NANO-SCALE EFFECTS IN THE ADHERENCE, SLIDING AND ROLLING OF A CYLINDER ON A SUBSTRATE

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Abstract

The behavior of a nano-scale cylindrical body (e.g. a fiber), lying on a substrate and acted upon by a combination of normal and tangential forces, is the subject of this investigation. As the scale decreases to the nano level, adhesion becomes an important issue in this contact problem. Thus this investigation treats the two-dimensional plane strain elastic deformation of both the cylinder and the substrate during a rolling/sliding motion, including the effect of adhesion using the Maugis model. Due to adhesion, the contact area increases and a finite force (i.e. pull-off force) is needed to separate the bodies. For the initiation of sliding, the Mindlin approach is used, whereas for rolling, the Carter approach is utilized. Each case is modified for nano-scale effects using the adhesion theory of friction for the friction stress. Results are given for the normal and tangential loading problems, including the pull-off force, the initiation of sliding and rolling in terms of dimensionless quantities representing adhesion, cylinder size, and applied forces.

Key words: Cylinder; Nano-scale; Adhesion; Contact; Maugis Model; Rolling; Sliding; Pull-off Force.

Introduction

At the macro-scale, adhesion between contacting bodies has a negligible effect on surface interactions. However, as the contact size decreases, adhesion between the bodies becomes significant, especially for clean surfaces and lightly loaded systems. As technology advances, the scale of some devices decrease and adhesion becomes an important issue, especially in such applications as information storage devices and microelectromechanical systems (MEMS).

In contacting bodies, atoms of materials are separated by an equilibrium distance z_0 of a few Angstroms. At that separation, van der Waals forces are dominant over electrostatic forces [1]. The derivative of the Lennard-Jones potential expresses the surface pressure in terms of the separation z between two parallel surfaces

$$p_a(z) = \frac{8w}{3z_o}\left[(\frac{z_0}{z})^9 - (\frac{z_0}{z})^3\right] \tag{1}$$

where w is the work of adhesion $w=g_1+g_2-g_{12}$. Here the surface energies of the contacting bodies are g_1 and g_2, and the interface energy of the two surfaces is g_{12}. The Lennard-Jones potential includes attractive forces, which act over a long-range and repulsive forces, which act over a short-range. If the separation is less than the equilibrium spacing, the repulsive (i.e., contact) forces are dominant, whereas if the separation exceeds the equilibrium spacing there will be a net attractive force (i.e., the adhesion force). The Lennard-Jones pressure vanishes at the equilibrium spacing.

Numerous studies have been conducted on the adherence of spherical bodies. Bradley used the attractive force between two molecules and, by integrating through the whole bodies, found the cohesive force between two rigid spheres of radii R_1 and R_2 [2]. The pull-off force F required to separate two rigid spherical bodies was found to be

$$F = 2p\,wR \tag{2}$$

where $R = R_1 R_2/(R_1 + R_2)$ is the equivalent radius of curvature.

Johnson, Kendall and Roberts (JKR) presented a theory on the adherence of deformable elastic bodies [3]. In the JKR approximation the adhesion outside the contact region is assumed to be zero. The balance between the elastic, surface and potential energies was used, and the resulting stresses at the edges were infinite. It was found that the contact area is larger than the contact area predicted by Hertz, and the pull-off force was found to be

$$F = (3/2)p\,wR\,. \tag{3}$$

In 1975, Derjaguin, Muller and Toporov (DMT) presented another theory on the adhesion of deformable elastic bodies [4]. In the DMT theory the adhesion force is taken into account outside the contact area, but the form of the stress distribution in the contact is assumed to be unaffected. The pull-off force was found to be the same as the Bradley relation. It is seen that for both the JKR and DMT theories, the pull-off force is independent of the elastic properties of the materials.

Although the JKR and DMT theories seem to be competitive, it was shown by Tabor that these two theories represent the extremes of a parameter m, which is the ratio of elastic deformation to the range of adhesive forces [5], i.e.

$$m = \left(\frac{Rw^2}{E^2 z_0^3} \right)^{1/3} \tag{4}$$

where E is the composite Young's modulus.

Thus large compliant bodies are in the JKR regime, whereas stiff and small solids correspond to the DMT regime. Greenwood constructed an adhesion map that covers these regimes [6]. Maugis presented a model for the transition between the JKR and DMT theories [7]. Similar to the Dugdale model of a crack, the Maugis model assumes a constant tensile surface stress s_0 in regions where the surfaces are separated by a distance less than h, where h is obtained from the work of adhesion through the relation $w = s_0 h$. When this work of adhesion is set equal to that for the Lennard-Jones potential, the maximum separation distance for adhesion h is found to be $0.97\ z_0$ where s_0 is taken as the theoretical strength.

In this paper the adhesion of contacting cylinders or equivalently a cylinder in contact with a half space at the nano-scale is considered. If the cylinder is subjected to a combined tangential and normal loading it may remain at rest, roll, slide or undergo a complex motion depending on the magnitudes and the application points of the loading. The elastic behavior of the cylindrical body and half-space with adhesion are investigated using the plane strain theory of elasticity.

This nano-scale analysis differs from the corresponding macro-scale problem in two important ways. First, due to the small scale of the contact area, adhesion becomes important. The Maugis model is used to approximate the adhesive stress outside the contact region. Second, in the macro scale Coulomb's friction, which states that frictional force is proportional to the normal load, is considered valid. Due to surface roughness in the contact of two nominally flat surfaces, the bodies touch at a discrete number of contacts, so the real contact area becomes much smaller than the apparent contact area. If a statistical distribution for the asperity peaks is assumed, it turns out that the real area of contact is approximately proportional to the normal force. This result is consistent with the adhesion theory of friction. According to the adhesion theory of friction, contacting asperities form strong adhesive junctions. In order to have a relative motion between the contacting bodies, these junctions need to be broken which occurs when the interfacial shear stress reaches the shear strength of the weaker material. Therefore a constant shear stress occurs in the real area of contact during the relative motion of contacting bodies. Thus constant shear stress along with a near proportionality between normal load and real contact area gives a friction force approximately proportional to the normal load, i.e. Coulomb friction. At the nano-scale, however, in which the contact radius is in the order of 10 nm or less, it seems more reasonable to assume a single real contact area, i.e. constant shear stress during sliding.

Adhesion of cylindrical bodies on a substrate is encountered in nano-wires, carbon nano-tubes and nano-fibers, and in different fields such as microbiology, microelectronics and MEMS/NEMS devices. Determination of the pull-off force and the forces necessary to roll or slide a cylindrical body on the substrate are important quantities to know in these applications. In some cases it is important to manipulate these single fibers to form a structure whereas in other instances the sliding and rolling motions are important in contamination removal processes.

Theory and Discussion of the Results

The contact of a cylindrical body of radius R with a flat surface is investigated. The results are equally valid for the contact of two cylinders by using the equivalent radius of curvature previously defined. Linear plane strain elasticity is used throughout, which implies that the forces are given per unit length. Normal loading, the initiation of sliding, steady sliding and steady rolling are investigated.

Normal Loading

Consider normal loading in which a normal load F is applied to a cylinder when tangential force T is zero. As shown in Fig. 1, there will be a central contact zone ($-a<|x|<a$) surrounded by two adhesion zones ($a<|x|<c$) in which the separated surfaces are under a constant tensile stress as described by the Maugis model [7]. The tensile adhesive stress is effective up to a separation h, beyond which it vanishes.

The geometric relation for the deformations in the normal direction ($u_y^{(1)}$ and $u_y^{(2)}$ for body "1" and "2" respectively) at the contact interface is

$$u_y^{(1)} - u_y^{(2)} = -d_0 + \frac{x^2}{2R} \ , \quad -a<x<a \tag{5}$$

in the contact zone, where d_0 is the maximum cylinder penetration which occurs at the center of the contact zone. According to plane strain linear elasticity (e.g. Barber [9]), the derivative of the surface normal displacements can be written in terms of normal and shear stresses as

$$\frac{d(u_y^{(1)} - u_y^{(2)})}{dx} = \frac{A}{4p}\int_{-c}^{c}\frac{p_y(x)dx}{x-x} - \frac{B}{4}p_x(x) = \frac{x}{R} \ , \quad -a<x<a \tag{6}$$

where p_x is the tangential traction in x direction and the contact pressure p_y is considered positive in compression. The material parameters A and B are given by

$$A = \frac{4(1-u_1)}{G_1} + \frac{4(1-u_2)}{G_2} = \frac{8}{E}, \qquad B = \frac{2-4u_1}{G_1} - \frac{2-4u_2}{G_2},$$

$$\frac{1}{E} = \frac{1-u_1^2}{E_1} + \frac{1-u_2^2}{E_2}, \qquad E_i = 2G_i\left(1+u_i\right) \tag{7}$$

where G_1, G_2 are the shear moduli, u_1, u_2 are the Poisson's ratios and E_1, E_2 are the moduli of elasticity of bodies “1” and “2” respectively.

Eqn. (6) can be simplified for either identical materials ($B=0$) or for the frictionless case ($p_x(x)=0$). Even if the materials are not identical, the effect of the constant B is usually small [8] and is often neglected. Thus normal/shear stresses do not produce tangential/normal relative displacements.

For mathematical ease we let

$$p_y(x) = \bar{p}_y(x) - s_0, \qquad -c \le x \le c \tag{8}$$

where $\bar{p}_y(x) = 0$ for $a < |x| < c$ resulting in

$$\frac{1}{Ep}\int_{-a}^{a} \frac{\bar{p}_y(x)}{x - x}dx = \frac{-x}{2R} + \frac{s_0}{Ep}\ln\left(\frac{c-x}{c+x}\right) \quad , \quad -a < x < a \tag{9}$$

The singular integration in Eqn. (9) is bounded at the end points. This equation can be converted to a system of linear equations using the numerical collocation technique of Erdogan, Gupta, and Cook [10], given in appendix A.

Since the contact half-width a and the adhesion half-width c are both unknown, two extra equations are needed. The geometrical relation for the separation of the two bodies at $x=c$ and $x=a$ can be used along with Eqn. (6), and leads to

$$\left(u_y^{(1)} - u_y^{(2)}\right)_a^c = \frac{2}{Ep}\int_a^c \int_{-c}^{c} \left(\frac{p_y(x)dx}{x-x}\right)dx = \frac{c^2 - a^2}{2R} - h \tag{10}$$

If the order of the integration is changed and Eqn. (8) is substituted into Eqn. (10), it takes the form

$$\frac{2}{Ep}\int_{-a}^{a} \bar{p}_y(x)\ln\left(\frac{c-x}{a-x}\right)dx = \frac{c^2 - a^2}{2R} - h - \frac{2s_0}{Ep}\int_a^c \ln\left(\frac{c-x}{c+x}\right)dx \tag{11}$$

The last integral in Eqn. (11) was evaluated analytically.

Using force equilibrium in the *y*-direction, the total applied load *F* can be related to the contact half-width (a), i.e.

$$\int_{-a}^{a} \bar{p}_y(x)dx - 2cs_0 = F \tag{12}$$

Equations (11) and (12) can be converted into two algebraic equations and along with Eqn. (9) solved using the Erdogan, Gupta and Cook relations.

If the integral equation (9) is solved for zero adhesion, and the force balance equation (Eqn. (12)) is used, two-dimensional Hertz contact solution is obtained and is given as [8]

$$F = \frac{a^2 p E}{4R} \tag{13}$$

The solution with adhesion included, shows that the pressure distribution is compressive in the central part of the contact zone but tensile at the ends of the contact zone. In Figs. 2 and 3 the dimensionless applied normal load (F/Eh) is given as a function of contact half-length (a/R) for different adhesion stresses (s_0/E). Fig. 2 corresponds to $R/h=100$ and Fig. 3 to $R/h=200$. Note that for small adhesion stress, the Hertz solution is approached. It can be seen that as the applied load increases, the contact width also increases monotonically. For larger values of s_0/E, the load reaches a local minimum, which corresponds to a snap-off from the surface, at a finite force and contact radius is observed. A negative load is always observed as the contact width approaches either zero, or a local minimum, which correspond to pull-off force. The value of this pull-off force increases with increasing adhesion stress. On the other hand, as the two bodies approach each other, they can be expected to attract each other because of the tensile adhesive stress, resulting in a snap-on. It is noted that the pull-off force increases slightly as the fiber radius doubles (Figs. 2 and 3). However this increase in surface force is much less than the corresponding increase in body force (i.e., the weight).

Figs. 4 and 5 show the variation of the dimensionless adhesion width (c/R) with the dimensionless contact width (a/R) for different adhesion stress values (s_0/E). As the applied load increases, the contact width and the adhesion width increase. It may seem counter-intuitive that for given a/R, stronger adhesion gives smaller c/R. However, recall that for stronger adhesion a smaller force is needed to obtain that value of a/R.

An algebraic relation can be found for the force corresponding to zero contact radius by combining Eqns. (11) and (12) and taking the limit as the contact zone shrinks to zero, giving

$$\frac{F}{Eh}=\frac{8\ln 2}{p}\left(\frac{R}{h}\right)\left(\frac{s_0}{E}\right)\left(\frac{s_0}{E}-\sqrt{\left(\frac{s_0}{E}\right)^2+\frac{1}{8}\left(\frac{p}{\ln 2}\right)^2\left(\frac{h}{R}\right)}\right) \tag{14}$$

Unless the adhesion stress (s_0/E) is sufficiently high, the value of F from Eqn. (14) corresponds to the pull-off force. Otherwise the pull-off force will be larger than that given by Eqn. (14) due to the snap-off as discussed previously. This equation shows that in the case of adhesional contact of a cylinder with a flat surface, the pull-off force strongly depends on the composite modulus of elasticity (E) in contrast to the JKR and DMT theories for spheres. Fig. 6 shows the variation of the "zero contact length force" (from Eqn. (14)) with adhesion stress for different cylinder radii (R/h).

Initiation of Sliding

Tangential forces can be transmitted by friction in contacting bodies. Consider a cylinder in contact with a half-space, which are compressed by a normal force F, and acted upon by a tangential force T. The problem is solved for the uncoupled case ($B=0$) in Eqn. (6). Thus the contact area remains in a state of stick during the application of the normal load and the contact area remains constant during the application of the tangential force. For the initiation of sliding, the tangential force is gradually increased.

Mindlin studied the initiation of macro-scale sliding of a cylinder using Coulomb friction without adhesion [13]. He first assumed that stick prevails everywhere during the tangential loading phase. But when that problem is solved, a singularity exists for the shear stress distribution at the edges $x=|a|$, whereas the normal stress is bounded. Hence the frictional inequality must be violated; so it was deduced that slip will occur at the edges of the contact zone. Thus, according to Mindlin's theory there is a stick zone in the central region and slip zones symmetrically located in both the leading and trailing edges.

In nano-scale contacts, adhesion between the two bodies will affect the contact width. This effect in cylindrical contacts is described in the previous section. In the macro-scale Coulomb friction is used, whereas at the nano-scale the friction stress in the slip zone is assumed to be constant, i.e. the adhesion theory of friction as discussed previously.

According to the plane strain linear elasticity formulations (e.g. Barber [9]), the relation between the relative deformations of the bodies in the tangential direction to the boundary stresses is:

$$\frac{d(u_x^{(1)}-u_x^{(2)})}{dx}=-\frac{A}{4p}\int_{-a}^{a}\frac{p_x(x)dx}{x-x}-\frac{B}{4}p_y(x)\ , \quad -a<x<a \tag{15}$$

where p_x, the shear stress distribution in the x-direction, is equal to the constant t_0 in the slip zone. For mathematical convenience we take $p_x=t_0-t$ in the entire contact region. Prior to the initiation of global sliding motion, the horizontal shift (i.e. $u_x^{(1)}$- $u_x^{(2)}$) of the bodies in the stick zone ($-d<x<d$) will be constant. Note that the relation $|d|<|a|$ holds for the limits of the stick zone. Then the equation for tangential deformations becomes

$$\frac{1}{p}\int_{-d}^{d}\frac{t(x)dx}{x-x}=\frac{t_0}{p}\ln\left(\frac{a-x}{a+x}\right), \quad -d<x<d \tag{16}$$

Note that the contact width $2a$ is found as described previously in the normal loading problem. The integral Eqn. (16) is singular and the solution is bounded at the end points. The numerical method of Erdogan, Gupta and Cook [10] is used to obtain a set of algebraic equations, which are solved numerically.

The force balance in the horizontal direction can be used to find the relation between the half-length of the stick zone (d) and the applied shear force (T). This procedure gives

$$2at_0-\int_{-d}^{d}t(x)dx=T \tag{17}$$

which can also be evaluated using the method in [10].

In Fig. 7 the relation between the non-dimensional tangential force (T/Ea) and the non-dimensional half-length of the stick zone (d/a) is shown for various values of constant shear stress in the slip zone. As it can be clearly seen from this figure, the slip zone vanishes in the limit as the applied tangential force approaches zero. As the tangential force reaches a critical value, a state of complete slip (i.e. global sliding) occurs.

Pure Sliding

If the contacting cylinder has a relative sliding motion with respect to the plane it is in contact with, there need not be symmetry due to the nonlinear nature of adhesion. The origin of the coordinate system will be chosen in the center of the contact region. An eccentricity e will indicate the value of x corresponding to the apex of the cylinder. The leading adhesion zone will be a strip ($a<x<c_2$) and the trailing adhesion zone will be another strip ($-c_1<x<-a$) as shown in Fig. 8.

The geometrical relation between the deformations of the bodies in the y-direction inside the contact is

$$u_y^{(1)} - u_y^{(2)} = -d_0 + \frac{(x-e)^2}{2R} \tag{18}$$

The same elasticity formulation used for the normal contact can be used here i.e.,

$$\frac{d(u_y^1 - u_y^2)}{dx} = \frac{A}{4p}\int_{-c_1}^{c_2} \frac{p_y(x)dx}{x-x} - \frac{B}{4}p_x(x) = \frac{x-e}{R} \quad , \quad -a<x<a \tag{19}$$

If the stress distribution given by Eqn. (8) is substituted into Eqn. (19), it takes the form

$$\frac{1}{Ep}\int_{-a}^{a} \frac{\overline{p}_y(x)}{x-x}dx = \frac{-x}{2R} + \frac{e}{2R} + \frac{s_0}{Ep}\ln\left(\frac{c_2-x}{c_1+x}\right) \quad , \quad -a< x <a \tag{20}$$

The geometrical relations for the separation of the two bodies can be used which is in the same form with Eqn. (10), but with different integration limits due to non-symmetry.

When the cylinder is in a state of steady sliding, the adhesion effect in the trailing edge is assumed to be larger than in the leading edge. This assumption is considered valid because adhesion between surfaces is affected by surface contamination. Since the surface will be partially cleaned due to the sliding motion of the contacting surfaces, adhesion in the trailing edge is expected to be larger than in the leading edge. This effect is accounted for by taking the separation in the trailing edge (h_1) larger than in the leading edge (h_2), whereas σ_0 is assumed unchanged. Due to the difference between h_1 and h_2 the adhesion width values will not be the same in the leading and the trailing edges. Since there are two more unknowns, the separation equations must be written for both the leading edge and the trailing edge. On the leading edge this gives

$$(u_y^{(1)} - u_y^{(2)})_a^{c_2} = \frac{2}{Ep}\int_{-a}^{a} \bar{p}_y(x)\ln\left(\frac{c_2 - x}{a - x}\right)dx = \frac{(c_2 - e)^2 - (a - e)^2}{2R} - h_2 - \frac{2s_0}{Ep}\int_{a}^{c_2}\ln\left(\frac{c_2 - x}{c_1 + x}\right) \tag{21}$$

and the trailing edge relation is

$$(u_y^{(1)} - u_y^{(2)})_{-a}^{-c_1} = \frac{2}{Ep}\int_{-a}^{a} \bar{p}_y(x)\ln\left(\frac{c_1 + x}{a + x}\right)dx = \frac{(-c_1 - e)^2 - (-a - e)^2}{2R} - h_1 - \frac{2s_0}{Ep}\int_{-a}^{-c_1}\ln\left(\frac{c_2 - x}{c_1 + x}\right) \tag{22}$$

The Erdogan, Gupta and Cook relations [10] are used and Eqns. (20-22) are solved numerically.

In Figs. 9 and 10 the variation of the difference of the adhesion widths between the trailing and leading edges as a function of a/R are given for maximum adhesion separation ratios (h_1/h_2) for given cylinder radius ($R/h_2=100$) and adhesion stress ($\sigma_0/E=0.01$ in Fig. 9 and $s_0/E=0.04$ in Fig. 10). As the h_1/h_2 ratio increases, the difference between the adhesion widths of the leading and the trailing edges also increases due to asymmetric loading. For $h_1 = h_2$ the results are symmetric.

The applied normal force can be found from the force equilibrium in the y-direction. Due to the asymmetry with $h_1/h_2 \neq 1$, a resultant moment will act on the upper body due to the asymmetric normal stress distribution. If moment equilibrium is written with respect to the center of contact, the resultant moment (clockwise direction taken to be positive) is found as shown in Figs. 11 and 12. As the maximum adhesion separation ratio increases or as the adhesion stress increases, the resultant moment also increases as shown clearly in these figures.

Rolling

The problem of steady state rolling of an elastic cylinder on an elastic half-space (or equivalently one cylinder rolling on another) with Coulomb friction was solved by Carter [12]. According to Carter's solution the leading edge of the contact zone ($d<x<a$) is in a state of stick and the trailing edge ($-a<x<d$) is in a state of slip. In Carter's approach, $x = x' - Vt$ is a coordinate moving to the right with the speed of the contact region, and x' represents the stationary coordinate system. As in the application to a locomotive wheel [12], the upper body is the driving cylinder. During this rolling motion, the linear velocity is not exactly equal to the wR, where w is the angular velocity. Similarly, for two rollers, the angular velocities of the rollers will not be inversely proportional to their radii. Due to slip, the driving roller velocity will have a greater magnitude than the velocity of the driven roller. The creep velocity represents this velocity difference.

At the nano-scale the shear stress (friction stress) in the slip zone is assumed to be constant, as previously discussed. As with sliding, adhesion affects the relation between the normal force and contact width.

The tangential relative displacement (shift) between the bodies can be expressed as

$$s(x,t) = u_x^{(2)}(x,0,t) - u_x^{(1)}(x,0,t) + C(t) \tag{23}$$

where C represents the rigid body motion of the upper body relative to the lower body. In the stick zone the time derivative of the shift in the moving coordinate system is zero. The stick condition can be written as

$$\dot{s}(x,t) = -V\frac{d}{dx}(u_{x2} - u_{x1}) + \dot{C} = 0 \tag{24}$$

where $\dot{C}(t)$ is the constant rigid body slip (or creep) velocity.

From linear elasticity, the relation for the deformations of the bodies in the tangential direction can be used (Eqn. (15)). The shear distribution is now written as

$$t(x) = -t_0\left(1 - \frac{x-d}{a-d}\right) + \bar{t}(x)\ , \quad d<x<a \tag{25}$$

in the stick zone. It has a value of $-t_0$ at $x=d$, a value of zero at $x=a$. Furthermore

$$t(x) = -t_0\ , \qquad -a<x<d$$

in the slip zone, and so the unknown function $\bar{t}(x)$ is zero at the end points of the stick zone.

In the stick zone the time derivative of the shift is zero. If Eqn. (15) is substituted into Eqn. (24) for the uncoupled case ($B=0$)

$$-V\left(-\frac{A}{4p}\int_{-a}^{a}\frac{t(x)}{x-x}dx\right)+\dot{C}=0\,,\qquad d<x<a \tag{26}$$

is obtained, which becomes

$$\frac{VA}{4p}\int_{d}^{a}\frac{\bar{t}(x)}{x-x}dx+\dot{C}=\frac{VA}{4p}t_0\left(\left(\frac{a-x}{a-d}\right)\ln\left(a-x\right)-\left(\frac{d-x}{a-d}\right)\ln\left(x-d\right)-1-\ln\left(a+x\right)\right)\quad,\ d<x<a \tag{27}$$

The integral equation is again singular and the solution is bounded at the end points.

If force equilibrium is written in the x-direction, the applied shear force can be related to the contact width parameter (d) by

$$T=-\int_{-a}^{a}t(x)dx \tag{28}$$

which along with Eqn. (25) becomes

$$T=\int_{d}^{a}\bar{t}(x)dx+\frac{3a+d}{2}t_0 \tag{29}$$

Eqns. (27) and (29) are solved numerically using the Erdogan, Gupta and Cook relations [10]. The problem is not symmetric so the middle equation cannot be neglected. Instead it provides the extra equation needed for the additional unknown, i.e. the creep velocity.

Fig. 13 shows the variation of the dimensionless applied shear force (T/Ea) and Fig. 14 is the variation of the dimensionless creep velocity, both with respect to the slip zone length parameter (d/a). As the applied shear force approaches $2a\tau_0$, the length of the stick zone approaches zero. At that point there is complete sliding and the creep velocity attains its maximum magnitude. As the applied tangential force approaches zero there is full stick zone and the creep velocity becomes zero. The meaning of a negative creep velocity is that wR for the driving wheel is greater than the velocity of the contact zone.

Also as the value of the constant shear stress in the slip region increases, the magnitude of the applied shear force also increases as expected. The constant shear stress has the same effect on the magnitude of the creep velocity as can be clearly seen from the figure. Note that as the stick zone vanishes (d? a) the creep velocity approaches a small fraction of the sliding velocity (V). Thus this state corresponds to complete sliding along with a rigid body rotation.

Conclusion

As the scale of contacting bodies decreases, adhesion effects become significant especially for smooth surfaces and lightly loaded systems. The behavior of a cylinder (e.g., a fiber) which adheres to a substrate and which is subjected to a combination of normal and tangential forces is the subject of this investigation. In the presence of a tangential load, the cylinder might slide, roll or may undergo a complex motion. This paper treats the two-dimensional elastic contact problem of the cylinder on the substrate during a rolling/sliding motion and includes the effect of adhesion using the Maugis model. In contrast to the spherical case, the pull-off force for a cylinder on a flat surface is shown to depend strongly on the combined modulus of elasticity of the contacting bodies.

Mindlin's classic investigation of the initiation of sliding of a cylinder on a half-plane used Coulomb friction. As the scale decreases, two effects should be included. First, adhesion increases the contact width, especially under light normal loading. Second, according to the adhesion theory of friction for a single real area of contact, the shear stress can be assumed to be constant in the slip regions. These effects are included for the uncoupled case in which normal/shear loading does not produce relative tangential/normal displacements. During the initiation of sliding, there is a central stick zone surrounded by slip regions in the leading and trailing edges. As the applied tangential load increases, the lengths of the slip zones increase, until, at a certain value of the tangential force, there is complete slip.

During steady sliding the abrasive action of the shear stress can be expected to partially clean the surface, resulting in different leading and trailing edge adhesive properties. This effect is included in the model of steady nano-scale sliding.

Carter investigated the rolling of a cylinder on a substrate at the macro-scale. Similarly to the sliding case, as the dimensions approach the nano-scale, adhesion under normal loading and constant shear stress in the slip regions under tangential loading are assumed. The variations of the creep velocity and the length of the stick zone with the applied shear force are determined for various values of the friction stress. As the traction force increases, the stick zone length decreases and the creep velocity increases eventually leading to pure slip with rotation.

Future work should include the analysis of the transition from sliding to rolling and applications to technological problems. It should also address the analysis of the three-dimensional problem (spherical case) for sliding and rolling motions.

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Appendix A

The Erdogan, Gupta and Cook relations [10] can be used to obtain a solution of a singular integral equation. Consider

$$\frac{1}{p}\int_{-1}^{1}\frac{P(t)}{t-x}dt = f(x) \tag{A1}$$

in which a solution bounded at both ends, is to be found. The solution is

$$P(x) = w(x)q(x), \qquad w(x) = \sqrt{1-x^2} \tag{A2}$$

where *g(?)* is a smooth function which is bounded and non-zero at the end points. The singular integral equation (A1) can be approximated by

$$\sum_{i=1}^{n}\frac{1-t_i^2}{n+1}g(t_i)\frac{1}{t_i - x_k} = f(x_k) \tag{A3}$$

where

$$t_i = \cos\left(\frac{ip}{n+1}\right), \quad i=1,2,..,n \tag{A4}$$

are the quadrature points, and

$$x_k = \cos\left(\frac{p}{2}\frac{2k-1}{n+1}\right), \quad k=1,2,..,n+1 \tag{A5}$$

are the collocation points. In this manner the singular integral equation (A1) is replaced with $n+1$ linear algebraic equations. For the symmetric problems, the middle equation (corresponding to $x_k=0$ for "n" even) is ignored [10].