Draft:WTC2005-63862

A MODEL FOR ANALYZING MULTI-ASPERITY CONTACT OF THIN SHEETS WITH REAL SURFACES ON BOTH SIDES

John J. Jagodnik and Sinan Müftü
Department of Mechanical and Industrial Engineering
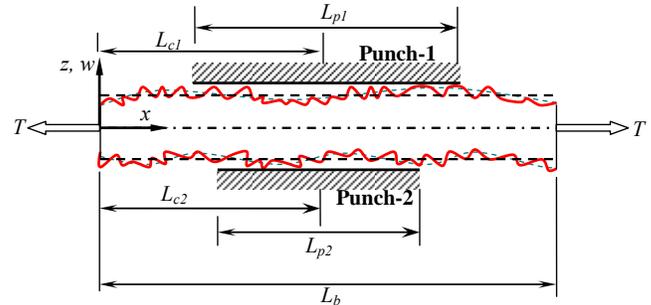Northeastern University
Boston, MA 02115, USA

ABSTRACT

A model for two-sided contact of a thin sheet of material, with real surfaces on both sides is presented. The model combines cylindrical-contact equations, with Euler-Bernoulli beam theory to examine the importance of substrate rigidity in two-sided contact problems. A finite difference program for solving this model is developed. Results for two-sided contact of numerically generated surfaces on thin tapes are presented. These results demonstrate that substrate rigidity is an important stress reducing phenomenon for typical two-side magnetic tape contact problems. The effects of tape thickness and tension are also explored. Here it is shown that substrate rigidty is an important phenomenon for all industrial tape thicknesses and tensions, however large thickness values exists for which substrate bending is negligible and elastic half-space solutions applied to both sides of the tape are adequate.

INTRODUCTION

Multi-asperity contact models can be categorized as *uncoupled* or completely *coupled*. For uncoupled multi-asperity contact models, such as Greenwood and Williamson's [1], the surface roughness is represented as a set of asperities, with statistically distributed parameters. The total effect is the sum of the actions of individual asperities. On the other hand, for the coupled contact models the equations of elasticity must be solved for the entire body, simultaneously.

Many authors have used elasticity methods to solve multi-asperity contact problems [2,3]. Webster and Sayles developed a 2D model where, two arbitrarily shaped surfaces are brought into contact, and the contact-region is broken up into small elements [4]. Pressure on each element is assumed constant. A Green's function is used to relate the pressure on each element to the overall displacements of the bodies, which are assumed to be a elastic half-spaces. The total force on the each surface is found until the two bodies are in equilibrium.

Recently, several authors have used fast Fourier transforms (FFT) to reduce the solution time [5]. Polonsky and Keer [6,7] proposed one of the most recent fast Fourier transform solution techniques. Peng and Bhushan [9,10] used quadratic programming, variational methods and FFT to solve multi-asperity layered contact problems.

Fig. 1 Schematic depiction of the model. Note that the simple-support boundary conditions are not shown.

Many authors have included the effects of substrate deformation in contact problems, however these solutions often carry restrictions. All of the above mentioned methods require an elastic half space approximation which is only a valid approximation for thick substrates.

In this work, two-sided contact of a thin sheet with two rigid punches (Fig. 1) is studied. The model includes the effects of substrate's bending rigidity [11]. Few limitations exist on the shapes, properties, and dimensions of the interfering bodies (punches) and the substrate surfaces. The substrate can be a single material or a composite structure; contact may be single sided or two-sided.

FORMULATION

Consider the beam shown in Fig. 1, subjected to tension T per unit width, and contact with the rigid punches on the top and bottom as shown. The beam is simply supported on both ends. On the top and bottom contact regions, pressure distributions p_{c1} and p_{c2}, will act on the beam, respectively. The beam deflection w can be obtained from:

$$EI\frac{d^4w}{dx^4} - T\frac{d^2w}{dx^2} = p_{c2} - p_{c1} \qquad (1)$$

where E is the elastic modulus and I is the moment of area of the beam [11]. The rough surfaces for the top and the bottom were numerically generated [11]. Contact at each peak was treated as a cylindrical contact [12]. Equation (1) was solved numerically using Newton's method, and a contact algorithm as described in [11].

RESULTS

Effects of beam thickness t and tension T on the overall equilibrium were investigated. The parameters used for the study are summarized in Table 1. In order to get statistically meaningful results thirty different surfaces were generated; and results were evaluated for each one separately. This work has shown that subsurface stresses and the total energy stored in the

system are greatly reduced when bending is considered, for typical two-sided contact of magnetic tapes.

Fig 2. shows tape deflection in the punch regions and the contact pressures, $t = 10$ μm, $T = 20$ N/m tension. The tape bends around the asperities and stores some strain energy. The total strain energy in the system is given by:

$$E_{total} = U_{bend} + U_{axial} + U_1^{asp} + U_2^{asp} \quad (2)$$

where U is the strain energy stored in the beam due to *bending* and *longitudinal* strains and U^{asp} is the strain energy stored in the asperities, on the top (1) and bottom surfaces (2). Fig. 3 shows the effect of tape thickness and tension on the total potential energy in the system.

It is shown that when the tape thickness exceeds 90 μm, the effects of bending are negligible and the tape may be solved using a static solver [11]. Thin tapes should be solved with the inclusion of substrate bending, as maximum subsurface stress can be reduced by as much as 92% when bending is included. Tension changes within a reasonable range are shown to have little effect on the bending solution.

REFERENCES

1. Greenwood, J, Williamson, J., 1966, "Contact of Nominally Flat Surfaces," *Proc Roy Soc.* A295, pp. 300-319.
2. Francis, H.A., 1982, "A Finite Surface Element Model for Plane-Strain Elastic Contact," *Wear*, 76, pp. 221-245.
3. Cole, S.J. and Sayles, R.S., 1992, "A Numerical Model for the Contact of Layered Elastic Bodies with Real Rough Surfaces," *ASME Journal of Tribology*, 114, pp. 334-340.
4. Webster, M.N. and Sayles, R.S., 1986, "A Numerical Model for the Elastic Frictionless Contact of Real Rough Surfaces," *ASME Journal of Tribology*, 108, pp. 314-320.
5. Hu, Y.-Z., Barber, G.C., and Zhu, D., 1999, "Numerical Analysis for the Elastic Contact of Real Rough Surfaces," *Tribology Transactions*, 42, 3, pp. 443-452.
6. Polonsky, I.A. and Keer, L.M., 2000, "Fast Method for Solving Rough Contact Problems: A Comparative Study," *ASME Journal of Tribology*, 122, pp. 36-41.
7. Polonsky, I.A. and Keer, L.M., 2000, "A Fast and Accurate Method for Numerical Analysis of Elastic Layered Contacts," *ASME Journal of Tribology*, 122, pp. 30-35.
8. Kral, E.R. and Komvopoulos, K., 1997, "Three-Dimensional Finite Element Analysis of Subsurface Stress and Strain Fields Due to Sliding Contact on an Elastic-Plastic Layered Medium," *J. Trib*, 119, pp. 332-341.
9. Peng, W. and Bhushan, B., 2001, "Three-Dimensional Contact Analysis of Layered Elastic/Plastic Solids with Rough Surfaces," *Wear*, 249, pp. 741-760.
10. Peng, W. and Bhushan, B., 2001, "A Numerical Three-Dimensional Model for the Contact of Layered Elastic/Plastic Solids with Rough Surfaces by a Variational Principle," *ASME Journal of Tribology*, 123, pp. 330-342.
11. Jagodnik, J.J., 2003, "A Model for Analyzing Multi-asperity Contact of Finite Thickness Structures with Real Surfaces on Both Sides," M.S. Thesis, Northeastern University, Boston, MA 02115.
12. Jagodnik, J.J., Müftü,, S, 2003, "A Cylindrical Contact Model for Two Dimensional Multiasperity Profiles," Proc. 2003 STLE/ASME Joint Int. Trib. Conf. Ponte Vedra Beach, Florida USA, October 26-29.
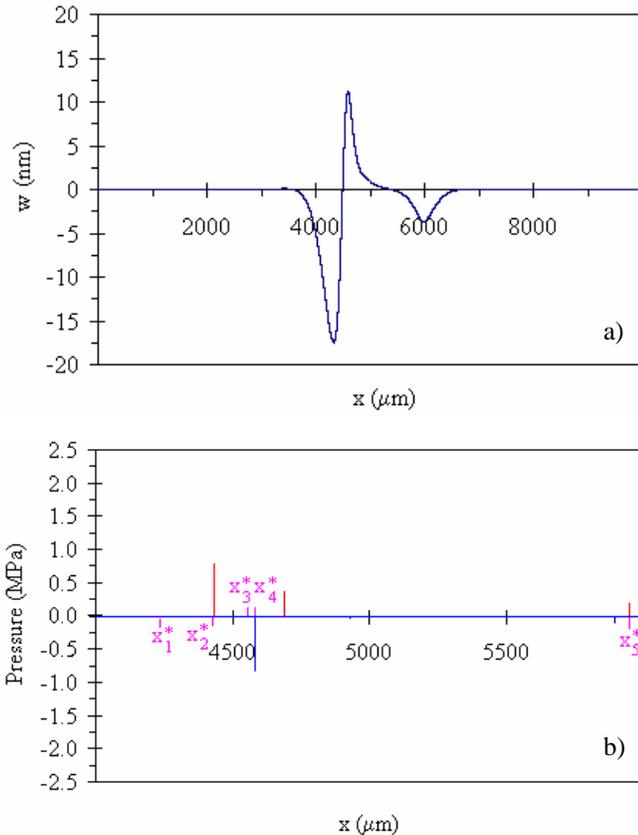
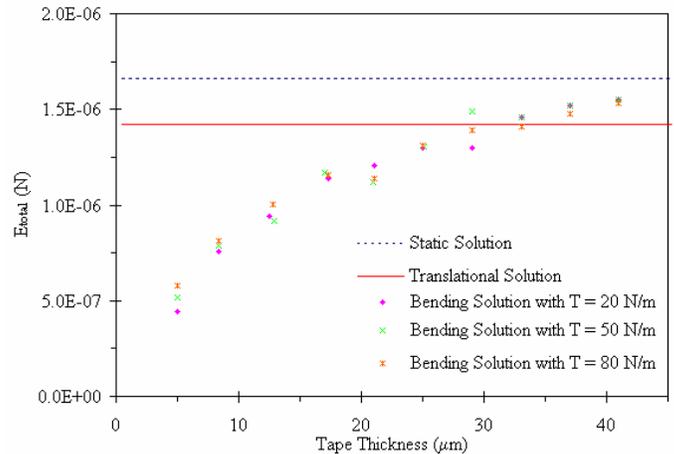**Fig. 2** Tape deflection under the punches and the contact pressure distribution.

Fig. 3 Total potential energy per unit width as a function of thickness and tension

Variable		Variable	
$L_{c1} = L_{c2}$	5 mm	E	5 GPa
$L_{1p} = L_{2p}$	2.02 mm	ν	0.3
L_b	10 mm	T	20, 50, 80 N/m
t	10 - 90 μm	$\delta_1 = \delta_2$	10 nm

Table 1. Parameters used in this work. Note that δ_1, δ_2 are the rigid body displacements of the punches toward each other.