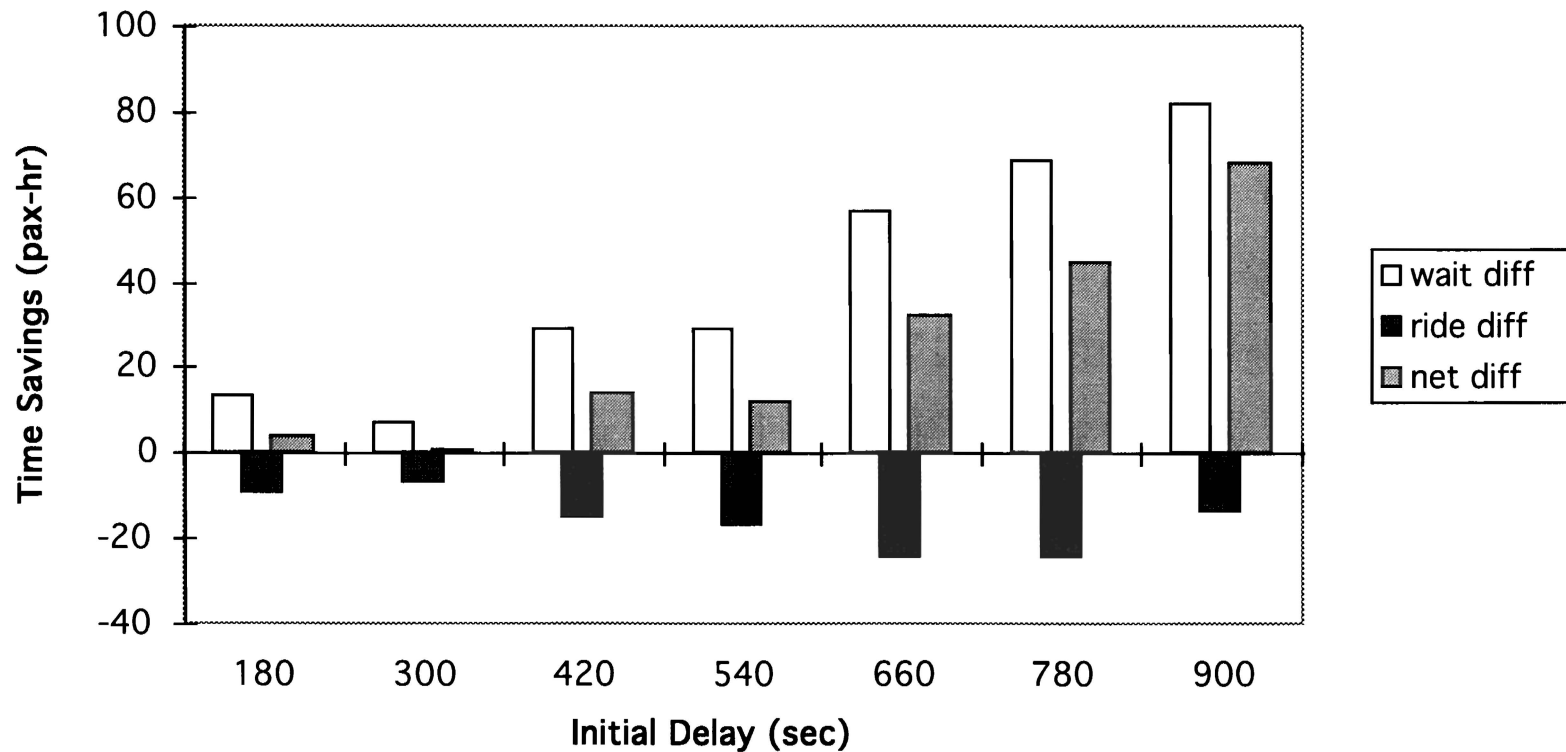


## Figure 2

### Recovery Distribution With Initial Delay at Station 8



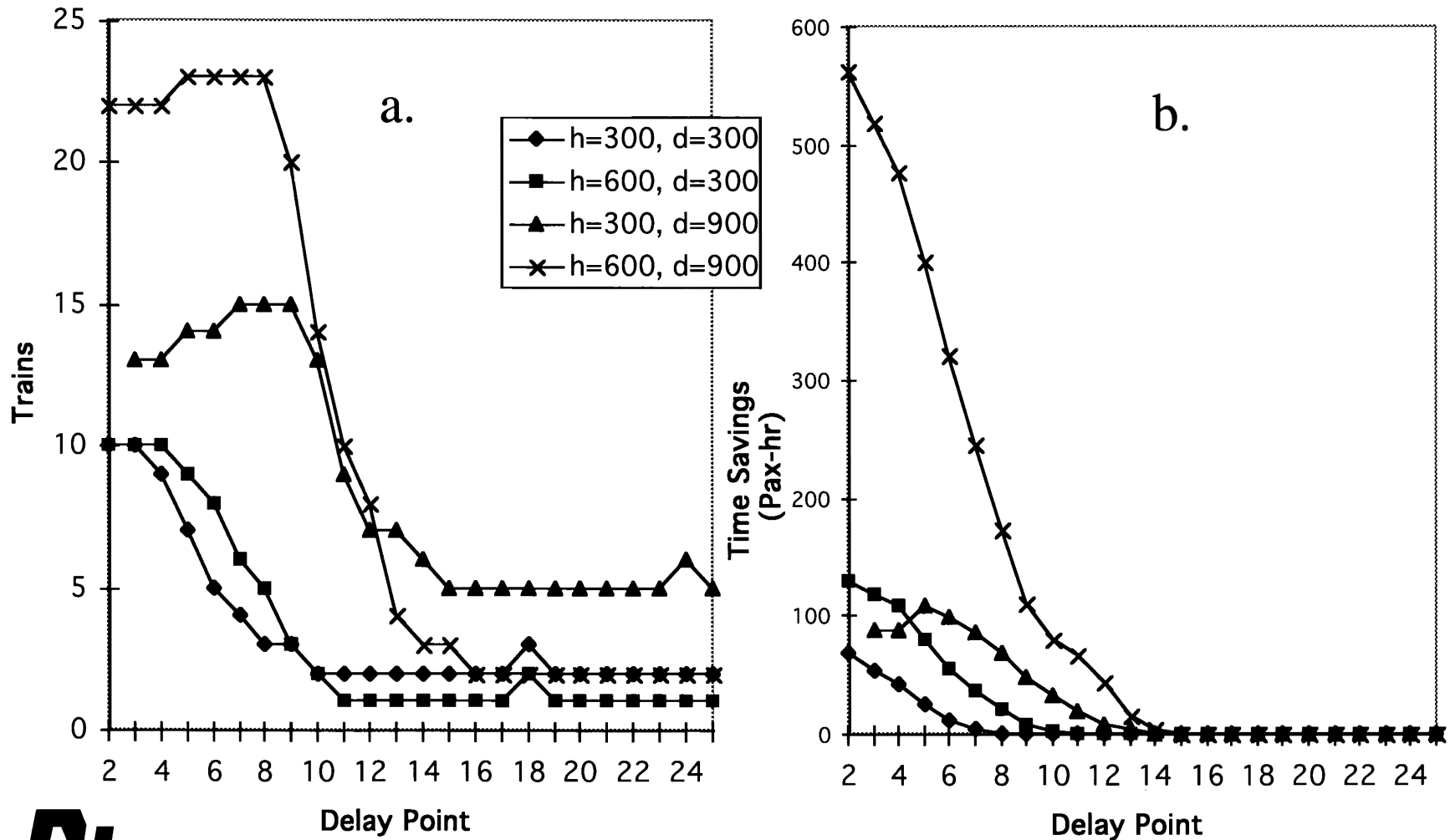
**Figure 3**  
**Travel Time Savings vs. Initial Delay**  
**for Initial Delays at Station 8**



# Figure 4

a. Number of trains used to recover vs. point of initial delay

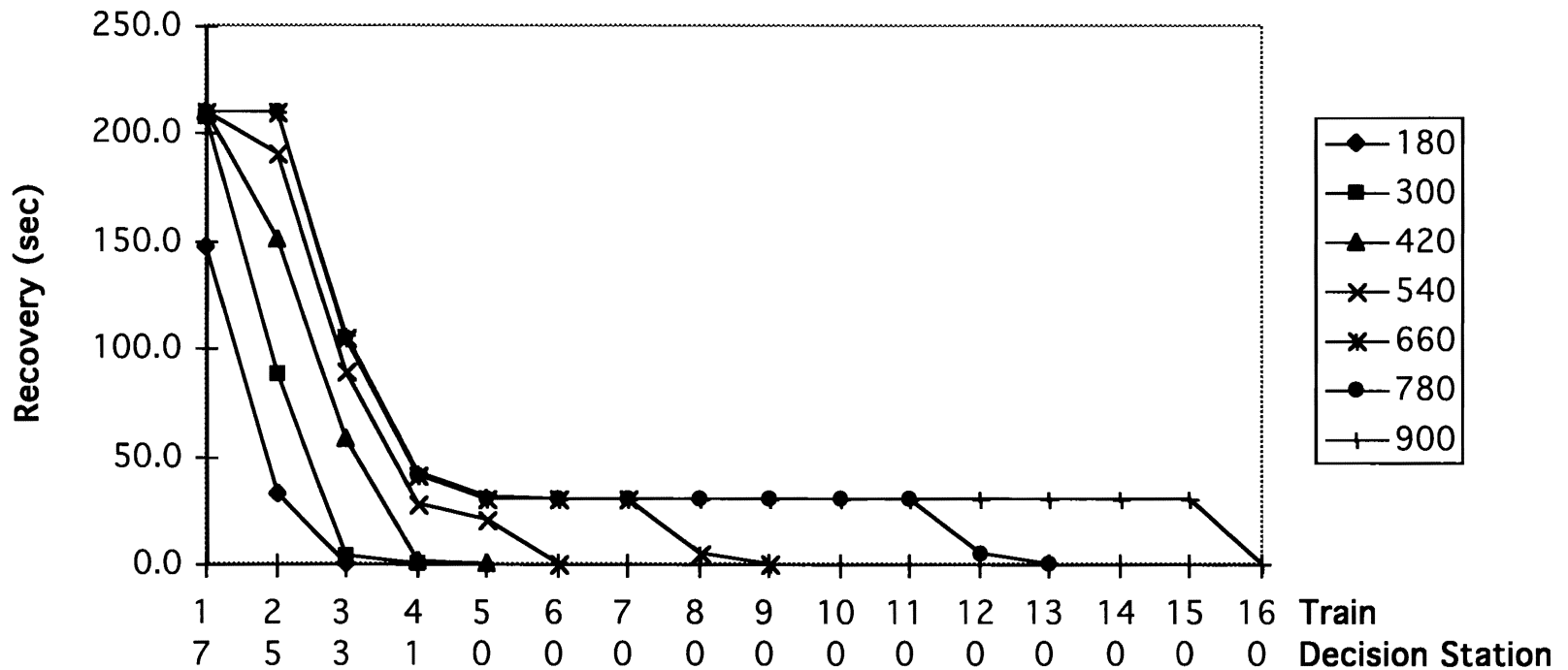
b. Travel time savings vs. point of initial delay





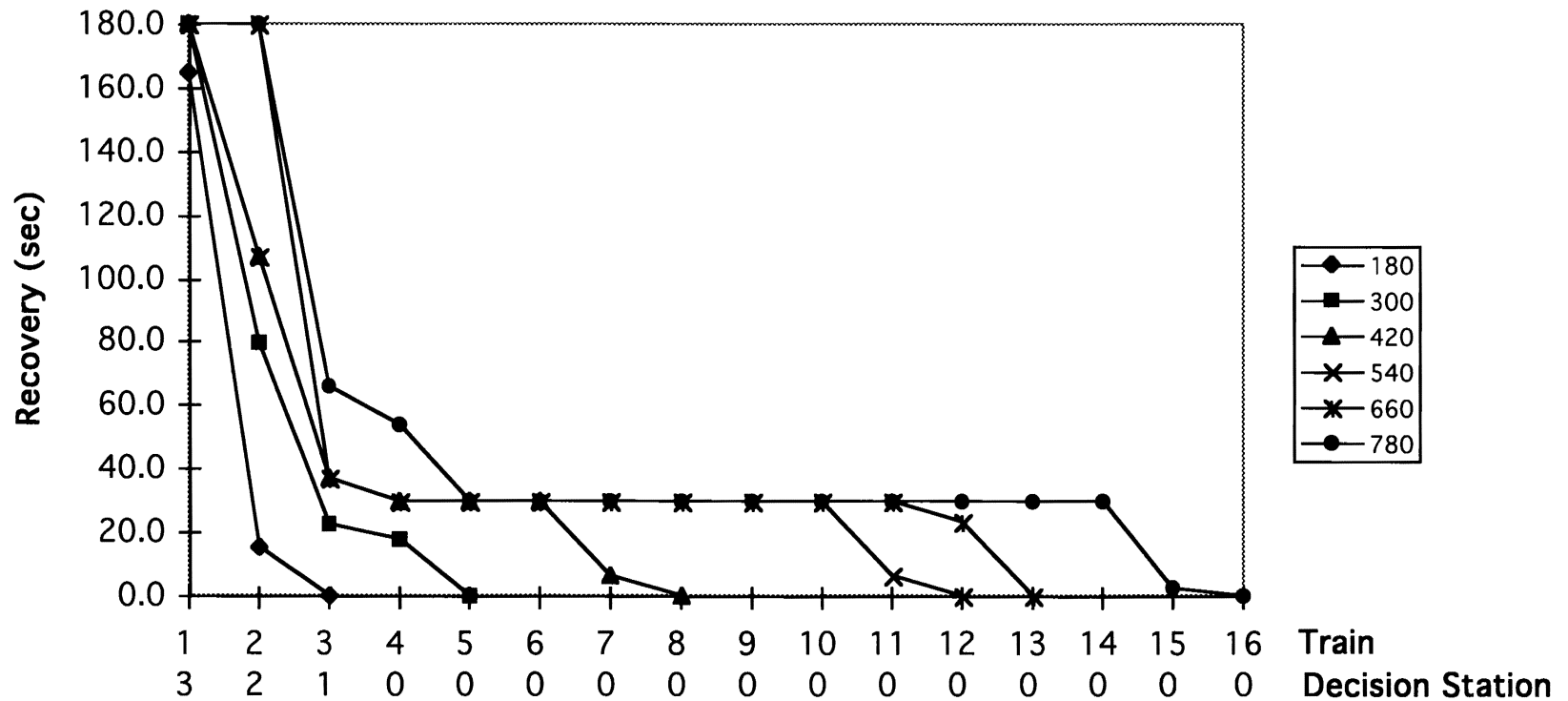
# Figure 5

## Recovery Distribution, Heavy Demand at Station 1



### Figure 6

## Recovery Distribution for Orange Line; Initial Delay at Station 4



# Figure 7

- a. Number of trains used to recover, Orange Line
- b. Travel time savings, Orange Line

