# Probabilistic Models for Seismic Design and Assessment of RC Structural Walls

Mehrdad Sasani Northeastern University Boston, Massachusetts

### Abstract

Probabilistic models for estimating lateral flexural displacement capacity, shear strength capacity, and shear deformation of reinforced concrete structural walls are presented. In developing all the models, available experimental data are utilized and the Bayesian parameter estimating technique is used. The model for estimating the shear deformation of structural walls is constructed based on the coupling between the flexural and shear inelastic deformations. Comparisons with some current models in seismic codes are made and significant improvements are shown.

**Keywords:** Displacement Capacity, Shear Strength Capacity, Shear Deformation, Structural Walls

## 1 Introduction

In probabilistic design and assessment of a structure, accounting for sources of uncertainties, the probability of demands being more than corresponding capacities (i.e. probability of failure) is estimated. In order to have meaningful estimation of the reliability of the structure under external actions (loads), one needs to not only account for uncertainties associated with the external actions and materials, but also consider other sources of uncertainties affecting estimations of demands and capacities. Furthermore, there is a need for the selection of proper measures of demands and capacities. In seismic engineering, these measures in addition to strength, could include displacement, energy, or more generally damage. In this paper strength and displacement capacities and demands are estimated and compared. To estimate structural capacities, different models at section, element, and structure levels are required whose uncertainties have to be accounted for. Furthermore, these models would estimate capacities more realistically, if they have been calibrated using available experimental data. To estimate structural demands proper models that incorporate both flexural and shear strengths and deformations are needed. In this paper different probabilistic models for estimations of seismic capacities of reinforced concrete (RC) structural walls are presented.

### 2 Flexural Displacement Capacity

Flexural capacity of a structural wall is limited by its strength as well as its displacement capacities. Mechanical models for estimating the flexural strength capacity of the structural wall are well developed and they usually predict the strength within a small error. Some models for estimating the displacement capacity that are available and even implemented in codes (UBC, 1997), however, may not

reasonably estimate the capacity of the wall (Sasani, 1998). The flexural displacement capacity of the wall is limited by the maximum acceptable concrete and steel strains. Displacement capacity of structural walls has been studied by Sasani and Der Kiureghian (2001) and below a summary of the study is presented. Figure 1 shows the elastic and inelastic deformations of a structural wall. The maximum displacement capacity at the top of a structural wall that has developed a plastic hinge near its base is approximately given by the expression



Fig.1. Deformation of structural wall

$$\hat{\Delta}_{f} = \alpha \Phi_{v} H^{2} + (\Phi_{u} - \Phi_{v}) L_{P} (H - L_{P} / 2)$$

(1)

where *H* is the height of the wall,  $L_p$  is the length of the plastic hinge,  $\Phi_y$  and  $\Phi_u$  respectively are the yield curvature (at the first yielding of the flexural reinforcement) and the ultimate cyclic curvature of the section near the base of the wall, and *a* is a coefficient that depends on the distribution of bending moments and the flexural stiffness along the height of the wall. The superposed hat on  $\hat{\Delta}_f$  is used to signify the fact that the above model is not exact. The first term in (1) represents the contribution of the elastic deformation at the first yielding of the flexural reinforcement, whereas the second term represents the contribution from the localized plastic deformation near the base of the wall. The various terms in the model are further discussed or developed below.

#### 2.1 Coefficient $\alpha$

The coefficient *a* depends on the distributions of the flexural stiffness and lateral load along the height of the wall. If one assumes a uniform flexural stiffness equal to that of the cracked section at the base of the wall, and a linear relation between the section curvature and bending moment, under an inverted triangularly distributed lateral load, Fig. 1, one obtains a = 11/40, whereas for a concentrated lateral load at the top one obtains a = 1/3. In reality, the top portion of the wall may not be cracked. While it is possible to use the uncracked section for the top portion, the effect on the coefficient *a* and, hence, the estimated top displacement is usually insignificant unless the amount of the flexural reinforcement and the axial compressive load of the wall are

small (Sasani 1998). Therefore, in the following analysis, cracked concrete sections are assumed and the inverted triangular load distribution is employed. Previous investigations have shown that this approach provides fairly accurate estimates of the elastic contribution to the top displacement (Sasani and Anderson 1996).

#### 2.2 Yield Curvature

For a given cross section of the wall and for known stress-strain relations of concrete and reinforcing steel, the yield curvature  $\Phi_y$  is easily determined by employing a kinematic assumption for the deformation of the wall, such as the assumption that plane sections remain plane. Usually the cracked section of the concrete is used for this analysis. This type of analysis is routine and needs no further investigation here. Note, however, that the prediction of  $\Phi_y$  is not free from error. The contribution of this error will be accounted for along with the errors in the other terms when the overall error in model (1) is assessed.

#### 2.3 Plastic Hinge Length

A term in (1) that needs to be developed is the plastic hinge length,  $L_p$  (see Fig. 1). According to Corely (1966),  $L_p$  is a distance such that when multiplied by the average plastic curvature (over a distance equal half the section depth) at the base of a cantilever member gives the plastic rotation of the hinge. Based on experimental results, Corley (1966) suggested the empirical expression  $L_P = 0.5d + 1.26 H/\sqrt{d}$ , where *d* is the effective depth of the section and *H* (the wall height) is the distance between the points of zero and maximum moment, in meters. In his discussion of Corley's paper, Mattock (1967) suggested the expression  $L_P = 0.5d + 0.05H$ . In order to develop a probabilistic model for  $L_p$  that is appropriate for RC walls, 29 of the test results reported by Corley (1966) and Mattock (1967) are selected that corresponded to beams with effective depths greater than 0.5m. This data was used in conjunction with the Bayesian method to estimate the parameters of the following: (a detailed description of the application of the method is presented later in this paper, where a shear strength capacity model is developed)

$$\frac{L_P}{d} = \alpha_1 + \alpha_2 \frac{\sqrt{H} l_s^{3/2}}{d^2} + \varepsilon_L$$
(2)

which provides a good fit to the data. In (2),  $l_s$  is a standard length equal to 1 meter (=39.4 inches) that is inserted to make the model parameters dimensionless, and  $\varepsilon_L$  in each equation is a model error term that is assumed to have the normal distribution with zero mean (so that one obtains unbiased models) and unknown standard deviation  $\sigma_L$ . (Strictly speaking,  $L_p$  being non-negative, the normal distribution for  $\varepsilon_L$  is not appropriate. However, the variability in the model is small in relation to its mean and the probability of having negative  $L_p$  is virtually zero.) The above models are linear in terms of the unknown parameters  $\alpha_1$  and  $\alpha_2$ . For such a case, with non-informative priors on  $\alpha_1$ ,  $\alpha_2$  and  $\sigma_L$ , closed form solutions of the posterior statistics are available (Box and

Tiao, 1992). The posterior mean values of the parameters based on the 29 test data are  $\overline{\alpha}_1 = 0.427$ ,  $\overline{\alpha}_2 = 0.077$  and  $\overline{\sigma}_L = 0.149$ . The standard deviations are 0.088, 0.019, and 0.021, respectively. The only considerable correlation coefficient is between  $\alpha_1$  and  $\alpha_2$ , which is equal to -0.3.

### 2.4 Cyclic Curvature Capacity

The cyclic curvature capacity,  $\Phi_u$ , of the cross-section of the RC wall is determined by modifying its monotonic curvature capacity,  $\Phi'_u$ . The monotonic curvature capacity is determined by calculating the moment-curvature relationship for the cross section. The ultimate monotonic curvature of the section is assumed to have been reached when any one of the following three criteria is satisfied: (1) concrete reaches its maximum usable strain, (2) any steel reinforcing bar reaches its fracture strain, (3) the moment strength of the section drops to 80% of the moment capacity. In addition to the geometry of the cross section and the placement and area of reinforcing bars, the monotonic curvature capacity depends on the stress-strain relationships of concrete and steel and on the magnitude of the axial load. For the present study, the modified Kent and Park model (Park et al. 1982) is employed to describe the stressstrain relations for unconfined and confined concrete. In the following section, a model for the maximum usable concrete strain is examined.

#### 2.4.1 Maximum Useable Concrete Strain

The maximum curvature capacity of a RC wall section may be limited by the maximum usable concrete strain,  $\varepsilon_c^{\max}$ , including the effect of confinement by the transverse reinforcement. A good measure for the confining action of the transverse reinforcement is  $f_{yh}\rho_{sh}$ , where  $f_{yh}$  is the yield stress and  $\rho_{sh}$  is the volumetric ratio of the confining steel hoops. Most available measurements of  $\varepsilon_c^{\max}$  are for columns under axial loads with uniform strain distribution, which is not representative of the strain distribution in the compressive zone of structural walls. Kaar et al. (1976) have tested specimens that have been specifically designed to model the compression zones of structural walls. To develop a probabilistic model based on this data, the idealized model

$$\varepsilon_{c}^{\max} = e^{\varepsilon_{c}} \left( \beta_{1} + \beta_{2} \frac{f_{yh} \rho_{sh}}{f_{ys}} \right)$$
(3)

is considered, where  $\beta_1$  and  $\beta_2$  are the model parameters and  $f_{ys} = 413$  MPa (=60 ksi) is the yield stress of grade 60 steel, which is used to make the model parameters dimensionless. Note that parameter  $\beta_1$  is identical to the maximum usable strain of unconfined concrete. The error term,  $\varepsilon_{\epsilon}$ , is assumed to have a normal distribution with zero mean and an unknown standard deviation,  $\sigma_{\epsilon}$ . The unknown parameters of the model are  $\beta_1$ ,  $\beta_2$  and  $\sigma_{\epsilon}$ .

In the Bayesian approach, one can easily incorporate any prior information on the model parameters. In the present case, while there is no information available about  $\beta_2$  and  $\sigma_{\epsilon}$ , prior information on  $\beta_1$ , i.e., the maximum useable strain of unconfined concrete is available. Most investigators would use a value of 0.003 to 0.004 for  $\beta_1$ . To incorporate such information,  $\beta_1$  is assumed to have a lognormal prior distribution with mean 0.0035 and a standard deviation 0.0005. For  $\beta_2$  and  $\sigma_{\epsilon}$ , non-informative priors are used (Box and Tiao 1992), which essentially imply locally uniform distributions for  $\beta_2$  and  $\ln \sigma_{\epsilon}$ . Using the experimental results of Kaar et al. (1976) and the computer program BUMP for Bayesian updating developed by Geyskens et al. (1993), the posterior statistics of the parameters are computed. The posterior mean values of the parameter are  $\overline{\beta_1} = 0.00355$ ,  $\overline{\beta_2} = 0.822$ , and  $\overline{\sigma_{\epsilon}} = 0.198$ . The standard deviations are 0.00039, 0.080, and 0.057, respectively. The only considerable correlation coefficient is found to be equal to -0.3 between  $\beta_1$  and  $\beta_2$ .

#### Effect of Compressive Strain Concentration

The traditional assumption that plane sections remain plane in flexure is not applicable to structural walls with deep sections, particularly within the hinging region. Unfortunately, sufficient data are not available to construct a probabilistic model for this mechanism. Instead, to account for the effect of strain concentration in the compression zone of concrete, the maximum useable concrete strain is modified to obtain

$$\left(\varepsilon_{c}^{\max}\right)_{\mathrm{mod}} = \theta \ \varepsilon_{c}^{\max} \tag{4}$$

where  $\theta$  is a correction parameter having a value less than unity. With this reduced concrete strain capacity, section analysis with a linear strain distribution is carried out to determine the moment-curvature relationship and, thereby, the yield curvature  $\Phi_y$  and the monotonic curvature capacity  $\Phi'_u$ . Since no data is available to directly assess the model in (4),  $\theta$  will be estimated in the course of assessing the global model for the displacement capacity of the wall, as described below.

#### Model for the Cyclic Curvature Capacity

Using the Park and Ang (1985) damage model, it can be shown that the reduction in the curvature capacity of a RC section due to the cyclic nature of the load depends on the curvature ductility of the section,  $\mu_{\Phi} = \Phi'_u / \Phi_y$ . Using that model with parameter values suggested by Fajfar (1992), Sasani (1998) suggested an empirical model for the curvature capacity under cyclic displacement having the form

$$\Phi_{u} = (\gamma_{1} - \gamma_{2} \mu_{\Phi}) \Phi_{u}^{\prime}$$
(5)

where  $\gamma_1$  and  $\gamma_2$  are unknown parameters. Unfortunately, no reliable data is available to directly estimate the parameters of this model. Hence, they will be estimated along with parameter  $\theta$  of the model in (4) in the course of assessing the global model for the displacement capacity of the structural wall, as described below. The reader will

note that the error terms are not included in the above two sub-models. This is because the errors in these sub-models are incorporated in the overall error term of the global model.

#### 2.5 Probabilistic Model for Flexural Displacement Capacity

Motivated by (1) and the sub-models described above, and noting that the displacement capacity must be non-negative, the following global probabilistic model is considered for the displacement capacity of RC structural walls:

$$\ln\left(\frac{\Delta_{f}}{H}\right) = \ln\left[\alpha\Phi_{y}H + \left(\Phi_{u} - \Phi_{y}\right)\overline{L}_{p}\left(1 - \frac{\overline{L}_{p}}{2H}\right)\right] + \mathcal{E}_{\Delta f}$$
(6)

In this model *a*,  $\Phi_y$  and *H* are as described earlier and contain no model parameters;  $\overline{L}_p$  is the mean estimate of the plastic hinge length obtained by using the mean values of  $\alpha_1$  and  $\alpha_2$  and setting  $\varepsilon_L = 0$  in (2);  $\Phi_u$  is computed from (5) and involves the unknown parameters  $\gamma_1$  and  $\gamma_2$  as well as the monotonic curvature capacity  $\Phi'_u$ , which in turn involves the modified maximum useable concrete strain in (4) involving the unknown parameter  $\theta$ . As usual,  $\varepsilon_{\Delta f}$  is the random correction factor of the model, which is assumed to have the normal distribution with zero mean and unknown standard deviation  $\sigma_{\Delta f}$ . This correction term includes not only the error in the form of the global model (6), but also the errors inherent in the sub-models for  $L_p$ ,  $\Phi_y$ ,  $\Phi_u$ ,  $\Phi'_u$  and  $(\varepsilon_c^{max})_{mod}$ .

The model in (6) involves four unknown parameters:  $\theta$ ,  $\gamma_1$ ,  $\gamma_2$  and  $\sigma_{\Delta f}$ . These parameters are estimated using data for 8 structural wall models, which were tested cyclically under displacement control and failed in flexure. At each displacement level, the walls were subjected to either 2 or 3 cycles. The relevant references and essential parameters of the tested wall models can be found in Sasani and Der Kiureghian (2001). Note that the volume fraction of the confining reinforcement varies from  $\rho_{sh} = 0.22$  to 2.08, the percentage of total longitudinal reinforcement in the section varies from  $\rho_t = 0.48$  to 1.95, the aspect ratio, i.e., the height from the base to the point of zero bending moment divided by the length of the wall in the plan, varies from  $Z/L_w = 1.8$  to 3.1, and the axial compressive load divided by the gross section area times the compressive strength of concrete varies from 0.3 to 10.2. In other words, the data covers a wide range of these important variables.

The measured flexural displacement capacity of each wall is found by linearly interpolating between the displacement experienced by the wall in the cycle where the force-displacement relationship shows a drop of more than 20% in the lateral load capacity, and the displacement experienced in the previous stable cycle. The interpolation is carried out based on the number of stable cycles before the mentioned drop in lateral load capacity. For example, if only 1 out of 3 cycles at displacement level  $\Delta = 0.10$  m is stable and the previous cycle is at  $\Delta = 0.07$  m, then

the displacement capacity of the wall is  $\Delta_f = 0.07 + (0.10 - 0.07) \times (1/3) = 0.08$  m. For the analytical predictions, the stress-strain relation for the steel is obtained from tension results reported for each tested wall. As indicated earlier, the concrete stress-strain relation was based on the modified Kent and Park model (Park et al. 1982) with the compressive strength of concrete as measured in each test.

Bayesian analysis by use of the program BUMP revealed strong correlation between  $\gamma_1$  and  $\gamma_2$ , suggesting that these parameters are approximately linearly dependent. Using the posterior statistics, the linear estimate  $\gamma_2 = 0.030\gamma_1 - 0.014$  was obtained. Substituting this relation in (5), one obtains the simplified model

$$\Phi_{u} = \left[\gamma - (0.030\gamma - 0.014)\mu_{\Phi}\right]\Phi_{u}^{\prime}$$
(7)

where  $\gamma_1$  is replaced by  $\gamma$ . The number of unknown parameters inherent in the model in (6) is now reduced to 3, i.e.,  $\theta$ ,  $\gamma$  and  $\sigma_c$ . Repeating the Bayesian analysis with the reduced model, the posterior mean values of the parameter are  $\overline{\theta} = 0.796$ ,  $\overline{\gamma} = 0.659$  and  $\overline{\sigma}_{\Delta f} = 0.149$ . The standard deviations are 0.023, 0.106, and 0.052, respectively. All the correlation coefficients are negligible.

Figure 2 shows a comparison of the measured versus predicted top displacement capacities for the 8 walls tested. On the horizontal axis the measured displacement capacity is shown. The vertical axis shows the predicted displacement capacities. Solid circular dots indicate median (50% fractile) estimates, whereas the I-bars indicate the 15-85% fractile ranges. It is noted that almost all the I-bars cover the 1/1 line.

Also shown in Fig. 2, as solid square marks, are estimates of the displacement capacity of the 8 tested walls obtained by using the current provisions of the Uniform Building Code (1997). These estimates are found to be grossly on the unconservative side. The author believes the reason for this overestimation of the displacement capacity by the UBC code provisions



displacement capacities

is the fact that these provisions neglect the effect of compressive strain concentration in the compression zone of concrete and the effect of the cyclic load in reducing the curvature capacity of the wall section.

## 3 Shear Strength Capacity

The shear failure of structural walls may arise from any combination of sliding shear, web crushing and shear-compression failure of the compression zone (Fig. 3). In order to develop a shear strength capacity model for structural walls, sixteen structural walls tested under cyclic loads are studied (see Sasani et. al., 2002). Among the sixteen walls, nine failed in shear and the remaining seven had flexural failures.



(c) shear-compression failure (Oesterle et. al., 1976)

Preliminary studies with a shear strength model revealed weak correlation between the displacement ductility at the failure and the shear strength of the wall. Based on this observation, the following probabilistic model is considered for shear strength capacity,  $V_{cap}$ , of RC structural walls:

$$V_{cap} = e^{\varepsilon_{V_c}} \left[ \left( \nu_1 a_{asp} + \nu_2 P / A_g f'_c \right) \sqrt{\left( f'_c f_{sc} \right)} b l + V_s \right]$$

$$V_s = \rho_h f_{vh} b l \le \nu_3 \sqrt{\left( f'_c f_{sc} \right)} b l$$
(8)

In the above,  $v_1$ ,  $v_2$  and  $v_3$  are the model parameters and  $\varepsilon_{Vc}$  is a normally distributed model error with zero mean and unknown standard deviation  $\sigma_{Vc}$ .  $a_{asp}$  accounts for the aspect ratio of the wall and linearly varies from 1.5 to 1.0 as the aspect ratio increases from 1.5 to 2.5. For aspect ratios larger than 2.5,  $a_{asp}$  is set equal to 1.0.  $f_{sc}$ is a scaling stress equal to 1 MPa (or its equivalent in other units), which is employed to make the parameters of the model dimensionless. *P* is the axial compression on the wall and  $A_g$  is the gross section area, *b* is the width of the web, *l* is the total length of the section,  $f'_c$  is the compressive strength of concrete and  $f_{yh}$  is the yield stress of the horizontal reinforcement in the web. Finally,  $V_s$  is the shear strength corresponding to the horizontal reinforcement and has an upper bound in order to inhibit web crushing of the wall due to large amount of shear reinforcement.

The Bayesian parameter estimation technique provides an effective tool for the development of probabilistic models (Der Kiureghian 1999). In this paper, the Bayesian technique is employed to develop different probabilistic models required for seismic design and assessment of reinforced concrete (RC) structural walls at the life safety level.

#### 3.1 Bayesian model assessment

Details of the Bayesian technique can be found in the existing literature (Box and Tiao 1992, Der Kiureghian 1999). Here, only a brief outline is presented. Let

$$y = \hat{g}(\mathbf{x}, \boldsymbol{\theta}) + \varepsilon \tag{9}$$

be a mathematical model for predicting variable v in terms of a set of observable variables  $\mathbf{x} = (x_1, x_2, ...)$ , in which  $\hat{g}(\mathbf{x}, \boldsymbol{\theta})$  is an idealized model (signified by the superposed hat),  $\theta = (\theta_1, \theta_2, ...)$  is a set of unknown model parameters, and  $\varepsilon$  is a random variable representing the unknown error in the model. We will assume that  $\varepsilon$ has a normal distribution (normality assumption) and that it has a constant standard deviation  $\sigma$ . (homoskedasticity assumption). If, for a given model  $\hat{g}(\mathbf{x}, \boldsymbol{\theta})$ , these assumptions are not satisfied, then it is possible to make a transformation of the model such that these assumptions are at least approximately satisfied. Box and Cox (1964) suggest a parametric family of transformations for this purpose. In the experimental results utilized in this paper, it is expected that the error in the capacity model will increase linearly with the capacity. Furthermore, the capacity being nonnegative is well represented by a lognormal distribution. Therefore, a logarithmic is selected to approximately satisfy the transformation normality and homoskedasticity assumptions. Finally, with the aim of developing an unbiased model, we assign a zero value to the mean of  $\varepsilon$ .

The set of unknown parameters of the model, thus, are  $\Theta = (\theta, \sigma)$ . The model is "assessed" by estimating  $\Theta$  based on the available information, which typically consists of a set of measured values of **x** and the corresponding *y*, and possibly subjective information on the likely values of the parameters. In the Bayesian approach, this is done by the use of the well-known updating rule

$$f(\boldsymbol{\Theta}) = c \ L(\boldsymbol{\Theta}) \ p(\boldsymbol{\Theta}) \tag{10}$$

where  $p(\Theta)$  denotes the prior distribution on  $\Theta$  reflecting the subjective information,  $L(\Theta)$  is the likelihood function, which is a function proportional to the conditional probability of making the observations on **x** and *y* for a given value of the parameters and reflects the objective information gained from the data, *c* is a normalizing factor, and  $f(\Theta)$  is the posterior distribution reflecting our updated information about  $\Theta$ . This rule is used to construct capacity and demand models and estimates of the fragility for RC structural walls based on observed laboratory test data. Formulations of the prior distribution and the likelihood function for specific models are presented throughout the paper.

### 3.2 Probabilistic Model for Shear Strength Capacity

The experimental information available for predicting the shear strength capacity of the walls are of two kinds: Measured shear strength, when shear failure has been

observed, and measured lower bound to the shear strength when the wall has failed in flexure. These two types of information are reflected in the likelihood function. Let

$$\hat{\mathbf{V}}_{cap} = \left[ \left( \nu_1 \, \mathbf{a}_{asp} + \nu_2 \, \mathbf{P} / \mathbf{A}_g \mathbf{f}_c' \right) \sqrt{\left( \mathbf{f}_c' \mathbf{f}_{sc} \right)} \, \mathbf{b} \, \mathbf{l} + \mathbf{V}_s \right] \tag{11}$$

denote the predicted shear strength capacity excluding the error term. In the *k*-th experiment, given the set of observable variables  $(a_{asp}, P, A_g, b, l, f'_c, f_{yh}, V_s)_k$ ,  $(\hat{V}_{cap})_k$  is calculated from (11). Having the measured value of the maximum applied shear force on the section, the *k*-th realization of the error term is

$$\left(\varepsilon_{V_c}\right)_k = \ln\left(V_{cap}\right)_k - \ln\left(\hat{V}_{cap}\right)_k \tag{12}$$

Considering the normal distribution of the error term with a zero mean, and assuming statistical independence between the observations, the likelihood function takes the form

$$L(\nu_{1},\nu_{2},\nu_{3},\sigma_{Vc}) = \prod_{\text{Shear Failure}} \left[ \frac{1}{\sigma_{Vc}} \varphi \left( \frac{(\varepsilon_{Vc})_{k}}{\sigma_{Vc}} \right) \right] \times \prod_{\text{Flexural Failure}} \left[ 1 - \Phi \left( \frac{(\varepsilon_{Vc})_{k}}{\sigma_{Vc}} \right) \right]$$
(13)

where the first product is for all the walls that failed in shear and the second product is for all the walls that failed in flexure. In the above expression  $\varphi(\cdot)$  is the standard normal probability density function and  $\Phi(\cdot)$  is the standard normal cumulative distribution function.

Not having prior information on the parameters of the model, a non-informative prior distribution is used (Box and Tiao, 1992). This essentially implies locally uniform distributions for  $v_1$ ,  $v_2$ ,  $v_3$ , and  $\ln(\sigma_{Vc})$ . This prior distribution together with the likelihood function in (13) is used in the Bayesian updating formula to estimate the posterior statistics of the parameters. The computer program BUMP (Geyskens et al., 1993) is used for this purpose. The posterior mean values of the parameters based are  $\overline{v}_1 = 0.067$ ,  $\overline{v}_2 = 2.240$ ,  $\overline{v}_3 = 0.500$ , and  $\overline{\sigma}_{vc} = 0.051$ . The standard deviations are 0.013, 0.244, 0.010 and 0.002, respectively. The only considerable correlation coefficient is between  $v_1$  and  $v_2$ , which is equal to -0.64. The standard deviation of the model error is small (equivalent to a coefficient of variation of about 0.051 in the capacity), which is an indication of the accuracy of the model. Based on the comparison between the mean values of  $v_1$  and  $v_2$ , for a  $P/(A_g f'_c)$  value of only 0.06, the effect of the axial load on the shear strength capacity of the wall is twice that of the first term on the right hand side of (11). The importance of the axial load on the shear strength of the wall is also reflected in the significant correlation between the shear deformation and the term  $P/(A_{a}f_{c})$ . The large negative correlation between  $v_{1}$ and  $v_2$  implies that the two terms can be combined with little loss of accuracy. This simplification is not used in this study.

Figure 4 compares the measured and predicted median shear strength capacities for the tested walls. As can be seen, the data points for walls that failed in shear are closely lined up along the 1:1 line that represents equal values for the measured and predicted shear strengths. The data points for walls that did not fail in shear fall below the diagonal line, indicating that the predicted median shear strength capacities are larger than the maximum applied shear force.



The form of the shear deformation of a RC structural wall is different from the form of the flexural deformation over its height. Fig. 5 shows an idealized shear distortion pattern of a structural wall. Experimental data shows that a significant part of the inelastic shear deformation takes place at the base of the wall over a height almost equal to the total depth of the section, length of the wall in the plan,  $L_W$ , (Oesterle et. al., 1976 and Vallenas et al., 1979). Therefore, in this section a model is proposed for estimating the shear distortion of the wall over this length, denoted as  $Drift_S^{L_W} = \Delta_S^{L_W}/L_W$ .

The test results show that the shear yielding (i.e.



Fig. 4. Measured versus mean predicted shear strength



Fig. 5. Shear deformation of RC structural wall

significant drop in shear stiffness) coincides with flexural yielding, which is not necessarily accompanied by yielding of horizontal reinforcement (Oesterle et. al., 1976 and 1979). Therefore, inelastic flexural and shear deformations are coupled. In a truss analogy, under the lateral loads, the longitudinal reinforcement (mainly in the boundary element region) forms the tensile element of the assumed truss system. Therefore, the yielding of the flexural reinforcement implies the yielding of the tensile element of the assumed truss system. This is demonstrated in Fig. 6. Figure 6(a) shows the deformation of a truss model for a structural wall due only to the yielding of the bottom left vertical element. This deformation is decomposed to flexural (Fig. 6(b)) and shear (Fig. 6(c)) deformations.



Fig. 6. Effect of flexural yielding in shear deformation

Furthermore, after yielding of the flexural reinforcement, the cracks (flexural and shear cracks) widen and the stiffness of the shear-transferred mechanism through aggregate interlock drops. As explained by Oesterle et. al. (1976), this is accompanied by a reduction in the dowel stiffness of the tensile boundary element.

Test results show a significant correlation between the amount of axial load and the shear distortion. Oesterle et. al. (1984) suggest the following relationship between the shear drift,  $Drift_{s}^{L_{W}}$  and total drift,  $Drift_{t}^{L_{W}}$ , over the height  $L_{W}$ 

$$Drift_{s}^{L_{W}} = \left(0.76 - 2.6\frac{P}{A_{g}f_{c}'}\right) Drift_{t}^{L_{W}} \ge 0.52 Drift_{t}^{L_{W}}$$
(14)

Another parameter that may affect the shear deformation of structural walls is the level of shear force demand,  $V_d$ , on the wall. Therefore, the following model is used to estimate the shear distortion of structural walls in the regions close to the base of the walls:

$$Drift_{s}^{L_{W}} = e^{\varepsilon_{Ds}} \left( \lambda_{1} - \lambda_{2} \frac{P}{A_{g} f_{c}'} - \lambda_{3} \frac{V_{Cap}}{V_{d}} \right) Drift_{t}^{L_{W}}$$

$$Set \quad \frac{P}{A_{g} f_{c}'} = \lambda_{4} \quad if \quad \frac{P}{A_{g} f_{c}'} \ge \lambda_{4}$$

$$Set \quad \frac{V_{Cap}}{V_{d}} = \lambda_{5} \quad if \quad \frac{V_{Cap}}{V_{d}} \ge \lambda_{5}$$

$$(15)$$

where  $Drift_{f}^{L_{W}}$  is the flexural drift over the height  $L_{W}$ ,  $V_{Cap}$  is the shear strength capacity of the section as given in (8), excluding  $V_{Cap}$ .  $\lambda_{1}$  to  $\lambda_{5}$  are the parameters of the model and  $\varepsilon_{Ds}$  is a random variable representing the unknown error in the model having the normal distribution with zero mean and unknown standard deviation  $\sigma_{Ds}$ . Using the Bayesian parameter estimation technique, the posterior statistics of the model parameters are estimated and given in Table 1.

Parameter	Mean	Standard Deviation	Correlation Coefficient					
			$\lambda_I$	$\lambda_2$	$\lambda_3$	$\lambda_4$	$\lambda_5$	$\sigma_{Ds}$
$\lambda_I$	2.15	0.25	1					
$\lambda_2$	7.50	1.13	0.59	1				
$\lambda_3$	2.30	0.17	0.68	0.36	1			
$\lambda_4$	0.07	0.00	-0.03	0.03	0.06	1		
$\lambda_5$	0.50	0.02	0.02	0.05	0.01	0.08	1	
$\sigma_{Ds}$	0.15	0.02	0.01	0.04	-0.01	0.02	0.00	1

 Table 1. Posterior statistics of shear distortion model parameters

Figure 7 compares the measured and predicted mean shear distortion values for the tested walls. The data for two walls are not included because of lack of information on the measured shear distortion. As can be seen, except for one wall (the only barbellsection wall with low flexural and high confining reinforcement boundary element and under low axial load that failed in flexure), in which the measured shear distortion is considerably larger than the estimated value, the results of other walls fairly closely follow the 1:1 line.





## 5 Application

The capacity models presented in this paper can be utilized in the probabilistic seismic design and assessment of RC structural walls. The models can also be used in the estimation of seismic fragility of RC structural wall. The seismic fragility of a structural system is defined as the conditional probability of failure of for a given intensity of the ground motion. Proper measures of the ground motion intensity need to be selected, in order to find better correlation between the seismic ground motion intensity and the response of structures. For long period RC structural walls it is found that the elastic response spectrum is a reliable measure of the ground motion intensity (Sasani and Der Kiureghian, 2001). For short period RC structural walls, a new measure of ground motion intensity, called significant peak ground acceleration, is found to be well correlated with the response of structures under severe pulse-type ground motions (Sasani et. al., 2002). Having the probabilistic models for demands and capacities and the proper measures of ground motion intensity, the fragility of RC structural walls can be estimated.

## 6 Summary

Incorporating mechanics of the shear and flexural behavior of RC walls, using the Bayesian parameter estimating technique, and utilizing available experimental data, capacity models for flexural deformation, shear strength, and shear deformation of RC structural walls are developed. Significant errors observed in some available models in current seismic codes suggest a need for accounting for the model errors in probabilistic design of structures.

## 7 References

- Box, G. E. P., and Cox, D. R., (1964). "An analysis of transformation," *Journal of the Royal Statistical Society. Series B* (Methodological), **26(**2), 211-252.
- Box, G. E. P., and Tiao, G. C., (1992). *Bayesian inference in statistical analysis*. Addison-Wesley, Reading, MA.
- Corley, W. G. (1966). "Rotational capacity of reinforced beams," *Journal of the Structural Division, ASCE,* **92**(10), 121-146.
- Der Kiureghian, A. (1999). "A Bayesian framework for fragility assessment," Proc. 8th Int. Conf. On Applications of Statistics and Probability (ICASP) in Civil Engineering Reliability and Risk Analysis, Sydney, Australia, December 1999, R. E. Melchers and M.G. Stewart, Eds., 2, 1003-1010.
- Fajfar, P. (1992). "Equivalent ductility factors, taking into account low-cycle fatigue," *Earthquake Engineering and Structural Dynamics*, **21**, 837-848.
- Geyskens, P., Der Kiureghian, A., and Monteiro, P., (1993). BUMP: Bayesian updating of model parameters. Report UCB/SEMM-93/06, Structural

Engineering, Mechanics and Materials, Department of Civil Engineering, University of California, Berkeley, CA.

- Kaar, P. H., Fiorato, A. E., Carpenter, J. E., and Corley, W. G., (1976). "Confined concrete in compression zones of structural walls designed to resist lateral loads due to earthquakes," Proc. International Symposium on Earthquake Structural Engineering, St. Louis, MI, 1207-1218.
- Mattock, A. H. (1967). Discussion of "Rotational capacity of reinforced concrete beams," by W.G. Corley, *Journal of the Structural Division*, ASCE, **93**(2): 519-522.
- Oesterle, R. G., Fiorato, A. E., Johal, L. S., Carpenter, J. E., Russell, H. E. and Corley, W. G., (1976). Earthquake resistance structural walls tests of isolated walls, Construction Technology Laboratories, PCA, Skokie, IL, 315pp.
- Oesterle, R. G., Aristizabal, J. D., Shiu, K. N., and Corley, W.G., (1984). "Web Crushing of Reinforced Concrete Structural Walls," *ACI Journal*, **81**(2), 231-241.
- Park, R., Priestley, M. J. N. and Gill, W. D. (1982). "Ductility of square-confined concrete columns," *Journal of the Structural Division, ASCE*, **108**(4), 929-950.
- Park, Y. J. and A. H-S. Ang (1985). "Mechanistic seismic damage model for reinforced concrete," *Journal of Structural Engineering, ASCE*, **111**(4), 722-739.
- Sasani, M. (1998). "A two-level performance-based design of reinforced concrete structural walls," Proc. 6<sup>th</sup> US National Conference on Earthquake Engineering, Seattle, Washington, Paper No. 78.
- Sasani, M. and Anderson, D. L. (1996). "Displacement-based design versus forcebased design for structural walls," Proc. 11<sup>th</sup> World Conference on Earthquake Engineering, Mexico, Paper No. 32.
- Sasani, M. and Der Kiureghian, A., (2001). "Fragility of reinforced concrete structural walls: displacement approach," *Journal of Structural Engineering, ASCE*, **127**(2), 219-228.
- Sasani, M., Der Kiureghian, A., and Bertero, V. V. (2002). "Seismic fragility of short period reinforced concrete structural walls under near-source ground motions" *Structural Safety*, **24**(2-4), 123-138.
- UBC (1997). *Uniform Building Code*, Volume 2. International Conference of Building Officials, Whittier, CA.
- Vallenas, J. M., Bertero, V. V., and Popov, E. P. (1979). Hysteretic behavior of reinforced concrete structural walls. Report UCB/EERC-79/20, Earthquake Engineering Research Center, University of California, Berkeley, CA.