

Stability analysis of a LTI-TDS dynamics with four independent delays

Dynamics with the following 2 by 2 state space representation is studied for stability:

$$\dot{x}(t) = A x(t) + B_1 x(t-\tau_1) + B_2 x(t-\tau_2) + B_3 x(t-\tau_3) + B_4 x(t-\tau_4)$$

where

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -4 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -6 & 0 \\ 1 & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 & 4 \\ 0 & -2 \end{bmatrix},$$

$$B_3 = \begin{bmatrix} 3 & -4 \\ -2 & 1 \end{bmatrix}, \quad B_4 = \begin{bmatrix} -9.45 & -9.1 \\ 5.5 & 2.55 \end{bmatrix}$$

When $\tau_3 = \tau_4 = 0$, the problem reduces to the one solved in [Sipahi, Olgac, 2006, Systems and Control Letters] (see the stability picture with boundaries in black) with the technique CTCR [Sipahi, Olgac, 2005 Automatica].

When τ_3 and τ_4 are *non-zero*, a 4-D stability picture has an interesting formation. With non-trivial extensions of CTCR, corresponding stability picture can still be extracted. For ease of display and for comparison with $\tau_3 = \tau_4 = 0$ case, τ_3 and τ_4 are chosen as 10 msec, and the boundaries of the stability picture are depicted by blue curves.

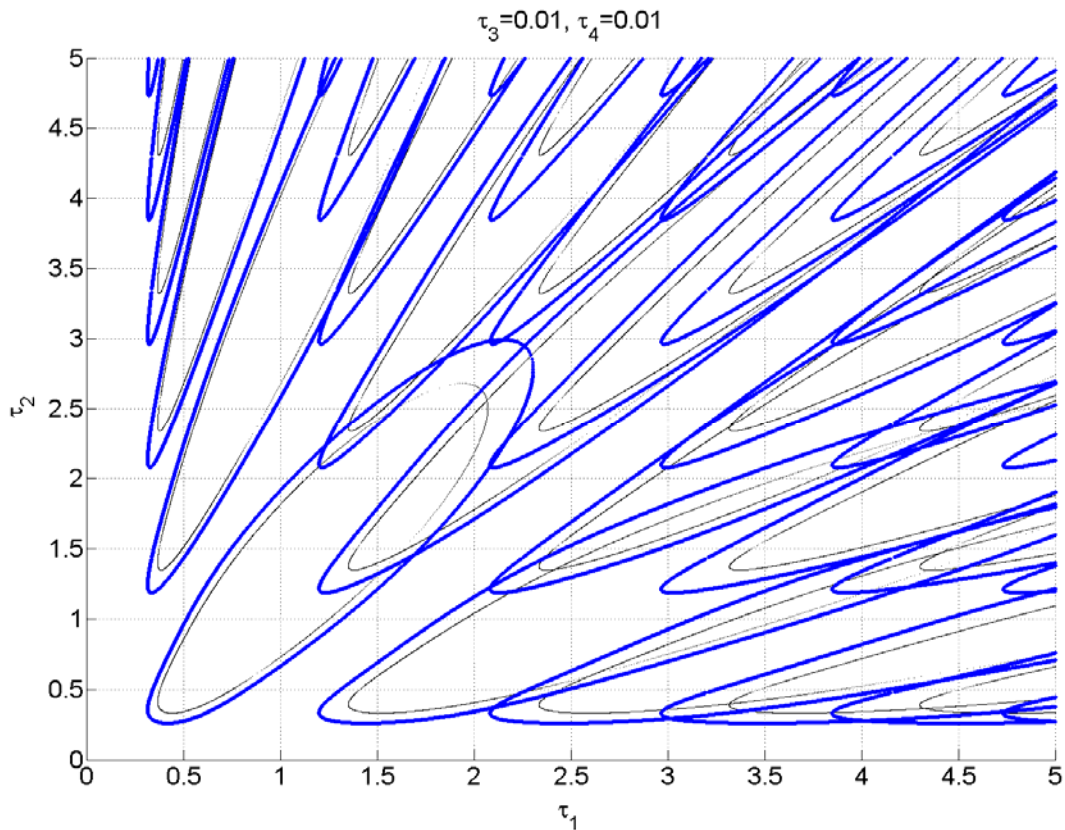


Figure obtained by non-trivial extensions of CTCR

For cross-validation, two different techniques (one based on detection of imaginary axis crossings by [Elias Jarlebring], and the other on the distribution of right most roots by [Dimitri Breda]) are presented below.

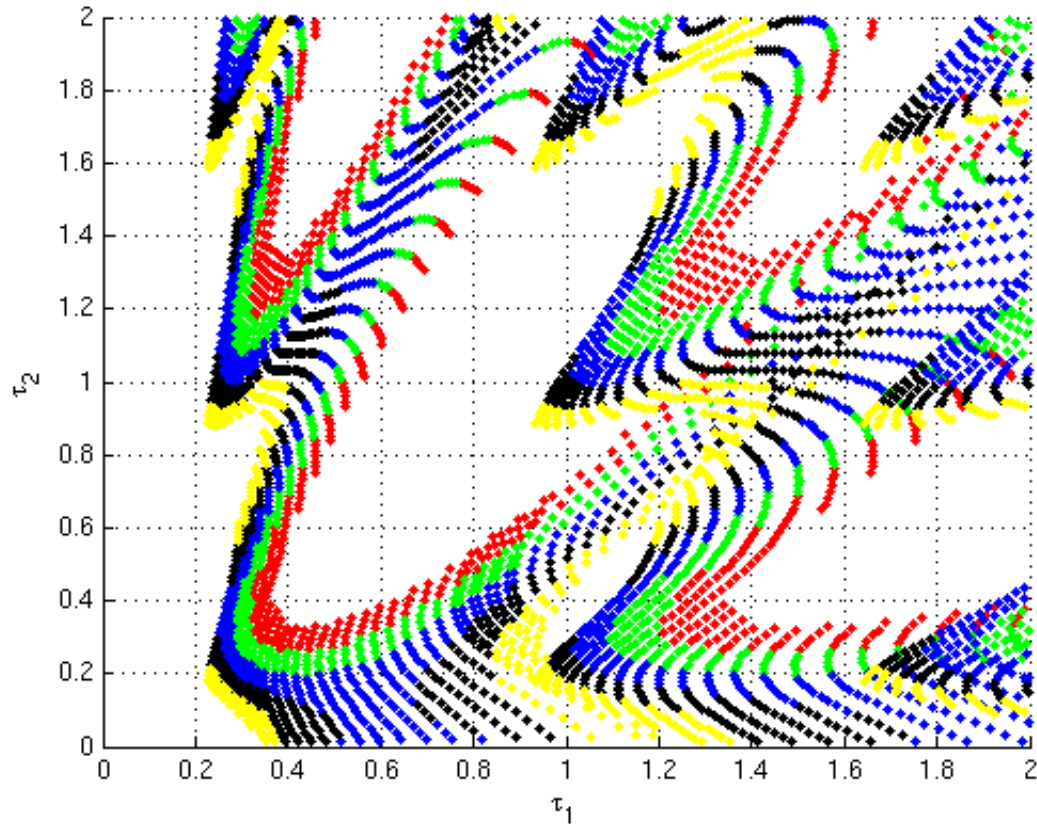


Figure obtained by the technique in [Jarlebring, Computing the Stability Region in Delay-space of a TDS using Polynomial Eigenproblems, 6th IFAC Workshop on Time-Delay Systems, 2006] (tau3=tau4 swept between 0 to 0.01)

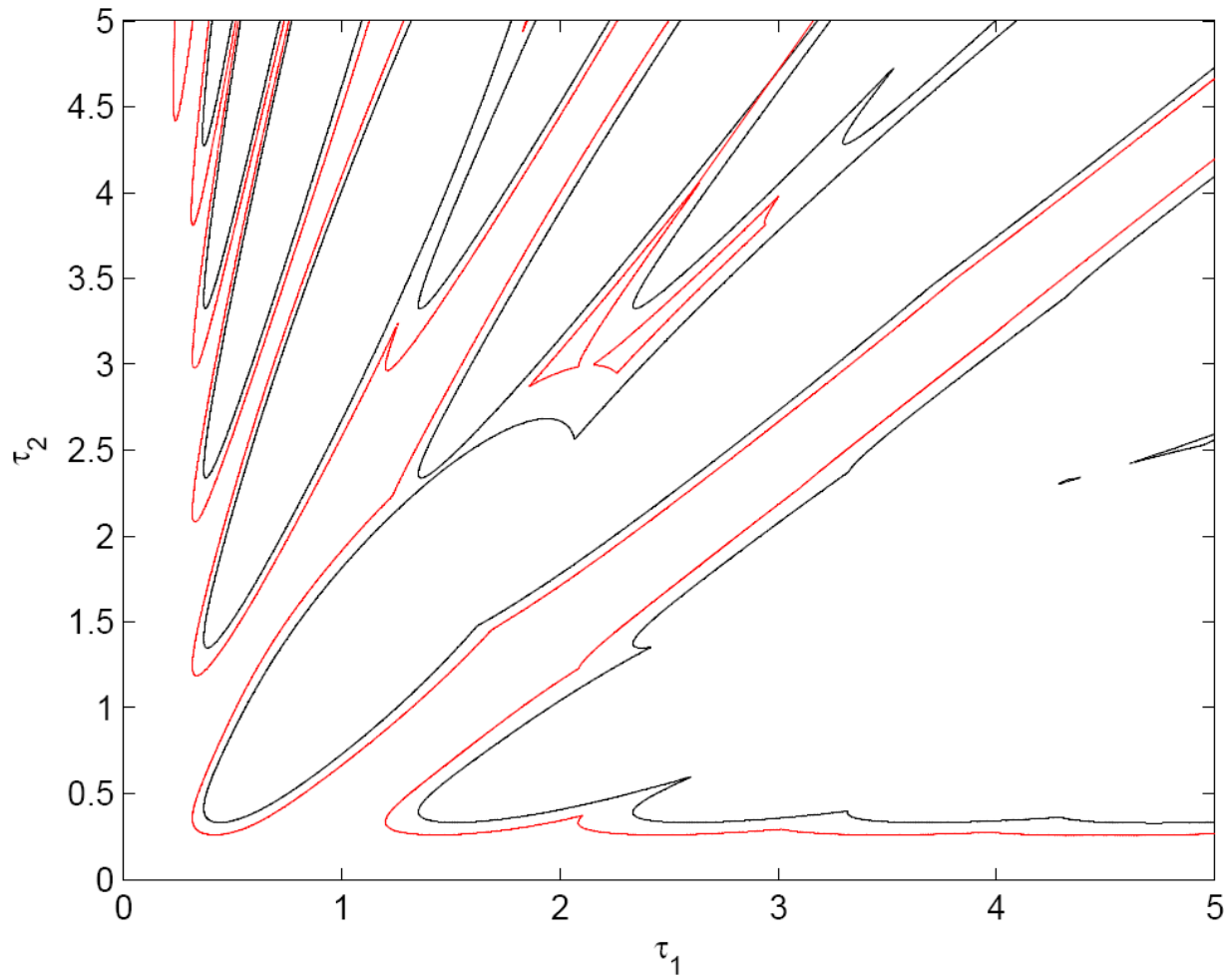


Figure obtained by the technique in [Breda, D., Maset, S., and Vermiglio, R., "Pseudospectral differencing methods for characteristic roots of delay differential equations", SIAM J. Sci. Comput., vol. 27(2), pp. 482-495, 2005.] (black curves $\rightarrow \tau_3 = \tau_4 = 0$, red curves $\tau_3 = \tau_4 = 0.01$)