Identifying Stability vs. Instability of Inventory Dynamics of Supply Chains in Presence of Delays

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Background

•Simon, 1952 (Nobel prize winner, 1978)

He was beginner researcher to use Laplace transform technique to analyze the supply chain stability.

•Forrester, 1961 (from MIT)

Forrester also derived first order differential equation for the same reason.

•Towill, 1982

Additionally, like Simon, Towill used to Laplace transform to study inventory and order based production control system.

He realized that inventory order based production control system has three crucial parameters that may cause instability: **DELAY, ALPHA AND BETA** (two control parameters of supply chain).

•Riddalls and Bennett, 2002; Lewis et al., 1995; Chandra, 2006 considered one delay.

•Many earlier work do numerical simulations, [Riddalls et al., 2000; Jayendran, 2007].

•There are few analytical approaches [Warburton, 2004]

Objectives

•Develop novel method for identification of supply chain stability map,

•Consider all delays & all system parameters,

•Avoid simulations,

•Provide robustness against delay uncertainties.

Inventory Models

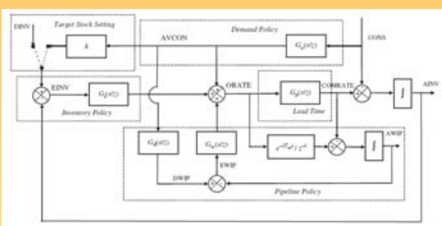


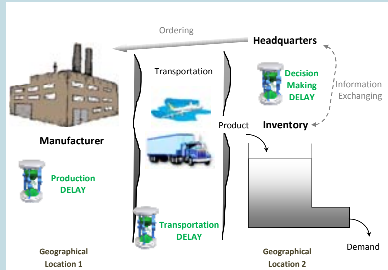
Figure - Model with AINV actual inventory holding, AVCON average consumption, AWIP actual WIP holding, COMRATE completion rate, CONS consumption or market demand, DINV desired inventory level, DWIP desired work in progress, EINV error in inventory holding, EWIP error in work in progress, ORATE order rate

Figure -Replenishment rule

Table –Several Inventory models

Model		Target stock setting	Demand policy	Inventory policy	Pipeline policy
IBPCS	Inventory based production control system	Constant	$G_a(s)=0$ $G_a(z)=0$	$G_i(s)=\frac{1}{T_i}$ $G_i(z)=\frac{1}{T_i}$	$G_w(s)=0$ $G_w(z)=0$
IOBPCS	Inventory and order based production control system	Constant	$G_a(s)=\frac{1}{T_a s+1}$ $G_a(z)=\frac{\alpha}{1-(1-\alpha)z^{-1}}$	$G_i(s)=\frac{1}{T_i}$ $G_i(z)=\frac{1}{T_i}$	$G_w(s)=0$ $G_w(z)=0$
VIOBPCS	Variable inventory and order based production control system	Multiple of average market demand	$G_a(s)=\frac{1}{T_a s+1}$ $G_a(z)=\frac{\alpha}{1-(1-\alpha)z^{-1}}$	$G_i(s)=\frac{1}{T_i}$ $G_i(z)=\frac{1}{T_i}$	$G_w(s)=0$ $G_w(z)=0$
APIOBPCS	Automatic pipeline, inventory and order based production control system	Constant	$G_a(s)=\frac{1}{T_a s+1}$ $G_a(z)=\frac{\alpha}{1-(1-\alpha)z^{-1}}$	$G_i(s)=\frac{1}{T_i}$ $G_i(z)=\frac{1}{T_i}$	$G_w(s)=\frac{1}{T_w}, G_d(s)=T_p$ $G_w(z)=\frac{1}{T_w}, G_d(z)=T_p$
APVIOBPCS	Automatic pipeline, variable inventory and order based production control system	Multiple of average market demand	$G_a(s)=\frac{1}{T_a s+1}$ $G_a(z)=\frac{\alpha}{1-(1-\alpha)z^{-1}}$	$G_i(s)=\frac{1}{T_i}$ $G_i(z)=\frac{1}{T_i}$	$G_w(s)=\frac{1}{T_w}, G_d(s)=T_p$ $G_w(z)=\frac{1}{T_w}, G_d(z)=T_p$

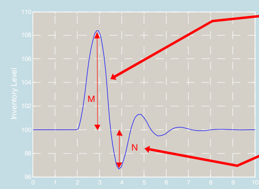
Automatic Pipeline, Inventory and Order based Production Control System



•How do we understand effects of delays?

• In systems thinking, supply chains can be conceived as a dynamic network where information and products flow between nodes to succeed multiple objectives of maintaining steady inventories around desired levels, responding consumer demand and minimizing costs while combating against uncertainties.

With Delays



Large peaks in the inventory response increases storage and handling costs

Inventory depletion increases reordering raw material which causes high cost of freight.

Figure - Inventory response

System level representation

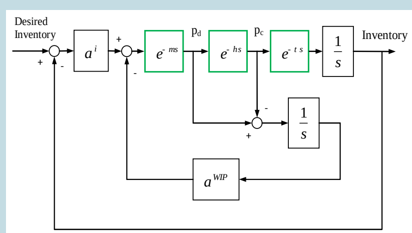


Figure - Block diagram of our model

Stability Analysis & Stability Maps

• Stability analysis is needed over mathematical models representing inventory dynamics. These models are cumbersome to study as they are in the form of infinite dimensional differential equations. In our work, we achieve this study via a non-trivially developed mathematical paradigm that enables the stability analysis of inventory levels.

$$CE=\frac{1}{\alpha^i}s+\beta e^{-\tau_1 s}-\beta e^{-\tau_2 s}+e^{-\tau_3 s}$$

where $\tau_1=\mu, \tau_2=\mu+h, \tau_3=\mu+h+\tau$

when $(\tau_1,\tau_2,\tau_3)=(0,0,0) \Rightarrow s=-\frac{1}{\alpha^i}$

Table - Anticipated computation times with respect to number of sweep parameters.

η, number of independent parameters	Number of grid points to sweep	Time needed to sweep
1	10^3	10^{-2} seconds
2	$(10^3)^2$	10 seconds
3	$(10^3)^3$	10^4 sec = 2.7 hours
5	$(10^3)^5$	10^{10} sec = 317 years

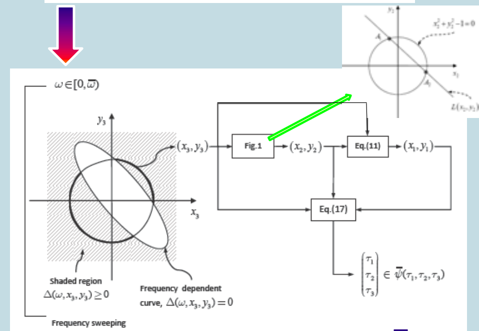


Figure – Flow chart of the proposed procedure.

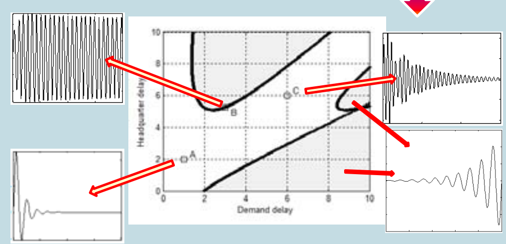


Figure – Typical stability map and inventory responses

Novel Procedure for Extracting Stability Switching Hypersurfaces

• **Step a.** For dummy variable $j=1,2,3$, define the real and imaginary parts of $P_j(s)$ in CE as:

$$P_{jR}(\omega)=\Re(P_j(i\omega)),\ P_{jI}(\omega)=\Im(P_j(i\omega)).$$

• **Step b.** For dummy variable $j=0,4,\ldots,\ell$, all the terms in the summation correspond to a numerically known complex number for a given sweep parameter ω.

$$\phi(i\omega)=\chi(\omega)+i\gamma(\omega)=P_0(i\omega)+\sum_{j=4}^{\ell}P_j(i\omega)e^{-i\tau_j\omega},$$

where the pair $(\chi(\omega),\gamma(\omega))\in\mathbb{R}^2$ is only a function of the sweep parameter ω.

• **Step c.** Define

$$e^{-i\tau_j\omega}=x_j+iy_j,\quad j=1,2,3.$$

where $(x_j,y_j)\in\mathbb{R}^2$. Since τ_j are unknown, so are the scalars x_j and y_j. Note also that the exponential terms define a unit circle in $\mathbb{C}$. In other words, the following holds

$$\left|e^{-i\tau_j\omega}\right|=1 \rightarrow x_j^2+y_j^2-1=0,\quad j=1,2,3.$$

Following steps **a, b** and **c**, real and imaginary parts of CE can be expressed as

$$\sum_{j=1}^{3}\left[A_j\begin{pmatrix}x_j\\y_j\end{pmatrix}\right]+\begin{pmatrix}\chi(\omega)\\\gamma(\omega)\end{pmatrix}=(0,0)^T,$$

where T represents the transpose, $A_j=\begin{pmatrix}P_{jR}&-P_{jI}\\P_{jI}&P_{jR}\end{pmatrix}$ and

$$\det(A_j)=P_{jR}^2+P_{jI}^2=|P_j(i\omega)|^2\neq0\quad\text{since } P_j(s)\neq0,\ j=1,2,3.$$

• **Step d.** The problem at hand reduces down to simultaneously solving (x_j,y_j) pairs from the above two equations. Since A_1 is invertible, we have

$$\begin{pmatrix}x_1\\y_1\end{pmatrix}=-A_1^{-1}\left[\sum_{j=2}^{3}\left[A_j\begin{pmatrix}x_j\\y_j\end{pmatrix}\right]+\begin{pmatrix}\chi(\omega)\\\gamma(\omega)\end{pmatrix}\right].$$

Back substituting (x_1,y_1) solution from above into circle equation for the counter $j=1$ yields

$$x_2\Gamma_1(\omega,x_3,y_3)+y_2\Gamma_2(\omega,x_3,y_3)+\Gamma_0(\omega,x_3,y_3)=0$$

• **Step e.** Notice that the coefficients of x_2 and y_2 in this equation are in terms of x_3 and y_3. For admissible solutions, it is necessary and sufficient that (x_3,y_3) pairs lie on the unit circle.

• **Step f.** For each (x_3,y_3) pair chosen on the unit circle, one can solve for (x_2,y_2) as follows.

$$x_2=\frac{-\Gamma_0\Gamma_1\mp\Gamma_2\sqrt{\Delta(\omega,x_3,y_3)}}{\Gamma_1^2+\Gamma_2^2},\quad \Delta(\omega,x_3,y_3)=\Gamma_1^2+\Gamma_2^2-\Gamma_0^2.$$

• **Step g.** Calculate y_2 from circle equation.

• **Step h.** Obtain (x_1,y_1) pairs.

• **Step i.** The corresponding (τ_1,τ_2,τ_3) triplets can be calculated as

$$\tau_j=\frac{-1}{\omega}\angle(x_j+iy_j),\quad j=1,2,3.$$

Preliminary Results

The contribution of the paper originates from a nontrivially developed mathematical approach which reveals a tableau on which stable and unstable operation of supply chain with respect to delays and system parameters are identified. This unique tableau is a valuable tool to aid managerial decisions and to train supply chain managers. Although, output of supply chain is always bounded due to some constraints in the network, managers can choose a proper combination of parameters from this tableau that leads to inventory dynamics less prone to oscillations.

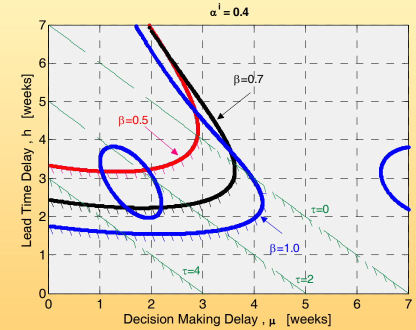


Figure – Stability map for $\tau_3 = 7$ weeks

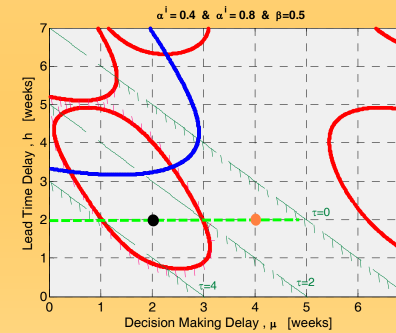


Figure – Stability map for $\tau_3 = 7$ weeks

Conclusion and Future works

1. We develop novel procedures for identifying stability characteristics of supply chain dynamics in presence of delays,

2. End results are stability maps on which robustness against delays are revealed.

3. Stability maps are seen as decision-making facilitators for managers as well as for training purposes.

4. In this regard, we see that managing supply chain is a combination of Operations Research, Systems Engineering and Mathematics.

1. Implementation of the ideas developed on larger supply chain networks and investigation of bullwhip effects.

2. Cost optimization and investigation of product price dynamics.

3. Potential applications in small businesses and start-up companies (with Prof. T.J. Marion).

4. Collaborations with School of Tech. Entr. (Prof. T. J. Marion) and Business School at NU.