A measured numerical value is known only to a specific number of decimal digits
   – According to how well the measurement has been made

We measure a quantity and determine that its value is between 1.87 and 1.85
   – We could report as 1.86, (i.e. 6 is uncertain)
   – The rightmost (third) digit is uncertain by one unit.

Calculations based on imprecise numbers are imprecise

\[
\begin{align*}
0.09 \times 4.26 &= 0.3834 \\
0.3 \times 4.26 &= 1.278 \\
4.26 \times 8.39 &= 35.7414
\end{align*}
\]

This presentation explains rules for different mathematical operations
SIGNIFICANT DIGITS

- The significant digits of a number are those that can be used with confidence.
  - Significant digits correspond to the number of certain digits plus one estimated digit
  - One and only one imprecise digit can be considered as significant
- The zero (0) is significant ONLY when it is NOT USED to set the decimal place

<table>
<thead>
<tr>
<th>Numerical Value</th>
<th>Significant Figures</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2</td>
<td>two</td>
</tr>
<tr>
<td>0.0012</td>
<td>two</td>
</tr>
<tr>
<td>0.001200</td>
<td>four</td>
</tr>
<tr>
<td>10.0</td>
<td>two</td>
</tr>
<tr>
<td>10</td>
<td>one</td>
</tr>
<tr>
<td>$1.31 \times 10^{21}$</td>
<td>three</td>
</tr>
<tr>
<td>1000</td>
<td>one</td>
</tr>
<tr>
<td>12300</td>
<td>three</td>
</tr>
</tbody>
</table>

The numbers
18, 18.00, and 18.000

First is recorded at **two**
Second recorded at **four**
Third recorded at **five**
SIGNIFICANT FIGURES - MULTIPLICATION AND DIVISION

- When multiplying or dividing two approximately known numbers,
  - The number of significant figures in the result = Smaller of the number of significant figures in the factors

<table>
<thead>
<tr>
<th>Calculation</th>
<th>Significant Digits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.23 x 0.9 = 1</td>
<td>One</td>
</tr>
<tr>
<td>1.23 x 0.9612 = 1.18</td>
<td>Three</td>
</tr>
<tr>
<td>1.23 x 0.9612 / 0.3 = 4</td>
<td>One</td>
</tr>
</tbody>
</table>

No of significant digits in results = min(no of sig. digits in all operands)
SIGNIFICANT FIGURES - ADDITION AND SUBTRACTION

- When adding or subtracting two approximately known numbers,
  - The number of significant figures in result decided by precision after decimal in each factor
  - The number of digits after the decimal points in result = Smallest number of digits to the right of the decimal point in all operands

<table>
<thead>
<tr>
<th>Calculation</th>
<th>Significant Digits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.23 + 0.9 = 2.1</td>
<td>Two</td>
</tr>
<tr>
<td>1.2 + 0.9811 = 2.1</td>
<td>Two</td>
</tr>
<tr>
<td>10 + 0.9811 = 10</td>
<td>One</td>
</tr>
</tbody>
</table>

Result is only supposed to have 2,1,1 digits after decimal
- Significant figures involving logarithms are subtle,
  - A logarithm tells us both the power of the base (usually 10 or \(e\)) and the number itself

\[
x = 1.234 \times 10^5
\]

\[
\log x = \log 1.234 + \log 10^5 = 0.0913 + 5.
\]

- Significant Digits in logarithms
  - As many digits to the right of the decimal point as there are significant figures in the original number
  - Infinite number of significant figures in the second term, since the exponent is exactly known.

\[
\log x = 0.0913 + 5 = 5.0913
\]

- Antilogs – Similar rules: Digits to the right of the decimal point of the original logarithm = Total number of significant digits in original value

\[
x = 10^{28.0913} = 1.234 \times 10^{28}.
\]
Expressing the result of a numerical calculation to the appropriate number of significant figures,

We round the result as necessary as follows.

- If a digit to be dropped is 0, 1, 2, 3, or 4, leave the next remaining digit unchanged,
- If a digit to be dropped is 5, 6, 7, 8, or 9, increase the next remaining digit by one.

<table>
<thead>
<tr>
<th>Calculation</th>
<th>No of Significant Digits</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9811 = 0.981</td>
<td>Three</td>
</tr>
<tr>
<td>1.745 = 1.75</td>
<td>Three</td>
</tr>
<tr>
<td>14.29 = 14.3</td>
<td>Three</td>
</tr>
<tr>
<td>21.74 = 21.7</td>
<td>Three</td>
</tr>
</tbody>
</table>
CONCLUSION

- This exercise was confusing and time wasting
- Confusion can be avoided by using scientific notation, for example

\[
\begin{align*}
2.41 \times 10^3 &= \text{it has three significant digits} \\
2.410 \times 10^4 &= \text{it has four significant digits} \\
2.4100 \times 10^4 &= \text{it has five significant digits}
\end{align*}
\]

- Providing only as many digits as per your input values is misleading

\[
\begin{align*}
12.0000 + 0.9811 &= 12.9811 \\
12 + 0.9811 &= 12.9811
\end{align*}
\]

- Contrary to everything mentioned, when performing a calculation, do not follow these guidelines for intermediate results - keep as many digits as is practical to avoid rounding errors
REFERENCES

- http://quantum.bu.edu/notes/GeneralChemistry/a-1-SignificantFigures.pdf
- Dr. Ibrahim A. Assakkaf, Spring 2001. ENCE 203 - Computation Methods in Civil Engineering II
- http://www.chem.sc.edu/faculty/morgan/resources/sigfigs/index.html

- If you really wish to be confused
  - http://antoine.frostburg.edu/chem/senese/101/measurement/faq/difference-sigfig.shtml
THANK YOU,
QUESTIONS ? COMMENTS ?

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