ADHESION OF A RECTANGULAR PLATE IN THE PRESENCE OF INTERSURFACE FORCE

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Abstract

A rectangular freestanding plate clamped at opposite ends adheres to the planar surface of a rectangular punch. Only bending exists in this plate and membrane stress is ignored. A tensile load applied to the punch causes the plate to deform and gradually delaminate from the substrate. At equilibrium, the applied load is balanced by the disjoining pressure at the plate-punch interface with range, $y$, and magnitude, $p$. Applying the Dugdale-Barenblatt-Maugis cohesive zone approximation, the disjoining pressure is taken to be uniform and confined to a finite cohesive length at the contact edge. For a fixed adhesion energy, $\gamma = p y$, we investigate the following: (i) the Derjaguin-Muller-Toporov (DMT) limit where $y \to \infty$ and $p \to 0$, (ii) the Johnson-Kendall-Roberts (JKR) limit where $y \to 0$ and $p \to \infty$, and (iii) the general case for intermediate but finite $y$ and $p$. Delamination continues until the contact area shrinks to a line prior to “pinch-off”.
1. Introduction

As the increasing focus on the MEMS/NEMS, the adhesion has an important impact on the application of them [1, 2]. Because the adhesion force will dominant when the dimension of the devise is very small. For example, when the MEMS devise has beam-like or plate-like components, they will contact and stick to each other or to other parts under the adhesion force. This adhesion force comes from many factors, such as the distance change between the two parts, the temperature, the humidity, etc. In some case, this extra contact or sticky phenomenon will affect the work of the devise. In a typical capacitor, the contact of the two plates will destroy the capacitor. On the other hand, it can be utilized. In a typical MEMS-RF switch, a mechanically bridge is suspended at two opposite ends and pulled by an electrostatic voltage. When the voltage exceeds the pull-in threshold, the bridges will contact with the pad and either an “on” or “off” signal will be obtained [3].

Since Hertz [4] founded the classical contact theory to account for contact between two spheres, many revised theories followed. Derjaguin et al [5] incorporated long-range intersurface forces into solid-solid adhesion interfaces and formulated the Derjaguin-Muller-Toporov (DMT) model. Johnson et al [6] further considered the local elastic deformation of the adhering solids in the presence of short-range (or zero range) interactions and perfected the adhesion theory of Johnson-Kendall-Roberts (JKR). Maugis [7] later adopted the Dugdule-Barenblatt cohesive zone approximation to model finite range and magnitude of intersurface forces, and therefore derived the transition from JKR to DMT limits.

Wan constructed adhesion models for the limiting case of zero-range intersurface forces (JKR-limit) and applied the model to (a) 1-D rectangular membrane clamped at the opposite ends and (b) 2-D axisymmetric membrane clamped at the circular periphery, where sample membranes are allowed to deformed by mixed bending-stretching with or without residual stress [8-10]. The mechanical response of the pure bending plate is obtained. However, no intersurface force is considered. Recently, Wan [11] established the model for a circular membrane clamped at the periphery and relaxed the JKR-DMT constraint by allowing finite magnitude and range of intersurface forces. DMT limit, JKR limit and the transition were discussed. We [12] studied a rectangular membrane clamped at the opposite two ends under the finite magnitude and range of intersurface force.
However, these 1D and 2D model only consider the membrane stretching and no bending effect is included. Plate like film referring to small deformation and pure stretching membrane having large deformation behave differently. These two limits cannot appear separately and they are always mixed. So the purpose of this project is to investigate the adhesion of the plates under pure bending in the presence of intersurface force, including DMT-limit, JKR-limit and DMT-JKR transition. We will discuss the measurement of adhesion response, like the applied external load, the distance between the plates or beams to the substrate or the contact area. The adhesion delamination mechanics will also will be discussed and the measurement of the adhesion potential will be obtained. As we only consider the bending effect, no membrane stress is included. Based on this assumption, we are not interested in the material property. So we will normalize all the variables in our calculation. The conclusion is valid for any material, as long as it has only bending effect.

2. Geometrical description

Figure 1 shows a rectangular plate of unit width being clamped at the opposite ends and adhered to the planar surface of a flat punch with the same length and width. Upon an external tensile load applied to the punch, delamination occurs reducing the contact area. Figure 2 shows a schematic of half the plate profile, $w(x)$, from the centerline to the clamped edge, $0 \leq x \leq 1$. All variables and their dimensionless equivalences are defined and listed in Table 1. All horizontal dimensions are scaled by the plate half length, $a$, while vertical dimensions by the plate thickness, $h$. The long-range intersurface force or disjoining pressure, $p$, acting across the plate-substrate gap gives rise to a cohesive zone immediately behind the delamination front ($x = c$). The cohesive edge ($x = b$) divides the freestanding plate into an inner cohesive zone where $p$ acts ($c \leq x \leq b$) and a traction free outer region ($b \leq x \leq 1$). A few basic assumptions are taken to construct the new adhesion model: (i) the plate is deformed by plate-bending only with negligible membrane-stretching (i.e. zero membrane stress), (ii) any residual stress and sliding in the contact region are ignored [13].

3. Delamination mechanics of a rectangular plate

3.1 Governing equation
\[
\begin{cases}
D \nabla^4 w = p & \text{for } c < x \leq b \\
D \nabla^4 w = (F/2) \delta(x) & \text{for } b < x \leq a
\end{cases}
\] (1)

where \( \nabla^4 = d^4/dx^4 \) is the Laplacian operator and \( \delta(x) \) is the Dirac’s delta function denoting the applied load at the centerline \( (x = 0) \), \( F \) is the applied force, \( D \) is the flexible rigidity of the plate, \( w \) is the displacement of the plate and \( p \) is the disjoining pressure. Equation (1) can be rewritten in normalized form:

\[
\begin{cases}
\frac{d^4w}{dx^4} = p & \text{for } c < x \leq b \\
\frac{d^4w}{dx^4} = 0 & \text{for } b < x \leq 1
\end{cases}
\] (2)

which can be solved exactly to yield

\[
w = \begin{cases}
\frac{F}{24(1-c)}(1-x)^2((1-c)(1-3c+2x)-(b-c)^2) & \text{for } c < x \leq b \\
\frac{p}{24(1-c)}((2b-2b^3+b^4-2b-8bc+6b^2c-b^4c+8c^2-4c^3) \\
+ (12bc-12b^2c+4b^3c-12c^2+8c^3)x \\
+ (-6b+6b^2-2b^3+6c-4c^3)x^2+(-4c+4c^2)x^3+(1-c)x^4) & \text{for } b < x \leq 1
\end{cases}
\] (3)

The punch displacement is given by:

\[
w_0 = w(x = c) = \frac{p}{24}(b^4 + b(2 - 6c) + 6b^2c - 2b^3(1 + c) + c(-2 + 6c - 4c^2 + c^3))
\] (4)

### 3.2 Energy balance

Total energy of the film-substrate system is given by \( U_T = U_E - U_S \) with \( U_E \) the elastic energy due to plate bending and \( U_S \) the surface energy to create new surfaces. As the surface energy is defined to be positive here, the sign before \( U_S \) is minus, which is different from the common definition. For delamination to occur, \( (\partial U_T / \partial c)_{w_0=\text{constant}} = 0 \). According to the theory of plate and
shells, the energy density of a deformed plate is given by \( \int \frac{1}{2} \left( \frac{d^2w}{dx^2} \right)^2 dx \) and \( U_E = \text{energy density} \times \text{area} \). Substituting \( w \) in equations (3) into (5), the elastic energy is therefore given by

\[
U_E = \begin{cases} 
2 \int_0^b \frac{1}{2} \left( \frac{d^2w}{dx^2} \right)^2 dx & \text{for } 0 < x \leq b \\
2 \int_c^b \frac{c}{2} \left( \frac{d^2w}{dx^2} \right)^2 dx & \text{for } b < x \leq c 
\end{cases}
\]

(5)

The surface energy per unit area decreases in the cohesive zone, and reaches zero at the cohesive edge. Therefore,

\[
U_S = 2c.\gamma + 2 \int_c^b p \cdot [y - (w_0 - w)] dx
\]

(6)

Substituting equation (3) and (4) into (6), surface energy of the plate is obtained.

4. JKR-limit

In the JKR-limit, \( y \rightarrow 0, p \rightarrow \infty \) and \( b \rightarrow c \). The disjoining pressure turns into a Dirac’s delta function at the contact edge, the cohesive zone width \( (b - c) \) reduces to zero, and the traction free freestanding plate is given by

\[
w = w_0 \frac{(1-x)^2}{(1-c)^4} ((1-c)(1-3c+2x)-(b-c)^2) \quad w_0 = \frac{F}{24}(1-c)^3
\]

(7)

The mechanical response \textit{without} delamination is found to be

\[
F = \frac{24 w_0}{(1-c)^3}
\]

(8)

The energy terms reduce to
\[ U_E = \frac{12 w_0^2}{(1-c)^3} \quad \text{and} \quad U_S = 2c \gamma \] (9)

An energy balance yields the delamination trajectory,

\[ w_0 = \frac{(1-c)^2}{3 \times 2^{1/2}} \gamma^{1/2} \quad \text{and} \quad F = \left( \frac{32}{3 \times 2^{1/2}} \right)^{1/2} \gamma^{3/4} w_0^{-1/2} \] (10)

This result is consistent with Wan’s calculation [8]. The mechanical response of the JKR limit is shown in Figure 3. We can see that the applied force will decrease as the punch moves from the membrane. However, the applied force cannot go to infinite at the beginning as shown in Figure 3, it always begins with some finite value and then keeps decreasing.

5. DMT-limit

In the DMT-limit, \( y \to \infty \), \( p \to 0 \) and \( b \to 1 \). Here we consider a pseudo-DMT-limit (abbreviated as DMT-limit hereafter) where \( b = 1 \), but \( p \) does not necessarily vanish and \( y \geq w_0 \). The cohesive zone is underdeveloped, and the disjoining pressure is present in the entire non-contact freestanding region. The plate profile is found from equation (3) to be

\[ w = w_0 \frac{(1-x)^2(1-2c+x)^2}{(1-c)^4} \quad \text{with} \quad w_0 = \frac{p}{24} (1-c)^4 \] (11)

The constitutive relation without delamination is therefore

\[ F = 2 \times 24^{1/4} p^{3/4} w_0^{1/4} \] (12)

The energy terms become

\[ U_E = \frac{1}{45} (1-c)^5 p^2 \quad \text{and} \quad U_S = 2 \gamma - \frac{14}{15} p w_0 (1-c) \] (13)

Energy balance requires
This result is the same as Wan’s calculation [3]. The curve of \( P = 2 \) shown in Figure 3 represents a typical DMT limit. The whole plate is under the disjoining pressure and the applied force keeps increasing as the punch moves away from the late until delamination occurs. \( P = 4.89898 \) is the maximum disjoining pressure for DMT limit. This means when the punch delaminates from the plate, the punch displacement just equals to the cohesive zone range. Once the disjoining pressure is beyond this value, the whole plate is not under the disjoining pressure before delamination and some part of the plate is in the free traction region. This will be discussed in the next part.

6. DMT-JKR transition

For disjoining pressure with intermediate range and magnitude like \( p = 10, 50, 200 \) shown in Figure 3, it is called general case and DMT-JKR transition is expected. Before the applied force reach the maximum point, the applied force increase with the displacement. Actually, at the maximum point, the punch displacement \( w_0 \) just equals to the cohesive zone range \( y \). This means before the maximum point, the whole plate is under disjoining pressure and this is the DMT limit. After that point, some part of the plate still stays in the cohesive zone, while some part goes into free traction region. The applied force decreases as the punch moves away from the plate. At some point, the punch delaminates from the plate. Obviously, this kind of configuration can only be tested in displacement controlled experiment.

7. Conclusion

The adhesion-delamination mechanics of a rectangular plate is obtained by linear elasticity and thermodynamic energy balance. Pinch-off with a line contact and the associated critical force and central displacement are determined. It shows that when the disjoining pressure is very small, DMT limit is valid. As the disjoining pressure has greater value, it approaches JKR limit. For a disjoining pressure with intermediate range and magnitude, DMT limit is valid before the punch goes beyond the cohesive zone range. The theoretical framework allows one to experimentally gauge the materials properties of thin film and interfaces and to track the delamination process.
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<th>Normalized Parameters</th>
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<td><strong>Geometrical parameters</strong></td>
<td>$x = \text{horizontal distance (m)}$</td>
<td>$x = \frac{x}{a}$</td>
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<tr>
<td></td>
<td>$w = \text{deformation profile (m)}$</td>
<td>$w = \frac{w}{h}$, $w_0 = \frac{w_0}{h}$</td>
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<td></td>
<td>$w_0 = \text{vertical displacement of punch (m)}$</td>
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<td></td>
<td>$a = \text{half width of sample membrane (m)}$</td>
<td>$c = \frac{c}{a}$, $b = \frac{b}{a}$</td>
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<td></td>
<td>$c = \text{half width of contact area (m)}$</td>
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<td></td>
<td>$b = \text{half width of cohesive edge (m)}$</td>
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<td></td>
<td>$h = \text{membrane thickness (m)}$</td>
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<tr>
<td><strong>Material Parameters</strong></td>
<td>$\nu = \text{Poisson’s ratio}$</td>
<td>$\gamma = \gamma \left[ \frac{12(1-\nu)a^4}{Eh^5} \right]$</td>
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<td>$E = \text{elastic modulus (N.m}^{-2})$</td>
<td>$p = p \left[ \frac{12(1-\nu)a^4}{Eh^4} \right]$</td>
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<tr>
<td></td>
<td>$\gamma = \text{interfacial adhesion energy (J. m}^{-2})$</td>
<td>$y = \frac{y}{h}$</td>
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<tr>
<td></td>
<td>$p = \text{disjoining pressure (N.m}^{-2})$</td>
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<td></td>
<td>$y = \text{surface force range (m)}$</td>
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<tr>
<td><strong>Mechanical loading</strong></td>
<td>$F = \text{applied external force (N)}$</td>
<td>$F = F \left[ \frac{12(1-\nu)a^3}{Eh^4} \right]$</td>
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<td></td>
<td>$U = \text{energy terms (J)}$</td>
<td>$U = U \left[ \frac{12(1-\nu)a^3}{Eh^4} \right]$</td>
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Table 1 Normalized coordinates and variables.
References


Fig. 1 Schematic of the clamped rectangular plate

Fig. 2 Plate in the normalized coordinates
Fig. 3 Mechanical response for a range of disjoining pressure